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Merging of binary white dwarfs, neutron stars and black holes under the influence of gravitational wave radiation

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ABSTRACT

By means of a numerical scenario code, we derive the rates of mergers of components of binary white dwarfs, neutron stars and black holes as functions of the ages of the progenitor populations. The rate of mergers of binary CO (or ONe) dwarfs with total mass of components $\gtrsim M_{\mathrm{Ch}}$ is consistent with the 'observed' rate of occurrence of Type Ia supernovae (SNeIa) in the Galaxy. We show that, for galaxies with continuing intense star formation, the history of star formation is not significant for the present rate of SNeIa, because about 60 per cent of SNeIa descend from populations younger than $\sim 10^9$ yr. Assuming that Type Ib and Type II supernovae (SNeIb and SNeII) descend, respectively, from compact helium remnants of initially massive ($M \gtrsim 10\ \mathrm{M}_\odot$) components of close binaries and from massive stars that managed to retain their extended hydrogen envelopes, we estimate their rates and derive linear relations between the rates of occurrence of SNeIa, SNeIb and SNeII that have the same trend along the portion of the Hubble sequence for non-elliptical galaxies as indicated by observations.

We compute the average distances that merging pairs with neutron star and/or black hole components can travel prior to mergers, and find that the overwhelming majority of merger events have to occur within the volume of the parental galaxies.

Key words: black hole physics – gravitation – methods: numerical – binaries: close – stars: neutron – supernovae: general.

1 INTRODUCTION

Gravitational wave radiation (GWR) may play an important role in the late stages of the evolution of close binaries. In particular, the merger of binary white dwarfs (WDs) under the influence of angular momentum loss via GWR, as first suggested by Tutukov & Yungelson (1979) and Webbink (1979), is currently considered as one of the most plausible scenarios for occurrence of SNe (for a detailed description of scenarios and analytical estimates, see Iben & Tutukov 1984a). Merging WDs may also be one of the most powerful sources of continuous gravitational wave radiation (Tutukov & Yungelson 1979). The merger of binary neutron stars is considered as one of the most promising sources of detectable bursts of GWR and neutrinos (Clark & Eardley 1977), and possibly γ-rays (Paczyński 1986; see, however, Woosley & Baron 1992).

In a recent series of papers (Tutukov, Yungelson & Iben 1992; Tutukov & Yungelson 1993a,b; Yungelson, Tutukov & Livio 1993; Yungelson et al. 1994), we have carried out population synthesis for binary stars of our Galaxy and for products of their evolution, including the formation of pre-SNeIa and merging pairs of neutron stars and/or black holes. To this end, we have used a numerical scenario code that combines data on the dependence of the birthrates of binaries on the masses of their components and on their separations with an analytical description of the evolution of binary and single stars. The latter is based on results of full-scale evolutionary computations (see, for details, the above-mentioned papers and references therein). The results of this modelling, relevant to the topic of present study, can be summarized as follows.

(i) If we identify the SNIa events with the merger of binary carbon–oxygen (CO) or oxygen–neon (ONe) white dwarfs with a total mass of components greater than the Chandrasekhar mass M_{Ch}, we are able to explain the rate of occurrence of SNeIa in a galaxy like ours (~ 0.001–0.003 per year per $10^{10}\ \mathrm{L}_\odot$ in the blue band).

(ii) The expected birthrate and number of double degenerate systems (DDs) in our Galaxy are consistent with the results of surveys which attempted to discover DDs. In particular, our model suggests the discovery of 5–6 DDs in the range of orbital periods from 0.5 h to 3 d, where Saffer, Liebert & Olszewski (1988), Bragaglia et al. (1990) and Bragaglia, Greggio & Renzini (1994) have discovered two certain and three 'candidate' DDs. The model also explains the null results of the surveys of Robinson & Shafter (1987) and Foss, Wade & Green (1991) as being due to an insufficiency of the samples of WDs explored by them for binarity in the period ranges where the DDs are very short-lived. We suggest the orbital period range 2–30 h as one in which the searches for DDs may be successful.

