Mon. Not. R. Astron. Soc. **280**, 1035–1045 (1996)

Double-degenerate semidetached binaries with helium secondaries: cataclysmic variables, supersoft X-ray sources, supernovae and accretion-induced collapses

A. Tutukov★ and L. Yungelson★

Institute of Astronomy of the Russian Academy of Sciences, 48 Pyatnitskaya Str., 109017 Moscow, Russia

Accepted 1995 December 18. Received 1995 December 11; in original form 1995 February 13

ABSTRACT

We model the galactic population of ultrashort-period binaries with helium white dwarf (WD) donors and carbon–oxygen WD accretors. Their total number is $\sim 10^8$. Our model suggests that ~ 10 systems of this kind, with $V \leq 13.0$ mag, $6 \lesssim P_{\rm orb} \lesssim 50$ min and $0.03 \lesssim M_2/{\rm M}_{\odot} \lesssim 0.15$, may be observed. The rate of accretion-induced collapses in all systems with helium donors is a strong function of the efficiency of the orbital energy dissipation in the common envelope stages preceding their formation, and may be as high as $\sim 10^{-5}$ yr^{-1}. The total galactic rate of supernova-scale events induced by helium shell flashes on accreting white dwarfs in binary systems with degenerate and non-degenerate helium donors may be as high as $\sim 5 \times 10^{-3}$ yr^{-1}, comparable to the estimates of the galactic rate of type Ia supernovae. We estimate that among semidetached double-degenerate systems with helium donors up to ~ 100 may be luminous supersoft X-ray sources which arise as a result of stationary helium burning at the surface of an accreting dwarf and/or of the presence of compact hot discs. Typical orbital periods of these systems are ~ 2–4 min.

Key words: accretion, accretion discs – binaries: close – novae, cataclysmic variables – supernovae: general – X-rays: general.

1 INTRODUCTION

The usual model of cataclysmic variables (CVs) envisions a close binary system containing a white dwarf (WD) primary and a low-mass secondary which fills its Roche lobe. It is usually assumed that the latter is a hydrogen-rich lower-main-sequence star which may be slightly evolved. Currently it is assumed that the driving force in the evolution of CVs is systemic angular momentum loss via gravitational wave radiation (GWR: Kraft, Mathews & Greenstein 1962) and/or magnetically coupled stellar wind (MSW: Verbunt & Zwaan 1981). This model easily explains the evolutionary status and physical properties of the overwhelming majority of CVs. In particular it predicts, in agreement with observations, the existence of the 'minimum' orbital period of a CV, which is, according to different authors, between 60 and 80 min (e.g. Paczyński 1981, Rappaport, Joss & Webbink 1982, Nelson, Rappaport & Joss 1986 and Fedorova & Tutukov 1994). The model value of the minimum period depends mainly on the opacity used in calculations. The 'absolute' minimum orbital period that may be achieved for non-degenerate hydrogen-rich secondaries is ~ 49 min, and it corresponds to the minimum mass (~ 0.084 M$_{\odot}$) for which hydrogen burning is possible (Faulkner 1971; Faulkner, Flannery & Warner 1972; Paczyński & Sienkiewicz 1981). The actual observed minimum orbital period for a CV with hydrogen features in its spectra is 59.04 min (V485 Cen: see compilation by Kolb & Ritter 1995; we refer to the latter for other observational data if not stated otherwise).

However, there exists a subpopulation of CVs with shorter orbital periods, which is often referred to, after its prototype, as 'AM CVn stars' (see Ulla 1994a; Warner 1995 for an extensive review): GP Com (46.5 min), CP Eri (28.7 min), V803 Cen (26.9 min), CR Boo (24.8 min), EC15330 – 1403 (18.65 min), AM CVn (17.2 min). In fact, with the exception of GP Com, it is still uncertain if the observed extremely short photometric variability of these stars really reflects the orbital motion. None of these systems have hydrogen features in their spectra. Absence of hydrogen features strongly suggests that donors in these

★E-mail: atutukov@inasan.rssi.ru; lry@inasan.rssi.ru

systems are extremely helium rich, because, e.g., for helium-rich accretion discs Balmer emission lines are stronger than He I lines, even when the He/H ratio is ~ 100 (Williams & Ferguson 1982). Ritter (1990) classifies GP Com, V803 Cen, CR Boo and AM CVn as nova-like stars. According to Ulla (1994b), however, the behaviour of CR Boo, CP Eri and V803 Cen resembles that of dwarf novae.

One can guess that the low-mass X-ray binaries 4U 1915 − 05 (V1405 Aql, $P=50.0$ min), 4U 1627 − 67 (KZ Tra, $P=41.4$ min) and 4U 1820 − 303 ($P=11.4$ min) are 'relatives' of AM CVn stars (for references to the observational data see Ritter 1990).

In the present paper we address the interpretation of these objects as close binaries with helium-rich secondaries, originally suggested by B. Warner and co-authors (Warner & Robinson 1972; Warner 1972; Faulkner et al. 1972), and discussed later by, e.g., Nather, Robinson & Stover (1981), Nather (1985), Wood et al. (1987), O'Donoghue, Menzies & Hill (1987), O'Donoghue & Kilkenny (1989) and Ulla (1994a,b). If this is really the case, these systems deserve special attention because accretion of pure helium may result in accretion-induced collapses (AIC) and helium shell explosions comparable in power with supernovae. If accretion of helium is within the range suitable for steady burning, these systems may manifest themselves as luminous supersoft X-ray sources. Also, the accretion rate in these systems may be high enough for the radiation of supersoft X-rays by accretion discs.

In Section 2 we consider evolutionary scenarios of the formation of semidetached systems with low-mass helium secondaries. In Section 3 we analyse the properties of the potentially most numerous family of them, namely, systems with helium-degenerate dwarfs. In Section 4 we discuss the possibility of AIC and explosive events in semidetached systems with helium-rich secondaries. Section 5 is devoted to a brief discussion of ultrashort-period double-degenerate systems as the sources of supersoft X-rays. We discuss our results in Section 6.

For brevity, we refer to semidetached systems with WD accretors and helium donors as 'CV' or 'AM CVn stars', although we do not discuss in detail their photometric behaviour.

2 EVOLUTIONARY SCENARIOS FOR THE HELIUM-RICH SEMIDETACHED LOW-MASS BINARIES

2.1 Computational algorithm

For the analyses of evolutionary routes that result in the formation of particular types of binaries we use a population synthesis code, which combines the statistical data on the birth rates of binaries, depending on the masses of their components and their separations, with the data on stellar evolution. The most crucial parameters entering the calculations, namely initial–final mass relations for components of binaries experiencing different cases of mass exchange and the algorithm for the treatment of common envelopes, were described in our earlier papers (e.g. Tutukov & Yungelson 1992, 1993, 1994, Yungelson, Tutukov & Livio 1993 and Yungelson et al. 1994, 1995). In particular, in this code, under the assumption of a constant star formation rate, we generate the population of zero-age semidetached systems with degenerate helium-rich secondaries. Then we compute analytically the evolution of each 'new-born' system for a Hubble time (15×10^9 yr), using the condition of continuous Roche lobe filling:

$$R_2=R_{\rm cr} \quad \text{and} \quad \frac{\mathrm{d}\ln R_2}{\mathrm{d}\ln M_2}=\frac{\mathrm{d}\ln R_{\rm cr}}{\mathrm{d}\ln M_2}, \tag{1}$$

as first suggested by Paczyński (1967), and later widely used by others (e.g. Faulkner 1971, Vila 1971, Tutukov & Yungelson 1979 and Rappaport et al. 1982).

As we will show below (see Fig. 1), the age of low-mass degenerate helium secondaries in the zero-age systems under consideration may vary between $\sim 10^4$ and $\sim 10^{10}$ yr. It would be desirable to use for computations the time-dependent radii for cooling WDs. However, the cooling of low-mass WDs ($M \le 0.3$ M$_\odot$) has as yet escaped the attention of modellers. Therefore, we assume that all white dwarfs, irrespective of their chemical composition, obey the approximate mass–radius relation for zero-temperature degenerate objects (Nauenberg 1972),

$$R \approx 7.84 \times 10^8 [(M/M_{\rm Ch})^{-2/3} - (M/M_{\rm Ch})^{2/3}]^{1/2}\ \text{cm}, \tag{2}$$

where M is the mass in solar masses and $M_{\rm Ch}=1.433$ M$_\odot$. In the derivation of equation (2) only the effect of the degeneracy pressure of the relativistic electron gas was taken into account. This equation does not reproduce the change of the sign of the derivative $\mathrm{d}\ln R/\mathrm{d}\ln M$ at masses ~ 0.001 M$_\odot$ owing to the rising influence of electrostatic interactions.

For the purpose of comparison we also performed a computation for the mass–radius relation for zero-tempera-

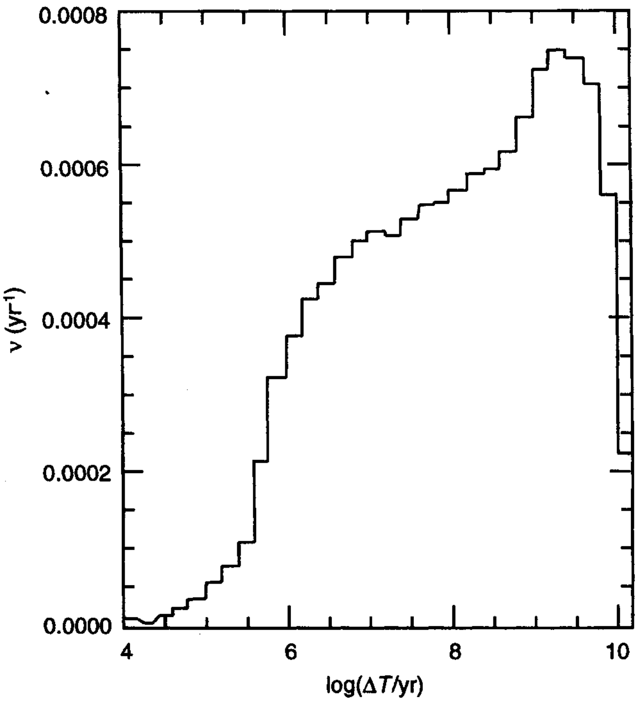

Figure 1. The age of white dwarf secondaries at the instant of RLOF (for 'standard' assumptions).

ture spheres obeying the Hamada–Salpeter equation of state (Hamada & Salpeter 1961), which takes into account corrections for electron correlation energy. The radii of the stars were computed by Zapolsky & Salpeter (1969). Their numerical results for helium low-mass zero-temperature spheres (both for the radii themselves and for their derivatives) are well represented if one uses a parabolic fit (Tutukov & Yungelson 1979):

$$\log R/\mathrm{R}_\odot \approx -1.92 - 0.41 \log (M/\mathrm{M}_\odot) - 0.0685 |\log (M/\mathrm{M}_\odot)|^2. \quad (3)$$

The derivative of expression (3) begins to deviate from that of expression (2) at $M \lesssim 0.1\ \mathrm{M}_\odot$. We shall discuss the implications of deviations from equation (2) below.

2.2 Origin of the systems with helium-rich secondaries

Current state of the art in the theory of stellar evolution suggests three kinds of helium-rich secondaries in semi-detached systems with WD primaries:

(i) helium-degenerate dwarfs;

(ii) helium-burning non-degenerate stars which are remnants of components of close binaries that experienced case B Roche lobe overflow (RLOF);

(iii) helium-mantle stars with CO cores which are remnants of components of close binaries with initial mass higher than $\sim 5\ \mathrm{M}_\odot$ that experienced case B or early case C RLOF ('helium algols').

The minimum mass of stars with CO cores and helium mantles which experience expansion in the helium shell burning phase is ~ 0.60–$0.64\ \mathrm{M}_\odot$ (Iben 1990; Iben & Tutukov 1989, 1993). Their luminosity exceeds $\sim 10^4\ \mathrm{L}_\odot$, i.e. it is much higher than the possible power of emission associated with mass accretion (hot spot, disc) and 'cataclysmic' activity.

The initial luminosity of the non-degenerate helium remnants of moderate-mass components of close binaries is $L_{\mathrm{He}} \gtrsim 10\ \mathrm{L}_\odot$ (Iben & Tutukov 1985). At the initial stage of mass exchange typical $\dot{M} \approx 3 \times 10^{-8}\ \mathrm{M}_\odot\ \mathrm{yr}^{-1}$ and the accretion luminosity is $\sim 100\ \mathrm{L}_\odot$ (Savonije, de Kool & van den Heuvel 1986; Tutukov & Fedorova 1989; Iben & Tutukov 1991). In this stage the orbital periods of stars with non-degenerate helium donors decrease from ~ 1 h to ~ 13 min. During the second phase of mass exchange the rate of the latter decreases to $\sim 10^{-10}\ \mathrm{M}_\odot\ \mathrm{yr}^{-1}$ in $\sim 10^8$ yr, and the period of the system increases to ~ 40 min. While in both stages orbital periods overlap with the orbital periods of ultrashort-period CVs, if one takes into consideration the bolometric corrections for a very hot donor and a compact helium disc, it appears that the donor is brighter than the disc in the first stage of mass exchange. Also, according to Smak (1983), helium accretion discs are stable if $\dot{M} \gtrsim 10^{-9}\ \mathrm{M}_\odot\ \mathrm{yr}^{-1}$. These two effects make the system in the first stage of mass exchange detectable not as a CV, but as a single blue star. In the second phase both donor and disc become dim, acting against discovery of the system. As we will show below, CV systems with non-degenerate helium donors must be much less numerous than systems with helium dwarf donors.

For the ultrashort-period cataclysmic variable V803 Cen, O'Donoghue et al. (1987) have shown that a non-degenerate helium secondary is incompatible with the characteristics of the system. Shara et al. (1991) did not discover any object with strong He II lines having equivalent width (EW) ≥ 15 Å in their deep ($\lesssim 17.0$ mag) sky survey. Thus, we still consider the question of the existence of 'helium star cataclysmic systems' as open.

Counterparts of systems with helium mantle donors and non-degenerate helium donors (if they exist) are, most probably, hidden among hot subdwarfs. Hence, among ultrashort-period CVs one can expect to observe only systems with helium dwarf donors. Nevertheless, for the sake of comparison, we list some numbers related to the properties of systems with helium mantle and non-degenerate helium donors in Table 1, and in the discussion compare

Table 1. Birth rates and numbers of low-mass semidetached systems and rates of related events. Systems are classified according to the nature of secondary.

Type of system	Birthrate, yr^{-1}	Number	AIC, yr^{-1}	Shell He flashes, yr^{-1}	Supersoft sources
Hydrogen MS stars, $\alpha_{ce}=1$	7.0(-3)	6.0(7)	2.6(-6)	0.0	$\sim$ 1000
Hydrogen subgiants, $\alpha_{ce}=1$	1.0(-5)	1.6(3)	0.0	0.0	460
He dwarfs, $\alpha_{ce}=1$	1.3(-2)	1.4(8)	2.5(-5)	0.0	$\sim$ 100
He dwarfs, $\alpha_{ce}=0.1$	1.1(-2)	8.3(7)	0.0	0.0	$\sim$ 100
He dwarfs, doubled R_{wd}	1.3(-2)	1.5(8)	2.6(-5)	1.0 (-3)	0.0
He dwarfs, ZS	1.4(-2)	1.6(8)	2.4 (-5)	0.0	$\sim$ 100
Nondegenerate He stars, $\alpha_{ce}=1$	4.9(-3)	4.6(5)	1.3(-4)	4.7(-3)	
Nondegenerate He stars, $\alpha_{ce}=0.1$	0.0	0.0	0.0	0.0	
Nondegenerate He stars, TK	4.9(-3)	1.9(5)	1.3(-4)	1.4(-3)	
He-mantle stars, $\alpha_{ce}=1$	8.5(-4)	2.0(2)	8.3(-6)	0.0	$\lesssim$ 200
He-mantle stars, $\alpha_{ce}=0.1$	0.0	0.0	0.0	0.0	

For the systems other than double degenerates the birth rates and numbers are estimated by the same population synthesis code (Yungelson et al. 1996). In systems with non-degenerate He donors supersoft sources do not exist. Numbers in parentheses give the power of decimal exponent.

them with the systems with dwarf donors. We clearly recognize that it is possible that not all binaries with helium dwarf donors, which we model, will manifest themselves as CVs. However, for brevity, we occasionally refer to them as CVs.