(iii) The estimate of the frequency of mergers of binary neutron stars (NSs) and black holes (BHs) in our Galaxy, when extrapolated to the Metagalaxy, suggests that, with the gravitational wave detectors now under development, one may hope to detect up to ~ 100 neutron star mergers and up to ~ 0.5 black hole mergers per year at distances $\lesssim 200$ Mpc. However, due to the high masses of black holes and therefore higher energy release by mergers, most numerous detectable GWR bursts can be generated by merging BH + BH binaries.

In relation to (i) and (ii) above, we have to note that our results indicate that the gravitational energy that is released during the common-envelope process is efficiently deposited into envelope ejection (i.e. the so-called common-envelope parameter $\alpha_{\rm ce}$ is ~ 1).

In the present paper, we extend the above-mentioned studies to explore the age dependence of the rate of occurrence of SNeIa and the relative frequency of occurrence of supernovae of different types (Section 2), and we explore the time dependence of mergers of neutron star/black hole binaries and their spatial distribution (Section 3).

2 THE AGE OF SNeIa AND THEIR RELATIVE FREQUENCY

2.1 The age of SNeIa

The occurrence of SNeI in both spiral and elliptical galaxies suggested that at least a proportion of their progenitors belong to very old populations ($\sim 10^{10}$ yr). In fact, Oemler & Tinsley (1979) have found evidence that a significant proportion of pre-SNeI belong to populations with ages of only 3×10^7–3×10^8 yr. Later, it was recognized that SNeI do not form a homogeneous class. Up to 50 per cent of them, classified as SNeIb (see e.g. van den Bergh & Tammann 1991; Cappellaro et al. 1993b), probably descend from helium remnants of massive ($M \gtrsim 10$ M$_\odot$) components of close binaries (see Tutukov et al. 1992 for a more thorough discussion). SNeIb do not occur in ellipticals, and almost certainly are related to the youngest stellar populations. The observed sample of SNeI, however, is dominated by SNeIa due to their higher brightness, and therefore the Oemler & Tinsley finding relates to SNeIa.

Different estimates of the rate of occurrence of SNeIa are in conflict. Van den Bergh & Tammann (1991) have found that it is almost constant along the sequence E–S0–Sa–Sb–Sd–Im. Cappellaro et al. (1993b) have found that it increases by a factor of 2 to 3 with the transition from E to Scd galaxies (see Fig. 3 below).

Tornambe' (1989) has attempted to get a theoretical estimate of the age dependence of the rate of CO + CO, CO + He and He + He white dwarf mergers under different assumptions on the common-envelope parameter $\alpha_{\rm ce}$. His results can be seen as giving the possible upper limit of the SNeIa rate due to white dwarf mergers.

In the present study we have computed the age dependence of the rate of occurrence of SNeIa. Our standard model assumes the following.

(i) All stars are born in binaries; within this model, components of 10 to 30 per cent of binaries merge in the course of evolution, giving birth to single stars.

(ii) One binary star with a mass of the primary component of between 0.8 and 100 M$_\odot$ is formed annually in the Galaxy; this corresponds to the observed rate of formation of planetary nebulae and white dwarfs. The birthrate function of binaries is (Popova, Tutukov & Yungelson 1982)

$$d^3\nu = 0.2\,d\log(A/R_\odot)\,M_1^{-2.5}\,dM_1 f(q)\,dq\ \mathrm{yr}^{-1}, \qquad (1)$$

with M_1 being the initial mass of the primary in M$_\odot$, q the initial mass ratio of the components, A the initial semimajor axis of the orbit, $f(q) = Cq^\alpha$ the initial distribution of binaries over q, and where $\int_0^1 f(q)\,dq = 1$. In the standard model we assume that $\alpha = 0$. With the given distribution of binaries over A, about 40 per cent of binaries are close, and the remaining ones are wide.

(iii) The common-envelope parameter $\alpha_{\rm ce} = 1$.

A thorough discussion of evolutionary scenarios leading to the formation of merging DDs in close binary systems was given by Iben & Tutukov (1984a). In fact, in the code we explore numerically these and several other evolutionary routes, with a fine spacing of the initial masses of the components, their mass ratios and separations.

In the present paper we again assume that SNeIa result from mergers of binary white dwarfs with CO (or ONe) components and with total mass exceeding $M_{\rm Ch}$. However, we recognize that the latter condition is neither necessary nor sufficient. First, in the process of merger the temperature of degenerate matter can rise high enough for thermonuclear explosion even for masses lower than $M_{\rm Ch}$. Secondly, a pair of white dwarfs with a total mass higher than $M_{\rm Ch}$ can lose part of its mass in the dynamical process of the merger and later, thus avoiding a SN explosion.

In Fig. 1(a) we show the contribution to the present annual rate of occurrence of SNeIa as a function of the age of the precursors under the assumption that the star formation rate was constant over the Hubble time of 1.5×10^{10} yr. The 'age of the precursors' includes the evolutionary stages prior to the formation of two WDs, plus the stage of 'delay' necessary for the merger of the WDs under angular momentum losses via GWR.

The age dependence of the rate of SNeIa for the case of an instantaneous star formation burst is shown in Fig. 1(b). To get the rate of SNeIa in an actual galaxy with a total mass of stars M, the rate given by Fig. 1(b) has to be multiplied by a factor $\frac{1}{2}(M/\mathrm{M}_\odot)(M_{\rm min}/\mathrm{M}_\odot)^{1/2}$, where $M_{\rm min}$ is the minimum mass of stars formed in the galaxy. (We assume here that the initial mass function is similar in all galaxies.)

Fig. 1(a) immediately shows that about half of all SNeIa occur in the $\sim(3$–$4)\times10^8$ yr after the birth of their precursors, in rough agreement with the findings of Oemler & Tinsley (1979). These 'young' pre-SNeIa with initial component masses 5–11 M$_\odot$ spend most of their lifetimes in the

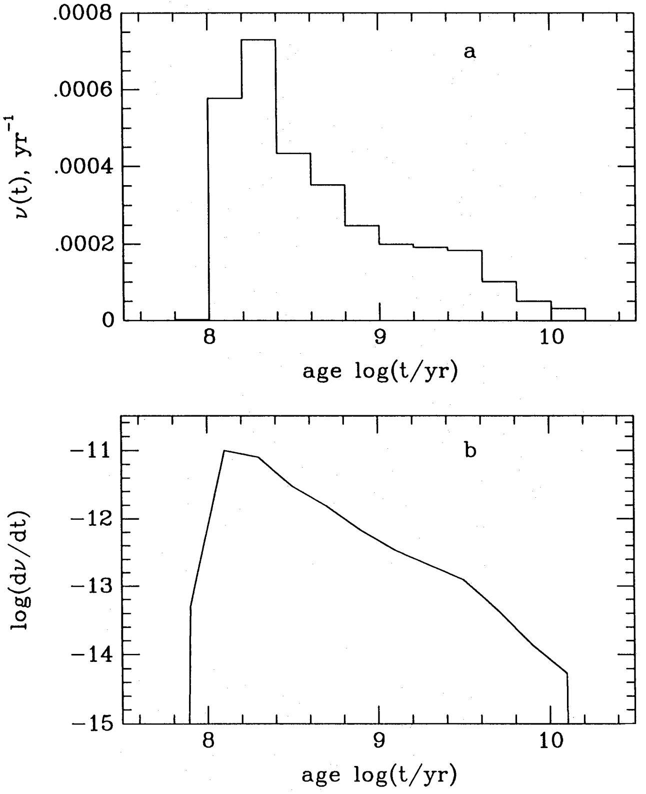

Figure 1. (a) The contribution to the present annual rate of occurrence of SNeIa as a function of the age of the precursor system for the case of continuous star formation. The rate is scaled to the formation of one binary star with the mass of the primary between 0.8 and 100 $M_\odot$. (b) The dependence of the rate of occurrence of SNeIa on age for the case of an instantaneous star formation burst. The rate is scaled to the instantaneous transformation of 1 $M_\odot$ of matter into binary stars with $0.8 \leq M_1 \leq 100$ $M_\odot$ and a Salpeter mass function of primaries.

main-sequence stage. About 30 per cent of SNeIa are really old: $\sim 10^9$–10^{10} yr. Their precursors spend most of their lifetimes in the 'delay' stage.