In the systems with helium dwarf secondaries the primaries in most cases form through RLOF in case C (i.e., after the exhaustion of helium in their cores). Their progenitors typically have masses of between 1.5 and 6 $M_\odot$, and separations of components in the initial systems are between 60 and 1000 $R_\odot$(Fig. 2). The typical mass of a forming WD is 0.65 $M_\odot$ (Fig. 3), while masses up to M_{Ch} are possible. Progenitors of the secondaries with masses below ~ 2.5 $M_\odot$ experience case B RLOF (i.e., in the hydrogen shell burning stage). In this second RLOF episode the system also passes through a common envelope. The time interval between formation of the secondary WD and RLOF by the latter may be anywhere between 10^4 and 10^{10} yr (Fig. 1). The range of possible initial masses of WD secondaries is limited from above by two conditions. The first is the condition of dynamically stable mass exchange

$$\left(\frac{\mathrm{d}\ln R_2}{\mathrm{d}\ln M_2}\right)_s > \left(\frac{\partial \ln R_{cr}}{\partial \ln M_2}\right)_J,$$

where $(\mathrm{d}\ln R_2/\mathrm{d}\ln M_2)_s$ is the adiabatic mass radius exponent of the secondary and J is angular momentum (e.g., Ritter 1988). If this condition is satisfied, the mass-exchange rate also has to be lower than the critical mass-exchange rate $\dot{M}_{RG}$ above which white dwarfs accreting helium develop extended envelopes. $\dot{M}_{RG}$ ranges from $\sim 10^{-6}$ $M_\odot$ yr^{-1} for $M_{wd} \approx 0.6$ $M_\odot$ to $\sim 3 \times 10^{-6}$ $M_\odot$ yr^{-1} for $M_{wd} \approx 1.3$ $M_\odot$. Otherwise we assumed that an extended common envelope engulfs both components and excluded such systems from consideration as progenitors of potential helium-rich CVs. (Components may merge in common envelopes, probably producing R CrB type stars: Iben & Tutukov 1984; Webbink 1984; Iben, Tutukov & Yungelson 1996). Thus we arrived at an upper limit for masses of secondaries, compatible with the stable mass exchange via RLOF driven by the radiation of gravitational waves, ~ 0.3 $M_\odot$. The lower limit ~ 0.13 $M_\odot$ is the lowest possible mass of a helium dwarf, which can be formed via case B RLOF in low-mass close binaries.

For comparison, systems with non-degenerate helium donors descend from systems with initial masses of primaries (progenitors of present accretors) of between 2.5 and 11.5 $M_\odot$, and orbital separations of $\sim$60–1000 $R_\odot$. Closer initial systems merge in common envelope events prior to formation of a WD; wider systems never become close enough to be brought into contact by angular momentum loss via GWR after formation of a helium secondary. Progenitors of the primaries experience case C or case BB mass exchange. The ranges of progenitors of systems with helium dwarf donors and helium non-degenerate donors in M_{10} and A_0 partly overlap. The difference is in the initial mass ratios of the components. The former systems have $M_{20} \lesssim 2.5$ $M_\odot$, while the latter have more massive secondaries. Primaries in the systems that have donors with helium mantles descend from stars more massive than ~ 4.8 $M_\odot$, because secondaries in these systems must have initial

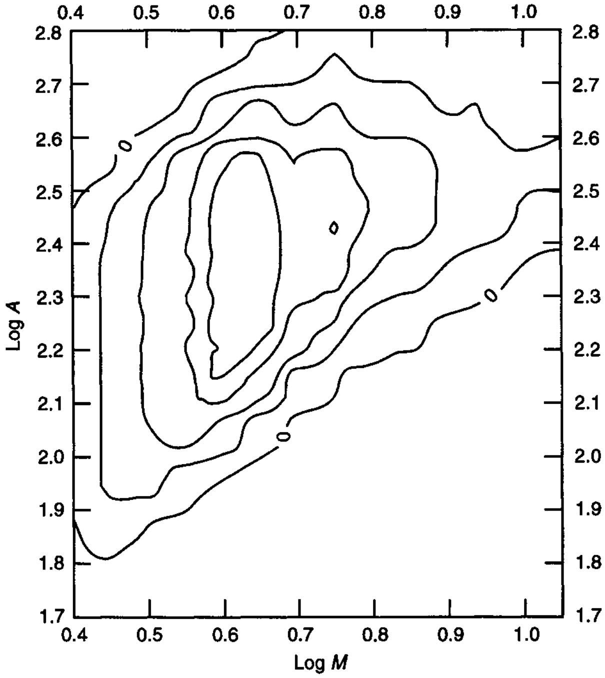

Figure 2. The position of the progenitors of systems with helium-degenerate secondaries in the initial mass of the primary (M) versus initial separation of the components (A) diagram. A and M are in solar units. Contours are lines of equal values of derivative of progenitor systems birth rate $(\partial^2\nu)/(\partial \log M\, \partial \log A)$. The step is equal to 0.005.

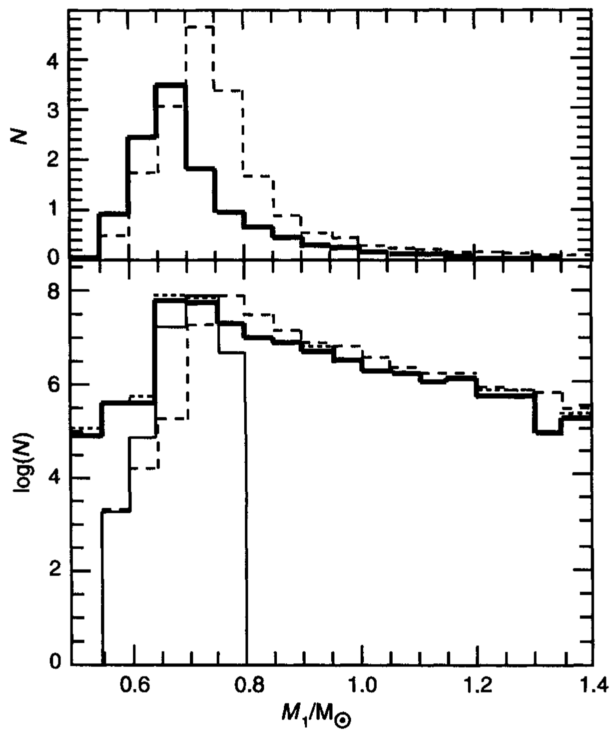

Figure 3. Distribution of systems with He dwarf donors with respect to masses of accretors for the total sample (lower panel) and for the sample limited by $V = 13.0$ mag (upper panel). In both panels the thick solid line corresponds to 'standard' assumptions, thin solid line to the sample with $\alpha_{ce} = 0.1$, the dashed line to the sample with doubled R_2, and the dotted line to the sample with Zapolsky–Salpeter radii of white dwarfs.

masses that are greater than this value in order to produce remnants with expanding helium envelopes following the RLOF. The lower limit of the initial separations of components in these systems is set by the requirement that no merging occurs in the common envelope phase that precedes the formation of the WD.

Table 1 gives the birth rates of systems of all three types for different model assumptions. In Table 1 we give also the birth rate and number of 'ordinary' semidetached hydrogen-rich systems with low-mass main-sequence and subgiant donors, as given by our scenario code. It is important that the birth rate of double-degenerate semidetached binaries with helium dwarf secondaries is about twice as large as the birth rate of hydrogen-rich candidate CV systems.

As Table 1 shows, the birth rate of systems with helium dwarf donors does not strongly depend on the common envelope parameter α_{ce}: as α_{ce} decreases the 'region' of their progenitors simply moves to higher initial separations in the diagrams similar to the one shown in Fig. 2. The birth rate of systems with non-degenerate helium donors and helium mantle donors is a stronger function of α_{ce}. For $\alpha_{ce}=0.1$ they totally disappear. The number of systems with non-degenerate helium donors decreases with decreasing α_{ce}, because more potential progenitors merge in common envelope phases. The reason for the decrease in the number of donors with helium mantles is rather peculiar. The lifetime of low-mass helium remnants of moderate-mass components of binaries is long: $T_{He} \approx 10^{7.3} (M_{He}/M_\odot)^{-3.1}$ yr (Iben & Tutukov 1987). Because of the lower value of α_{ce}, systems become tighter and more of them merge because of GWR before exhaustion of helium in their cores and thus become systems with non-degenerate donors instead of helium mantle donors.

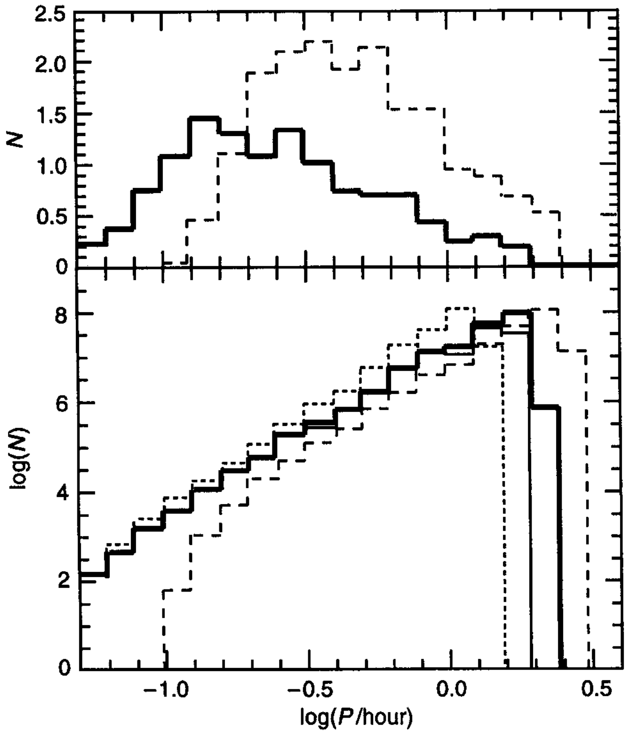

Figure 4. Distribution of systems with He dwarf donors with respect to orbital periods. Lines denote the same as in Fig. 3.