Fig. 1(a) shows that, for galaxies like our own, the influence of the old stellar population on the current rate of SNeIa is weak. Generally speaking, for massive spiral galaxies the history of star formation is still poorly known. It is quite possible that, in the early stages of their evolution, it was several times higher than at present, because with the present rate it is impossible to get the actual mass of the Galaxy (Gallagher, Hunter & Tutukov 1984). Let us assume that the bulk of the stars in the Galaxy were formed in the first 5×10^9 yr of its existence. Then, in order to get the actual mass of the Galaxy, we need to assume that the star formation rate in this period was ~ 10 times higher than at present. In the histogram in Fig. 1(a), this will result in a tenfold growth of the last bin. However, this will mean an increase in the current rate of occurrence of SNeIa by only ~ 10 per cent. Thus the estimates of the SNIa rate depend very little on the history of star formation, and can be obtained on the basis of the current star formation rate, if the latter has not varied significantly in the last $\sim (3$–$4) \times 10^8$ yr. For massive disc galaxies this is certainly the case, with the possible exception of their nuclei (Gallagher et al. 1984).

In Fig. 2 we show the dependence of the SNIa occurrence rate on the delay time after formation of the second WD in the system. Fig. 2 shows again that most mergers have ages of $\sim 10^8$ yr.

The age of a WD is related to its stellar magnitude. In absence of cooling calculations covering a wide enough range of masses and chemical compositions and having similar input physics (see e.g. D'Antona & Mazzitelli 1990 for a review), we assume for simplicity that all white dwarfs follow the relation between cooling time Δt and absolute bolometric magnitude M_b obtained by Iben & Tutukov (1984b). The latter can be approximated by a simple formula:

$$M_\mathrm{b}\,(\mathrm{mag}) \approx -16.7 + 3.17 \log(\Delta t),$$

where the cooling time is in years. It is evident that, while younger cooling dwarfs of pre-SNIa double degenerates immediately prior to the merger are distributed over a wide interval of M_b from 0.0 to 15.0 mag, most of them have $M_\mathrm{b} \approx 10.0$ mag. The difference in bolometric magnitude of the components of merging pairs immediately prior to the merger is up to ~ 6.0 mag; only 5 per cent of them may have double-lined spectra (if $\Delta M_\mathrm{b} \lesssim 1.0$ mag). (We neglect here the possible effect of tidal heating of strongly interacting stars immediately prior to merger.)

2.2 The relative frequency of SNeI and SNeII

The relative frequency of SNeIa and SNeII can be estimated by means of a simple model. To this end we can, as suggested by Figs 1(a) and (b), represent the stellar population of a galaxy by two components: 'old' stars with a total mass of M, born in the first 5×10^9 yr of the galaxy's history, and 'young' stars, born continuously with a rate $\dot{M}$ during last 10^{10} yr. Again, we assume that all stars are born in binaries. Both populations are assumed to have the same birthrate function, given by equation (1).

Let us remember that in our model we identify SNeIa with merging CO or ONe white dwarfs with total masses exceeding M_Ch, SNeIb with exploding remnants of massive ($M \gtrsim 10$ $M_\odot$) components of close binaries with compact hydrogen-deficient envelopes, and SNeII with exploding massive ($M \gtrsim 10$ $M_\odot$) stars that have retained their tenuous hydrogen envelopes to the instant of explosion. The rates of occurrence of SNe of different types under different assumptions on α_ce and the criterion of convective stability in the semiconvective zones are listed in Table 1. Columns labelled L correspond to the case when stars ignite and burn helium as red supergiants, while columns labelled S correspond to the case when they ignite He as blue supergiants and spend most of the core-He-burning stage in the blue supergiant region of the HR diagram. The difference arises from the application of either the Ledoux (L) or Schwarzschild (S) criterion for convective stability in the semiconvective zones of massive stars. The problem of the proper treatment of mixing in massive stars still is not solved. This difference between the cases L and S influences the relative frequency of SNeIb and SNeII events. As a 'standard' model, we consider the one corresponding to $\alpha_\mathrm{ce} = 1$ and the Schwarzschild criterion. Using the data given in Fig. 1(a) and Table 1 for the total rate of SNeIa in our Galaxy (~ 0.0031 yr^{-1}) and for the rate of SNeIa produced by 'old' stars [the last bin of the histogram ($\sim 0.000\,03$ yr^{-1})], one can obtain the following equation for the rate of occurrence of SNeIa:

$$\nu_{\rm Ia} \approx 3\times10^{-15}\left(\frac{M}{{\rm M}_\odot}\right)\left(\frac{M_{\rm min}}{{\rm M}_\odot}\right)^{1/2} + 0.0016\left(\frac{M_{\rm min}}{{\rm M}_\odot}\right)^{1/2}\dot{M}\,{\rm yr}^{-1}. \tag{2}$$

The first term on the right-hand side of equation (2) is due to the old population, while the second term is due to the young one ($\dot{M}$ is in units of $\rm M_\odot\ yr^{-1}$).

For the estimate of the current rate of star formation $\dot{M}$, we use the rate of occurrence of SNeII, which are, obviously, related to the young stellar population. According to our model, $\nu_{\rm II} \approx 0.014\ {\rm yr}^{-1}$. Using this value for the normalization, we obtain the following relation between $\nu_{\rm II}$, $M_{\rm min}$ and $\dot{M}$:

$$\nu_{\rm II} \approx 0.006\left(\frac{M_{\rm min}}{{\rm M}_\odot}\right)^{1/2}\dot{M}\,{\rm yr}^{-1}. \tag{3}$$

Usually the rate of occurrence of SNe in galaxies is expressed in 'supernova units' (SNU), giving the rate per century per $10^{10}\ {\rm L}_\odot$ in the blue band. Denoting SN rates in SNU as $(\nu_{\rm Ia}/L_B)$ and $(\nu_{\rm II}/L_B)$, and combining equations (2) and (3), we obtain

$$\left(\frac{\nu_{\rm Ia}}{L_B}\right) \approx 0.003\left(\frac{M_{\rm min}}{{\rm M}_\odot}\right)^{1/2}\left(\frac{M}{L_B}\right) + 0.23\left(\frac{\nu_{\rm II}}{L_B}\right). \tag{4}$$

Now, taking $(M/L_B) \approx 10$, typical for old elliptical galaxies (Tully 1988), and $M_{\rm min} = 0.2\ {\rm M}_\odot$ (Tutukov & Krügel 1980), we obtain

$$\left(\frac{\nu_{\rm Ia}}{L_B}\right) \approx 0.013 + 0.23\left(\frac{\nu_{\rm II}}{L_B}\right). \tag{5}$$

In Fig. 3 we show the relation given by equation (5) along with observational data (for $H = 75\ {\rm km\ s^{-1}\ Mpc^{-1}}$) from van den Bergh & Tammann (1991) and Cappellaro et al. (1993b). It is evident that the observational data from the two sources are highly uncertain and discrepant. This reflects the uncertainties in input parameters involved in the derivation of 'observed' rates of occurrence of SNe [see the discussions in van den Bergh & Tammann (1991) and Cappellaro et al. (1993a,b)]. Keeping in mind also the uncertainties in our model, we may note that the model relation (5) has the same trend as indicated by observations for non-elliptical galaxies. The reasons for the strong disagreement with the data for E and S0 galaxies are unclear.

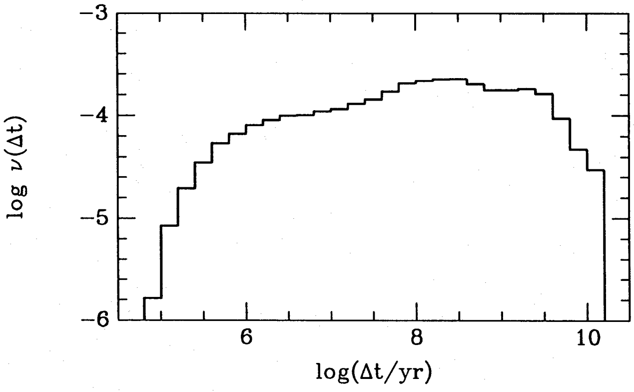

Figure 2. The dependence of the SNIa occurrence rate on the delay time after formation of the second WD in the system.