3 CATACLYSMIC VARIABLES WITH HELIUM DWARF DONORS

Let us discuss now the properties of semidetached systems with helium dwarf donors, because, as we have already mentioned, they are the only objects with helium-rich secondaries which one has a chance to observe as a CV. The lower panel of Fig. 4 gives the distribution of all these systems over orbital periods. Evidently, as the mass-loss rate decreases with time, systems accumulate at the highest periods accessible in Hubble time. This distribution weakly depends on α_{ce}, as is shown by a test computation performed for the extreme value $\alpha_{ce}=0.1$ (Table 1). Decline of the birth rate and of the number of helium-rich systems that have helium dwarf secondaries with decreasing α_{ce} reflects the 'shift' to wider progenitor systems for lower α_{ce}. For certain values of α_{ce} systems with the most massive initial primaries do not, after two common envelope events, become close enough to become semidetached in Hubble time. This is clearly seen from Fig. 3, which shows the absence of systems with $M_1 \geq 0.8$ M$_\odot$ for $\alpha_{ce}=0.1$.

There do exist, however, two important parameters which determine the mass-loss rate by the donor; these are the radius of the donor and its derivative. As Fig. 1 shows, in at least half of all systems, the RLOF occurs less than $\sim 10^8$ yr after formation of the WD donor. Despite the fact that computations of cooling curves for WDs of ~ 0.1 M$_\odot$ do not exist, one can easily guess that this time is too short for a WD to reach the state in which it obeys the mass–radius relation for the zero temperature completely degenerate configurations given by equation (1). Also, just after emerging from a common envelope, a helium WD may possess a thin hydrogen envelope, which makes its radius greater than the radius of the pure helium configuration. With the aim of testing the possible influence of the white dwarf radius being higher than given by equation (1), we computed a model with radius R_2 for helium donors twice as large as that given by equation (1). From the well-known equation for angular momentum loss via GWR (e.g., Landau & Lifschitz 1962) it follows that the mass-loss rate is $\propto R_2^{-3}$. This results in a lower value for $\dot{M}$ at the instant of RLOF. Hence, as compared with the 'standard' case for radii given by equation (1), secondaries of higher mass can transfer matter to the companions at a rate lower than the Eddington one (Fig. 5). Also, the initial period of a system $P \propto (R_2^3/M_2)^{1/2}$ is higher, and donors do not reach such low masses and long periods as in the 'standard' case. If we apply mass–radius relation (2) based on the Hamada–Salpeter equation of state, which at $M \lesssim 0.1$ M$_\odot$ gives lower radii and larger derivatives of them, the result is, evidently, opposite (Figs 4, 5 and 6). The birth rate and the total number vary by ~ 10 per cent only, because they depend on the evolution of the system prior to contact of components and on their radii.

If the lifetime of the systems with helium dwarf donors is limited by Hubble time, masses of their secondaries in our model never decrease below ~ 0.005 M$_\odot$ (Fig. 6). Therefore, they are probably stable against tidal disruption owing to the finite time required for transfer of angular momentum to the disc and from the disc back to the orbit. This instability ensues when $M_2 \lesssim 0.002$–0.004 M$_\odot$ (Ruderman & Shaham 1983, 1985; Hut & Paczyński 1984). The precise knowledge of this limit is very important for the determina-

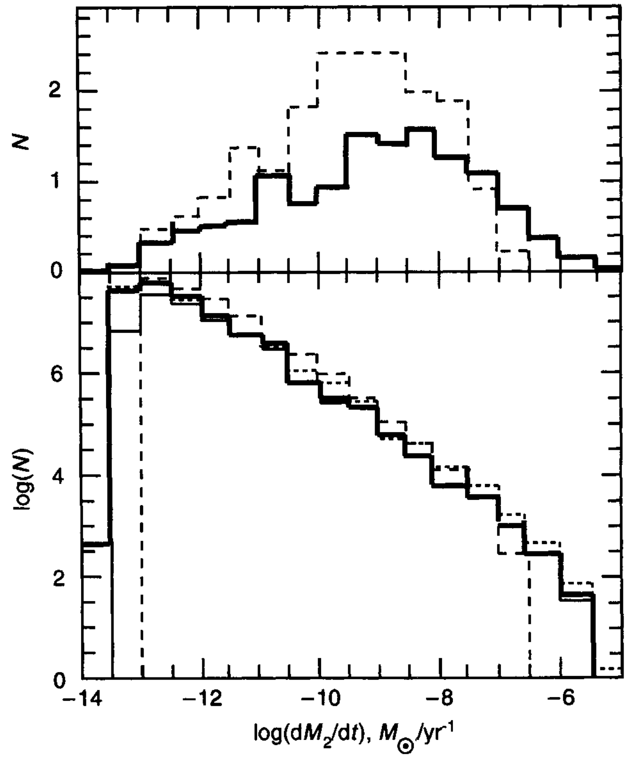

Figure 5. Distribution of systems with He dwarf donors with respect to mass exchange rate. Lines denote the same as in Fig. 3.

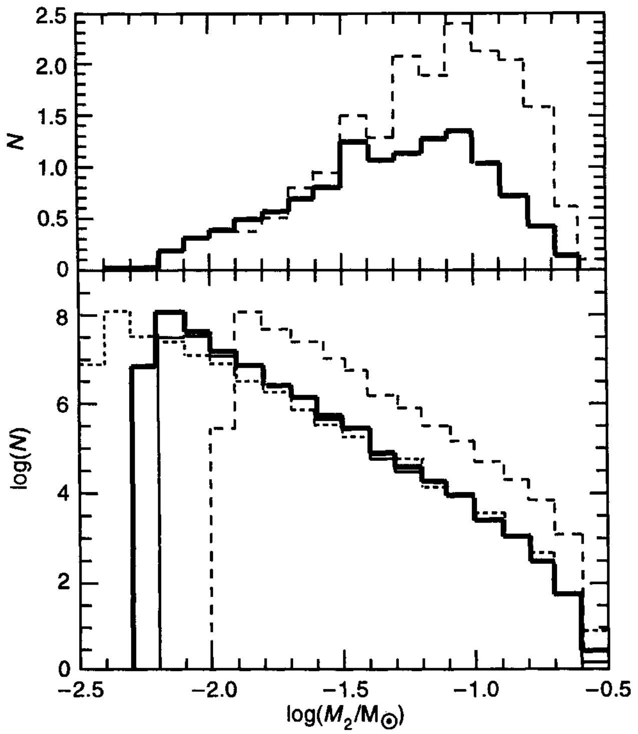

Figure 6. Distribution of systems with He dwarf donors with respect to masses of secondaries. Lines denote the same as in Fig. 3.

tion of the total number of systems with degenerate donors, because most of those in our model have masses below $\sim 0.01\ M_\odot$ (Fig. 6).

Interestingly, the mass function of the accretors in the systems with helium dwarf donors is peaked at 0.65–0.75 $M_\odot$ (Fig. 3). This is 0.05 $M_\odot$ higher than the mass at the peak of the distribution in the model of the population of all WDs of the Galaxy (Tutukov & Yungelson 1992) and in the sample of the observed WD. We can ascribe this difference to the descent of present accretors from stars with initial masses of components higher than $\sim 2.5\ M_\odot$, while common WDs descend (owing to the power-law initial mass function) from the stars with $M \sim 1\ M_\odot$, and to the mass-transfer effect.

4 ACCRETION-INDUCED COLLAPSES AND HELIUM SHELL OUTBURSTS IN SYSTEMS WITH HELIUM-RICH DONORS

In the present study we assumed, on the basis of the results of Iben & Tutukov (1985) and of Dominguez, Tornambé & Isern (1993), that all components of close binaries with initial masses between 9 and 11.4 $M_\odot$ evolve into oxygen–neon (ONe) WDs with masses $M_{wd} \gtrsim 1.19\ M_\odot$. This assumption may overestimate the range of progenitors of ONe WDs. For example, if progenitors of ONe WDs have masses confined to 10.3–10.6 $M_\odot$, as was guessed originally by Iben & Tutukov (1985), the birth rate of these WDs will be almost an order of magnitude lower than the one given by the present model.

If in the course of accretion mass of the ONe accretor grew to 1.39 $M_\odot$. the ensuing event was termed an accretion-induced collapse (AIC) and it was assumed that a neutron star is a product of AIC. In fact, the effect of the growth of the WD mass to the Chandrasekhar one (total disruption of the dwarf or formation of a neutron star) may depend on the accretion rate, the chemical composition and the mass of the dwarf at the onset of accretion (see, e.g., Nomoto & Kondo 1991 and Dominguez et al. 1993 for the most recent discussion).

As concerns carbon–oxygen dwarfs, which in our model are produced by stars initially less massive than 9.0 $M_\odot$, we assumed that after accumulation of M_{Ch} they explode without leaving a remnant.