The variation of the common-envelope parameter $\alpha_{\rm ce}$ can improve the agreement with observations. If we increase the value of $\alpha_{\rm ce}$ to 2, the total rate of SNeIa increases by a factor of ~1.5 (Table 1), while the rate of mergers of pairs with age greater than 10^{10} yr increases by about a factor of 8. This happens because now more time is necessary for the merger of pairs. If we again assume that, in the first 5×10^9 yr of galactic history, the star formation rate was enhanced by a factor of ~10 and $\alpha_{\rm ce} = 2$, the total rate of SNeIa increases by ~50 per cent. Thus the history of star formation will still not be very important for the SNIa rate in galaxies with continuing star formation. With the latter changes, instead of equation (5) for $\alpha_{\rm ce} = 2$ we obtain

$$\left(\frac{\nu_{\rm Ia}}{L_B}\right) \approx 0.11 + 0.35\left(\frac{\nu_{\rm II}}{L_B}\right). \tag{6}$$

Table 1. The galactic rate of occurrence of SNe (in $\rm yr^{-1}$).

Type	α_{ce}=1 S	α_{ce}=1 L	α_{ce}=2 S	α_{ce}=2 L
SNIa	0.0031	0.0031	0.0046	0.0045
SNIb	0.0054	0.0073	0.0063	0.0083
SNII	0.014	0.012	0.016	0.014
SNIb/SNII	0.39	0.61	0.39	0.59

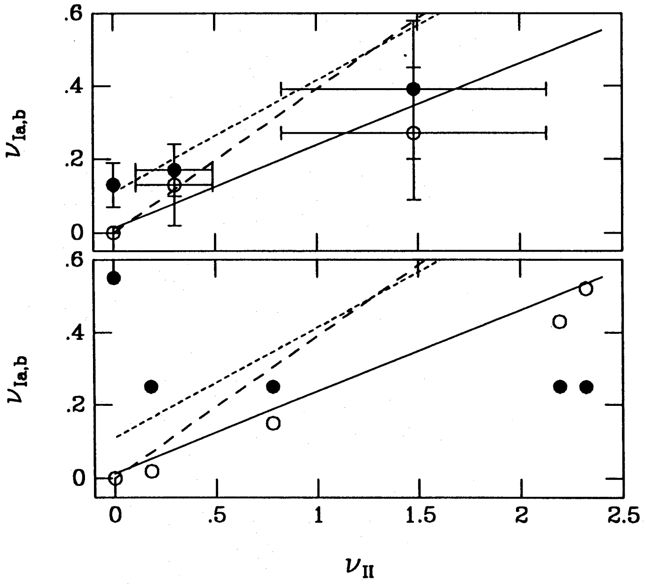

Figure 3. The relation between the rates of occurrence of SNeII and SNeI. Frequencies are given per 100 yr per $10^{10}\ {\rm L}_\odot$ in the blue band. Filled circles are SNeIa; open circles are SNeIb. In the upper panel, observational data from Cappellaro et al. (1993b) are plotted; in the lower panel, data from van den Bergh & Tammann (1991) are shown. The continuous line is the model relation between $\nu_{\rm II}$ and $\nu_{\rm Ia}$ for $\alpha_{\rm ce} = 1$; the short-dashed line shows the same relation for $\alpha_{\rm ce} = 2$; the long-dashed line shows the model relation between $\nu_{\rm II}$ and $\nu_{\rm Ib}$ for $\alpha_{\rm ce} = 1$.

The equation gives a rate of SNeIa that better agrees with the observational data of Cappellaro et al. (1993b) for elliptical galaxies with $(\nu_{\rm II}/L_B)=0$ (Fig. 3). The increase of $\alpha_{\rm ce}$ to a value of about 2 is quite reasonable, if we consider all the uncertainties in the evaluation of $\alpha_{\rm ce}$ based on energy conservation considerations, and allow the action of sources other than the liberation of orbital energy in the ejection of common envelopes (see a discussion of this point by, for example, Livio 1993). However, the discrepancy with van den Bergh & Tammann's data still remains. At present, we see no easy way to explain the extra-high (if real) SNIa occurrence rate in elliptical and S0 galaxies.