Table 1 shows that the total birth rate of semidetached systems with helium donors and WD accretors is about 2.5 times higher than that of the systems which are traditionally discussed as possible sites of AIC, namely, ordinary CVs and CV-like systems with subgiant donors. For these two types of systems Table 1 gives the rates of AIC estimated taking into account the efficiency of accumulation of helium influenced by the hydrogen shell flashes and stellar wind of accretor, as suggested by Iben & Tutukov (1996).

Usually, two limits are considered in the discussion of the behaviour of the accreted layer at the surface of a WD. The first assumes, based on calculations of novae eruptions and on data on the chemical composition of novae ejecta, that the WD is 'eroded'. The other limit is retention of all accreted matter. As usual, both limits are not accurate.

Analysis of existing calculations of helium accreting WDs allows us to suggest the following schematic picture. If the dwarf is cold and $\dot{M} \lesssim (3\text{–}5) \times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$, about

0.1–0.2 $M_\odot$ of inert helium must be accumulated at the surface of the WD prior to shell explosion, depending on the mass of the accretor (Fujimoto & Taam 1982; Limongi & Tornambé 1991; Iben et al. 1987; Iben & Tutukov 1991; Tutukov & Khokhlov 1992; Woosley & Weaver 1994). If the mass of the dwarf exceeds $\sim 0.6\ M_\odot$ (Woosley & Weaver 1994) or $\sim 0.8\ M_\odot$ (Tutukov & Khokhlov 1992), burning of helium is violent and energy release is on a supernova (SN) scale. The computed light curves resemble SN Ia in shapes, but the objects are less than half as luminous as a typical SN Ia (Woosley & Weaver 1994).

If $\dot{M}$ exceeds $(3\text{–}5)\times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$, helium burns in flashes, the violence of which decreases with increasing $\dot{M}$. At $\dot{M}_{\rm stat}\sim 3\times 10^{-6}\ M_\odot\ \mathrm{yr}^{-1}$ stationary burning of helium is possible. If $\dot{M}$ is higher, the accreted envelope expands. If, in the course of the flashes or at accretion rates higher than $\dot{M}_{\rm stat}$, the accretor expands beyond its Roche lobe, a common envelope presumably forms, in which all accreted matter may be lost. If the star burns helium stably or it is in a post-explosion state it may lose matter via radiatively driven wind with $\dot{M}\propto L/(v_{\rm esc}c)$. Therefore, He is never converted into carbon and oxygen with 100 per cent efficiency (for more detailed discussion see Iben & Tutukov 1996).

Table 1 shows the rates of AIC and helium shell flashes in systems with hydrogen and helium donors under different assumptions about the most crucial input parameters. In the 'standard' case we assumed that all WDs which were initially more massive than 0.6 $M_\odot$ and were able to accrete 0.2 $M_\odot$ of helium experience explosions on an SN scale. Furthermore, our assumption is that the dwarfs survive the flash, but become hot and burn all incoming helium in weak flashes. In the case marked TK in the Table, we assumed, after Tutukov & Khokhlov (1992), that for WD masses of between 0.6 and 0.8 $M_\odot$ accretion of 0.2 $M_\odot$ of helium at $\dot{M}\sim 3\times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ results in a mild expansion of the envelope and transformation of the system into an R CrB star. Evidently, in this case the number of semidetached systems with helium-rich donors and the rate of explosive shell flashes are strongly reduced. If we relax the assumption of the survival of WDs after helium shell flashes (and this may be a more realistic assumption: see, e.g., Livne 1990; Livne & Glasner 1991; Woosley & Weaver 1994), the number of systems with semidetached non-degenerate helium donors reduces by a factor of ~ 8, as compared to the 'standard' case. The reason is quite obvious: most of the 'critical' explosion mass of the shell is accumulated in the stage with relatively high ($3\times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$) mass-transfer rate.

It is clear that systems with non-degenerate helium donors may be the site of the most numerous AIC, by more than an order of magnitude, outnumbering systems with hydrogen-rich donors. If in the systems with non-degenerate helium donors the collapse of ONe dwarfs results in the formation of neutron stars, and these systems become low-mass X-ray binaries (LMXBs) and lose $\sim 0.3\ M_\odot$ with $\dot{M}\approx 3\times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ (Savonije, de Kool & van den Heuvel 1986; Tutukov & Fedorova 1989), we arrive at $\sim 10^3$ LMXBs of this type with $L\sim L_{\rm Edd}$ and orbital periods of between 50 and 10 min. No such system has been observed as yet. An effect which can prevent transformation of systems with non-degenerate helium donors into LMXBs is the 'spike' in the mass-exchange rate over $\dot{M}_{\rm Edd}$ just after RLOF by the donor, which is clearly seen in the computed evolutionary tracks (Tutukov & Fedorova 1989). This 'spike' becomes more pronounced if the donor is at least slightly evolved. If $\dot{M}$ exceeds $\dot{M}_{\rm Edd}$, it is possible that a common envelope forms, in which components rapidly merge with production of Thorne–Żytkow object. If they form nevertheless, irradiation of the donor by X-rays, which results in strong stellar wind, may significantly reduce the lifetime of the donor and, hence, the number of such systems (e.g., Podsiadlowski 1991; Harpaz & Rappaport 1991; Hameury, King & Lasota 1993; Tavani & London 1993; for a more complete bibliography and discussion of the problem of irradiation-induced wind, see Iben, Tutukov & Yungelson 1995a, b).

If AIC of ONe dwarfs results in the formation of neutron stars, the total number of LMXBs in systems with helium-degenerate donors for $\alpha_{\rm ce}=1$ is $N_{\rm X}\approx 3.5\times 10^5$. This number in our model is a strong function of $\alpha_{\rm ce}$. For example, for $\alpha_{\rm ce}=0.5$ the rate of AIC is $4.9\times 10^{-6}\ \mathrm{yr}^{-1}$, $N_{\rm X}\approx 6.5\times 10^4$ and for $\alpha_{\rm ce}=0.2$ the rate of AIC becomes $1.5\times 10^{-7}\ \mathrm{yr}^{-1}$, $N_{\rm X}\approx 1600$. For $\alpha_{\rm ce}=0.1$ AIC does not occur at all. While typical X-ray luminosity of X-ray sources with helium-degenerate donors is 0.2–2 $L_\odot$, our model predicts (in the absence of induced stellar wind and for $\alpha_{\rm ce}=1$) the existence of about 100 'bright' sources, with $10^3\lesssim L_{\rm x}/L_\odot\lesssim 10^4$. Regretfully, the effect of irradiation of WDs by X-rays has not been investigated as yet. Therefore, our estimates of the numbers of LMXBs with helium dwarf donors may be treated as the upper limits.

Only one bright source of X-ray emission with an ultrashort period of 11.4 min – 4U 1820 – 30 – is known. This source is in the globular cluster NGC 6624. The source was probably formed in a two- or three-body collision, but collision itself is not a sufficient explanation of the current state of the system, since the AIC is the most efficient way of obtaining an ultrashort-period X-ray binary with helium-degenerate donor. An evolutionary path including an AIC seems inevitable for this system, because if in the capture process (or after it) a system composed of a neutron star with a helium-degenerate dwarf companion forms, even if the mass of the latter is minimal (0.13 $M_\odot$), immediately after RLOF mass-exchange rate exceeds $\dot{M}_{\rm Edd}$ (Tutukov & Yungelson 1979). Thus formation of the common envelope and, subsequently, transformation of the system into a Thorne–Żytkow object may be expected.

Previously it was suggested (Tutukov et al. 1985) that in 4U 1820 – 30 secondary is a helium dwarf which descended from a low-mass main-sequence star, which almost completely exhausted hydrogen in the core ($X_{\rm c}\lesssim 0.01$) at the instant of RLOF. Evidently, such systems may originate from an extremely narrow interval of semimajor axes.

Two more ultrashort-period X-ray systems with $L_{\rm X}\sim 10^2\ L_\odot$ – 4U 1915 – 05 ($P=50.0$ min) and 4U 1627 – 67 ($P=41.4$ min) – may also have degenerate helium donors. Dimmer model sources with helium-degenerate donors ($L_{\rm X}\approx 0.2\text{–}2\ L_\odot$) may be hidden among approximately 10 000 weak *ROSAT* X-ray sources, which are not active galactic nuclei, usual stars or clusters of galaxies (Trümper 1994).

In systems with helium mantle donors mass-exchange rate ($\sim 10^{-6}\ M_\odot\ \mathrm{yr}^{-1}$) exceeds the Eddington one. These systems probably transform into Thorne–Żytkow objects immediately after collapse because of the formation of a common envelope which engulfs the whole system.

Summing up, we can conclude that our model seemingly overproduces the number of LMXBs with helium-degenerate donors, as compared with the observed number of relevant systems. Among the possible reasons for this discrepancy we may list the following, as yet unresolved, factors in the model: uncertainty in α_{ce}; uncertainty in the range of masses of progenitors of ONe dwarfs; the unknown influence of X-ray irradiation on the lifetime in the X-ray stage; and the unknown mass limit of degenerate donors that are stable against tidal disruption. The effect of several still uncertain model assumptions is illustrated by the entries in Table 1. Last, but not least, it is still unclear what may be the outcome of the accumulation of M_{ch} by an ONe dwarf. Recent computations of Dominguez et al. (1993) show that complete disruption of the dwarf is quite probable. If we accept, contrary to the assumption of inevitability of neutron star formation, the hypothesis of complete disruption, the apparent problem of overproduction of helium donor LMXBs disappears.

Only systems with helium non-degenerate donors appear to produce helium shell flashes with a rate (see Table 1) that is comparable to the observational estimate of the rate of SN Ia in our Galaxy, which is ~ 0.003 yr^{-1} (van den Bergh & Tammann 1991). Even if we assume, following Tutukov & Khokhlov (1992), that supernova-scale helium shell flashes occur only on the dwarfs that are more massive than 0.8 M$_\odot$, the rate of strong flashes reduces to ~ 0.001 yr^{-1}, still providing a considerable fraction of the 'oberved' SN Ia rate. The model rate of helium shell explosions may be ~ 25 per cent higher if we allow repetition of explosions after accumulation of another 'critical' layer of the same mass, 0.2 M$_\odot$.

Thus, in principle, the rate of strong helium flashes in systems with helium donors is high enough to explain the 'observed' rate for SN Ia. If one considers the age of the systems in which these flashes occur (Tutukov & Yungelson 1994), however, it appears that systems with non-degenerate helium donors are not older than 10^9 yr, and systems with helium dwarf donors are younger than 6×10^9 yr. Thus, they cannot challenge merging CO + CO, ONe + ONe or ONe + CO double-degenerate systems as candidate pre-SN Ia in early-type galaxies.

5 SEMIDETACHED DOUBLE-DEGENERATE BINARIES AS SOURCES OF SUPERSOFT X-RAY RADIATION

Supersoft X-ray sources have typical luminosities of $(4\text{–}10) \times 10^{37}$ erg s^{-1} and energy distributions peaked between 20 and 50 eV [effective temperatures above $\sim (2.5\text{–}3.0) \times 10^5$ K].

The most popular model of them presents a semidetached cataclysmic-like binary with a hydrogen-rich secondary exchanging matter on the thermal time-scale of the donor (van den Heuvel et al. 1992; Rappaport, Di Stefano & Smith 1994). Large Magellanic Cloud (LMC) sources CAL 83 and 87, with estimated orbital periods of 1.04 d and 10.4 h, respectively, fit well into this model. The post-nova supersoft X-ray source GQ Mus ($P_{orb} = 85.5$ min) may be identified with a WD burning the remainder of the accreted hydrogen envelope (Orio & Ögelman 1993). This model may be also applied to supersoft X-ray sources identified with symbiotic post-novae (Iben & Tutukov 1996; Yungelson et al. 1996).

Iben & Tutukov (1994) have shown that helium mantle stars with CO cores which are remnants of components of close binaries with initial mass higher than ~ 5 M$_\odot$ that experienced case B or early case C RLOF ('helium-algols') may also be donors in semidetached binaries emitting supersoft X-rays, because they can supply matter at a rate $\sim 10^{-6}$ M$_\odot$ yr^{-1}, which is appropriate for (quasi)stationary helium burning [in fact, these systems were briefly discussed as item (iii) in Section 2.2 of this paper]. Quasistationary burning of helium at the surface of WDs that have a mass exceeding 0.6 M$_\odot$ is possible if the accretion rate is $10^{-6.5}\text{–}10^{-5.5}$ M$_\odot$ yr^{-1} (see, e.g., fig. 3 in Iben & Tutukov 1989). In our 'standard' model of systems with dwarf donors about 70 objects have $\dot{M}$ above $10^{-6.5}$ M$_\odot$ yr^{-1}. This number probably gives the upper limit for the incidence of supersoft sources with helium-rich dwarf donors, because not all accreting dwarfs in our systems have effective temperatures in excess of 2.5×10^5 K. For models with other than standard assumptions the numbers of potential supersoft sources are similar to within factor $\lesssim 2$; the exception is the case of doubled radii, for which they almost disappear because of lower $\dot{M}$ at the instant of RLOF (Fig. 5). Systems with helium-rich WD donors may provide a non-negligible contribution to the total population of galactical supersoft sources, which is estimated as $\sim 100\text{–}1000$ (van den Heuvel et al. 1992; Rappaport et al. 1994; Di Stefano & Rappaport 1994; Yungelson et al. 1996). The number of helium dwarf supersoft X-ray systems may be comparable with the number of the 'helium-algol sources' ($\sim 100\text{–}200$: Iben & Tutukov 1994; Yungelson et al. 1996). One must bear in mind that the 'observed' galactical incidence of sources was obtained by a rough scaling of the estimated incidence of sources in the LMC. A more accurate estimate of the incidence is prevented by extremely effective absorption of supersoft X-rays by circumbinary and interstellar matter.

Orbital periods of helium dwarf systems with mass-exchange rate $\dot{M} \gtrsim 10^{-6.5}$ M$_\odot$ yr^{-1} are below ~ 4 min (Tutukov & Yungelson 1979). The variability related to the orbital motion may be detectable in X-rays.

Systems with helium dwarf donors may manifest themselves in supersoft X-rays also through the radiation of their accretion discs. The radial dependence of the temperature in the standard blackbody discs is given by (Shakura & Sunyaev 1973)

$$T(x) = \left(\frac{3GM\dot{M}}{8\pi\sigma R^3_{wd}}\right)^{0.25} x^{0.75}(1 - x^{0.5})^{0.25}, \tag{4}$$

where $x = R_{wd}/r$. The maximum value of $T(r)$ exceeds 250 000 K if $\dot{M} \gtrsim 4 \times 10^{-6}$ M$_\odot$ yr^{-1} for $M_{wd} = 0.6$ M$_\odot$ and $\dot{M} \gtrsim 6 \times 10^{-7}$ M$_\odot$ yr^{-1} for $M_{wd} = 1.0$ M$_\odot$. This means that a part of the stationary helium-burning dwarfs (~ 100) may also have discs radiating supersoft X-rays. In systems, with relatively massive WDs and $\dot{M}$ slightly below the rate necessary for stationary helium burning, which experience weak flashes, emission of supersoft X-rays may be expected both in the 'off' state and in the post-flash 'on' state when a part of the helium accumulated between flashes is burned at the hottest portion of the plateau stage of the evolutionary track of WD. The number of 'accretion disc' supersoft sources may be lower than our estimate if accretion of helium on to an initially cold dwarf at the rate $\sim 10^{-5}$–10^{-6} $M_\odot$ yr^{-1} results in significant expansion of the accretor (Nomoto, Nariai & Sugimoto 1979).

For comparison, we list in Table 1 the numbers of supersoft X-ray sources of different sorts estimated by means of the same population synthesis code (Yungelson et al. 1996).

6 DISCUSSION

Orbital periods of the model systems with helium-degenerate donors overlap with the interval of the orbital periods of known ultrashort orbital period cataclysmic variables (of AM CVn type). Consideration of numbers and properties of systems with helium-degenerate dwarf donors suggests that they are the most probable 'theoretical' counterparts of AM CVn stars. It would be interesting to look deeper into the properties of accretion discs in such systems and their possible observational manifestations.

Considering accretion disc radiation as the main source of luminosity of semidetached systems with helium dwarf secondaries, it is possible to estimate the numbers of systems and their distributions over different parameters for the samples limited by certain apparent stellar magnitudes $V_{\rm lim}$. In order to construct magnitude-limited samples we assumed that the Galaxy may be represented by a 400 pc thick homogeneous circular layer with $r=15$ kpc. Interstellar absorption was assumed to be equal to 1.9 mag kpc^{-1} (Allen 1973). The mean absolute stellar magnitude M_v of an infinite radius standard blackbody accretion discs may be approximated as

$$M_v = -9.48 - \tfrac{5}{3}\log(M_{\rm wd}\dot{M}), \qquad (5)$$

where $M_{\rm wd}$ is in $M_\odot$ and $\dot{M}$ in $M_\odot$ yr^{-1} (Webbink et al. 1987). Comparison with the $M_v - \dot{M}$ relation for finite-size blackbody standard discs (Paczyński & Schwarzenberg-Czerny 1980; Smak 1989) shows that for a typical model system under consideration, the mass-exchange rates shown in the lower panel of Fig. 5 and $R_{\rm disc}/R_{\rm star} \lesssim 10$, the values of M_v are higher than given by equation (5) by ≈ 2.0 to 3.0 mag. We conservatively correct equation (5) by 2.0 mag.

The sample of observed AM CVn stars (as well as of CV of any other types) is far from being complete in any respect, owing to numerous selection effects. If we limit the model samples by $V_{\rm lim}=13.0$ or 17.0 mag, we obtain the numbers of 'observable' systems with helium dwarf donors ~ 10 and ~ 1000, respectively. For the sample with doubled R_2 these numbers are ~ 18 and ~ 5000, respectively. Bearing in mind that even the sample of the brightest 'ordinary' CVs is significantly incomplete, we consider any comparison of these numbers with the number of known systems as premature. We also would like to mention (as suggested by the anonymous referee) that, in fact, the observed sample of AM CVn stars is a colour-selected one: with exception of GP Com all of them were discovered owing to extremely blue colours. For illustrative purposes in the upper panels of Figs 3–6 we show the distributions over M_1, $P_{\rm orb}$, $\dot{M}_2$ and M_2 in the standard model and in the model with doubled R_2 for the sample limited by 13.0 mag as suggested by V of the brightest AM CVn type stars CR Boo ($V=13.0$ mag), V803 Cen ($V=13.2$ mag), EC 15330 − 1403 ($V=13.7$ mag) and AM CVn ($V=14.2$ mag). Most systems in the magnitude-limited sample have $P_{\rm orb}$ between 6 and 50 min, just in the range of periods of interest for the present study (Fig. 4).

The data in Table 1 gives the space density of systems with helium dwarf donors equal to 4.7×10^{-4} pc^{-3}. This is two orders of magnitude larger than the estimate of space density of AM CVn type stars given by Warner (1995): 3×10^{-6} pc^{-3}. The latter estimate was obtained assuming that for all AM CVn stars the mean $M_{v,\rm disc} \sim 9.5$ mag and $\dot{M}=3 \times 10^{-9}$ $M_\odot$ yr^{-1}. In our model typical $\dot{M} \sim 10^{-12}$–10^{-13} $M_\odot$ yr^{-1} (Fig. 5). Also, we use a combination of Webbink et al. (1987) and Smak (1989) data on the stellar magnitudes of the discs to obtain the estimate of $M_{v,\rm disc}$, which for $\dot{M}=3 \times 10^{-9}$ $M_\odot$ yr^{-1} give $M_{v,\rm disc} \sim 2.5$ mag lower than Cannizzo's (1984) value used by Warner. Thus, it becomes evident that a crucial role in comparison of theoretical predictions and observations is played by disc models.

Shara et al. (1991 and private communication) did not discover any candidate helium-rich CVs in a 17.0-mag deep survey of sky which covered ~ 100 deg^2. The survey was aimed at the discovery of stars with strong helium photometric and spectroscopic features. According to our model, two to three helium-rich systems with $V \lesssim 17.0$ mag could be expected within the area covered by the Shara et al. survey. Continuation of this survey may be an efficient way of quantitatively checking the suggested model.

The available information on the properties of low-mass helium dwarfs and helium discs allows us to list several effects that may influence their number.

(i) In zero-temperature stars the effects of recombination, which reduce the pressure of electrons and 'invert' the sign of $d\ln R_2/d\ln M_2$ to a positive value, become prominent at $M_2 \sim 0.003$ $M_\odot$. If dwarfs are hot, these effects may begin to act earlier, e.g., if inversion begins to act at $M_2 \sim 0.01$ $M_\odot$ the number of helium-rich semidetached systems may reduce by an order of magnitude and their periods will not exceed ~ 1 h.

(ii) Accretion of helium on to a WD may slow down the cooling of the WD. Therefore, it may be visually brighter than the accretion disc and the system will be undistinguishable from ordinary relatively young single WDs.

(iii) According to Smak (1983), helium accretion discs are stable if $\dot{M} < 10^{-12}$ $(M_{\rm wd}/M_\odot)^{-2}$ $M_\odot$ yr^{-1}. Systems with lower $\dot{M}$ will not manifest themselves as dwarf novae and may avoid discovery as CVs. As Fig. 5 shows, the overwhelming majority of helium dwarf donor stars may have stable discs. Ultrashort-period binaries with helium-rich components can be easily confused with ordinary single DB dwarfs with helium emission lines. The history of discovery of AM CVn type stars (see Warner 1995) shows that AM CVn itself and GP Com were initially identified with DB dwarfs. The discovery of rapid flickering may be helpful in identification of candidate AM CVn stars among DB WDs (Warner 1972). On the other hand, in relatively bright systems ($V \lesssim 13.0$ mag) the mass-exchange rate exceeds $\sim 10^{-9}$ $M_\odot$ yr^{-1} and discs may be stable (Smak 1983), thus acting against the discovery of these systems. In our model sample of the brightest stars with helium dwarf donors only about one-half may have unstable discs and manifest themselves as CVs (Fig. 5).

(iv) Deviations of the spectral energy distributions of accretion discs from the blackbody one may influence their absolute stellar magnitudes.

The birth rate of systems with helium non-degenerate donors does not differ dramatically from the number of other helium-rich systems and ordinary CVs. They can have periods between 20.0 and 30.0 min. Therefore, one may expect to observe at least some of them as eclipsing systems, and we do not observe any of them. However, they must typically be in the evolutionary stage beyond the turning points of the tracks at masses below $\sim 0.18\ M_\odot$ and have typical $\dot{M} \sim 10^{-10}\ M_\odot\ yr^{-1}$. Then, if we take typical values of $M_{wd}=0.6\ M_\odot$, $\dot{M}=10^{-10}\ M_\odot\ yr^{-1}$, compute M_v according to equation (5) and correct the latter by 2.0 mag, we discover that systems brighter than 13.0 mag are visible up to the distance of ≈ 80 pc. This volume contains only 7×10^{-6} of the whole volume of the Galaxy. Therefore, there may be only three 'observable' systems with helium non-degenerate donors brighter than 13.0 mag, three times less than systems with degenerate helium donors. This number may be much lower if helium shell flashes completely destroy exploding accretors.

Semidetached systems with helium dwarf donors may form a previously unrecognized class of supersoft X-ray sources, because just after RLOF most of them have mass-exchange rates within the range where quasistationary helium burning is possible and some of them may have accretion discs hot enough to radiate supersoft X-rays. Despite this stage of high $\dot{M}$ being relatively short ($\sim 10^4$ yr), because of the high birth rate of helium dwarf donor systems ~ 100 sources of this kind may be currently present in the Galaxy, or ~ 1 source within their detection limit in the Solar vicinity. Their peculiarity is very short orbital periods: 2 to 4 min. The number of such sources may be non-negligible as compared with the number of sources suggested by models in which X-ray emission results from quasistationary burning of accreted hydrogen.

Some DB WDs may be related to the AM CVn type stars genetically. First, compressional heating of the WD owing to accretion may support relatively high luminosity of the dwarf. If binarity is not recognized, one would observe a usual 'single' DB WD. Secondly, if donors are disrupted upon their mass decreasing below a certain limit, single DB dwarfs have to descend from AM CVn stars. The surface density of DB WDs brighter than $B=17.0$ mag is ~ 0.1 deg^{-2} (Green, Schmidt & Liebert 1986). The surface density of systems with helium dwarf secondaries, which have discs brighter than $V=17.0$ mag, is ~ 0.024 deg^{-2}. As $B-V$ is positive for a cold WD, this gives the lower limit for the possible surface density of helium rich CVs brighter than $B=17.0$ mag. Thus, (i) a significant (but still uncertain) proportion of DB white dwarfs may actually be unrecognized AM CVn type stars, and (ii) some single DB WDs may descend from AM CVn type systems.

Short-period semidetached binaries with helium dwarf secondaries must be powerful emitters of gravitational waves. For frequencies $\nu \gtrsim 10^{-4}$ Hz the flux of their GWR exceeds that of 'ordinary' CVs and W UMa stars (Tutukov & Yungelson 1979; Hils, Bender & Webbink 1990; Warner 1995). Our trial computations of the flux of GWR from detached and semidetached systems with WD components show that the flux from close detached double-degenerate systems for the same frequencies is several times higher than the flux from semidetached binaries.

If our model describes properly the characteristics of the systems with helium donors, these may be the site of more frequent AIC than systems with hydrogen-rich donors (see Table 1). Theoretically, is still uncertain if accumulation of Chandrasekhar mass by ONe WDs results in the formation of a neutron star or in an explosion which does not leave any remnant.

In the case of neutron star formation, post-collapse systems must be LMXBs for a rather long time. Under 'standard' assumptions our model seemingly overproduces the number of LMXBs with helium donors. This problem for systems with helium non-degenerate donors may be non-existent if there is a spike in the mass-transfer rate over $\dot{M}_{Edd}$ immediately after RLOF by the donor. The number of LMXBs with helium donors may be strongly reduced if α_{ce} is sufficiently low (see Table 1) and X-ray-irradiation-induced stellar wind considerably shortens the lifetimes of systems (Iben, Tutukov & Yungelson 1995b).

If AIC results in disruption of an accreting ONe dwarf, the problem of overproduction of LMXBs with helium donors would not arise. Also, if for sufficiently massive WDs accreting at the rates of $\sim 10^{-8}\ M_\odot\ yr^{-1}$ the mass of the accreted helium layer necessary for shell flash is sufficiently lower than the value of 0.2 $M_\odot$ adopted by us, and helium ignition results in double detonation, which disrupts the dwarf, the predicted number of LMXBs with helium donors may be sufficiently reduced. Thus, proper statistics of short-period (presumably, with helium donors) LMXBs can provide additional clues for solving the problem of AIC and shell helium detonations.

Shell helium explosions in systems with helium donors may provide a significant contribution to the rate of hydrogen-deficient supernovae from stellar populations with the age below $\sim 10^9$ yr (this is, of course, if all our assumptions about proportion of binaries, mass function of stars, star formation rate etc. may be applied to other galaxies).

ACKNOWLEDGMENTS

We are indebted to N. Chugai, I. Iben, Jr, H. Ritter, M. Shara and J. Smak for stimulating discussions on the properties of CVs and, especially, the selection effects governing their discovery, and to B. Warner for sending his review of AM CVn stars prior to publication. We thank anonymous referees for their useful comments on the earlier version of the paper. We thank Mrs S. V. Sushko for her help in the preparation of the manuscript. This study was supported in part by Russian Foundation for Fundamental Research grant 93-02-2893, ESO C&EE Programme grant A-01-019, and International Science Foundation grants MPT000 and MPT300.

REFERENCES

Allen C. W., 1973, Astrophysical Quantities. Athlone Press, London

Cannizzo J., 1984, Nat, 311, 443
Di Stefano R., Rappaport S., 1994, ApJ, 437, 733
Dominguez I., Tornambé A., Isern J., 1993, ApJ, 419, 268
Faulkner J., 1971, ApJ, 170, L99
Faulkner J., Flannery B. P., Warner B., 1972, ApJ, 175, 179
Fedorova A. V., Tutukov A. V., 1994, Astron. Zh., 71, 431
Fujimoto M. Y., Taam R. E., 1982, ApJ, 260, 249
Green R. F., Schmidt M., Liebert J., 1986, ApJS, 61, 305
Hamada T., Salpeter E., 1961, ApJ, 134, 683
Hameury J.-M., King A. R., Lasota J.-P., 1993, A&A, 277, 81
Harpaz A., Rappaport S., 1991, ApJ, 383, 739
Hils D., Bender P. L., Webbink R. F., 1990, ApJ, 360, 75
Hut P., Paczyński B., 1984, ApJ, 284, 685
Iben I., Jr, 1990, ApJ, 353, 215
Iben I., Jr, Tutukov A. V., 1984, ApJS, 54, 335
Iben I., Jr, Tutukov A. V., 1985, ApJS, 58, 661
Iben I., Jr, Tutukov A. V., 1987, ApJ, 313, 727
Iben I., Jr, Tutukov A. V., 1989, ApJ, 342, 430
Iben I., Jr, Tutukov A. V., 1991, ApJ, 370, 615
Iben I., Jr, Tutukov A. V., 1993, ApJ, 418, 343
Iben I., Jr, Tutukov A. V., 1994, ApJ, 431, 264
Iben I., Jr, Tutukov A. V., 1996, ApJ, in press
Iben I., Jr, Nomoto K., Tornambé A., Tutukov A. V., 1987, ApJ, 317, 717
Iben I., Jr, Tutukov A. V., Yungelson L. R., 1995a, ApJS, 100, 217
Iben I., Jr, Tutukov A. V., Yungelson L. R., 1995b, ApJS, 100, 233
Iben I., Jr, Tutukov A. V., Yungelson L. R., 1996, ApJ, 456, 750
Kolb U., Ritter H., 1995, in Lewin W. H. G., van Paradijs J., van den Heuvel E. P. J., eds, X-ray Binaries. Cambridge Univ. Press, Cambridge, p. 578
Kraft R., Mathews P., Greenstein J., 1962, ApJ, 136, 312
Landau L. D., Lifschitz E. M., 1962, Teorija Polya. Fizmatzig, Moscow (in Russian)
Limongi M., Tornambé A., 1991, ApJ, 371, 317
Livne E., 1990, ApJ, 354, L59
Livne E., Glasner A., 1991, ApJ, 370, 272
Nather R. E., 1985, in Eggleton P. P., Pringle J. E., eds, Interacting Binaries. Reidel, Dordrecht, p. 349
Nather R. E., Robinson E. L., Stover R. J., 1981, ApJ, 244, 269
Nauenberg M., 1972, ApJ, 175, 417
Nelson L. A., Rappaport S., Joss P., 1986, ApJ, 304, 231
Nomoto K., Kondo Y., 1991, ApJ, 367, L19
Nomoto K., Nariai K., Sugimoto D., 1979, PASJ, 31, 287
O'Donoghue D., Kilkenny D., 1989, MNRAS, 236, 319
O'Donoghue D., Menzies J. W., Hill P. W., 1987, MNRAS, 227, 347
Orio M., Ögelman H., 1993, A&A, 273, L78
Paczyński B., 1967, Acta Astron., 17, 287
Paczyński B., 1981, Acta Astron., 31, 1
Paczyński B., Schwarzenberg-Czerny A., 1980, Acta Astron., 30, 127
Paczyński B., Sienkiewicz R., 1981, ApJ, 248, L27
Podsiadlowski P., 1991, Nat, 350, 136
Rappaport S., Joss P. C., Webbink R., 1982, ApJ, 254, 616
Rappaport S., Di Stefano R., Smith M., 1994, ApJ, 426, 692
Ritter H., 1988, A&A, 202, 93
Ritter H., 1990, A&AS, 85, 1179
Ruderman M., Shaham J., 1983, Nat, 304, 425
Ruderman M., Shaham J., 1985, ApJ, 289, 244
Savonije G. J., de Kool M., van den Heuvel E. P. J., 1986, A&A, 155, 51
Shakura N. I., Sunyaev R. A., 1973, A&A, 24, 337
Shara M. M., Moffat A. F. J., Smith L. F., Potter M., 1991, AJ, 102, 716
Smak J., 1983, Acta Astron., 33, 333
Smak J., 1989, Acta Astron., 39, 317
Tavani M., London R., 1993, ApJ, 410, 281
Trümper J., 1994, in Wamsteker W., Longair M. S., Kondo Y., eds, Frontiers of Space and Ground-Based Astronomy. Kluwer, Dordrecht, p. 47
Tutukov A. V., Fedorova A. V., 1989, Astron. Zh., 66, 1172
Tutukov A. V., Khokhlov A. M., 1992, Astron. Zh., 69, 754
Tutukov A. V., Yungelson L. R., 1979, Acta Astron., 29, 665
Tutukov A. V., Yungelson L. R., 1992, SvA, 36, 266
Tutukov A. V., Yungelson L. R., 1993, MNRAS, 260, 675
Tutukov A. V., Yungelson L. R., 1994, MNRAS, 268, 871
Tutukov A. V., Fedorova A. V., Ergma E. V., Yungelson L. R., 1985, SvA Lett, 11, 52
Ulla A., 1994a, Space Sci. Rev., 67, 241
Ulla A., 1994b, Mem. Soc. Astron. Ital., 65, 231
van den Bergh S., Tammann G. A., 1991, ARA&A, 29, 363
van den Heuvel E. P. J., Bhattacharya D., Nomoto K., Rappaport S. A., 1992, A&A, 262, 97
Verbunt F., Zwaan C., 1981, A&A, 100, L7
Vila S., 1971, ApJ, 168, 217
Warner B., 1972, MNRAS, 159, 315
Warner B., 1995, ApSS, 225, 249
Warner B., Robinson E. L., 1972, MNRAS, 159, 101
Webbink R. F., 1984, ApJ, 277, 355
Webbink R. F., Livio M., Truran W., Orio M., 1987, ApJ, 314, 653
Williams R. E., Ferguson D. H., 1982, ApJ, 257, 672
Wood M. A., Winget D. E., Nather R. E., Hessman F. V., Liebert J., Kurtz D. W., Wesemael F. G., Wegner G., 1987, ApJ, 313, 757
Woosley S. E., Weaver T., 1994, ApJ, 423, 371
Yungelson L. R., Tutukov A. V., Livio M., 1993, ApJ, 418, 794
Yungelson L. R., Livio M., Tutukov A. V., Saffer R., 1994, ApJ, 420, 336
Yungelson L. R., Livio M., Tutukov A. V., Kenyon S. J., 1995, ApJ, 447, 656
Yungelson L. R., Livio M., Tutukov A. V., Truran J. W., Fedorova A. V., 1996, ApJ, in press
Zapolsky H.S., Salpeter E. E., 1969, ApJ, 158, 809