For the sake of completeness in comparing the predicted and observed SN rates, we add the relation between the rates of SNeIb and SNeII. In our standard model,

$$\left(\frac{\nu_{\rm Ib}}{L_B}\right)\approx 0.39\left(\frac{\nu_{\rm II}}{L_B}\right). \tag{7}$$

This relation is also plotted in Fig. 3 for $\alpha_{\rm ce}=1$ and the Schwarzschild criterion for convective stability. It shows the same trend as the correlation of SNII and SNIb rates indicated by the van den Bergh & Tammann set of data. The model relation of SNeII and SNeIb, both of which types descend from the same extremely young population, depends on assumptions on the position of helium-burning stars in the HR diagram and on the stellar wind mass loss, both of which are still rather uncertain. Table 1 provides evidence in favour of using the Schwarzschild criterion for convective stability, because use of the Ledoux criterion results, in our model, in overproduction of SNeIb.

It is worth mentioning that uncertainties in other model input parameters can lead to variations of predicted SN rates within a factor ~ 2 (see Table 1 and discussion in Tutukov et al. 1992).

On the observational side, possible reasons for the obvious discrepancy may be an overestimate of the Hubble constant, misclassification of SNe and of galaxies, or an underestimate of the M/L_B ratio.

Let us discuss briefly the possible influence of the uncertainty in the value of the Hubble constant. The blue luminosity of a galaxy with a constant star formation rate may be written as (Tutukov & Krügel 1980; Firmani & Tutukov 1993)

$$\frac{L_B}{\rm L_\odot}\approx 10^{10}\left(\frac{M_{\rm min}}{\rm M_\odot}\right)^{1/2}\dot{M}. \tag{8}$$

[Here, we take into account that $L_B\approx 0.5L_{\rm bol}$ (Firmani & Tutukov 1993) and $\dot{M}$ is in $\rm M_\odot\ yr^{-1}$.] Inserting equation (8) into equation (3), we get the limiting value of the SNII occurrence rate (in SNU):

$$\left(\frac{\nu_{\rm II}}{L_B}\right)\lesssim 0.6. \tag{9}$$

If the discrepancy between the maximal observed rate of SNeII and the limit given by equation (9) is real, its existence can mean that the Hubble constant has to be lowered to ~ 50 km $\rm s^{-1}$ $\rm Mpc^{-1}$. The latter value is consistent with recent determinations of the Hubble constant using SNeIa as 'standard candles' (Sandage & Tammann 1993; Müller & Höflich 1993). For the diagram in Fig. 3, this will result in the reduction of all 'observed' occurrence rates by a factor of ~ 2. Simultaneously, it will result in improved agreement of the observed maximal $(\nu_{\rm II}/L_B)$ value with the theoretical limit of 0.6. For data on elliptical galaxies from van den Bergh & Tammann (1991), this will reduce the discrepancy with the 'standard' model from a factor of ~ 10 to a factor of ~ 5, still not solving the problem.

Absorption by the dust in disc galaxies can reduce the observed L_B by a factor of ~ 2, thus increasing the limit given by equation (9) to ~ 2. This can bring into better agreement the maximal value of the observed $(\nu_{\rm II}/L_B)$ and the theoretical limit.

Another important aspect of the SNIa problem arises from the possibly significant scatter in the expansion velocities. According to Branch & van den Bergh (1993), the blueshift velocity v of the Si II λ6355 line at 10 d after maximum strongly correlates with the Hubble type of the parental galaxy. Seven SNeIa with $8000\lesssim v\lesssim 9500$ km $\rm s^{-1}$ occurred in E, S0 and Sa galaxies only, while 22 events with $9500\lesssim v\lesssim 14\,000$ km $\rm s^{-1}$ occurred in Sb, Sc, Sd and Irr galaxies. If this blueshift velocity reflects the expansion velocity of supernova shells, the systematized data on v allows us to suspect the existence of a systematic trend of $v_{\rm exp}$: old populations give SNeIa with low $v_{\rm exp}$,while young populations give SNeIa with higher $v_{\rm exp}$ and a greater spread of $v_{\rm exp}$. What can cause this difference? In the three panels of Fig. 4 we show the distribution of merging double degenerates over the sum of the masses of their components for three age intervals: $\log T\leq 8.5$, $8.5<\log T\leq 9$, and $\log T>9$. The three distinct groups of objects in Fig. 4 are formed by CO + CO, CO + He and He + He (or ONe) pairs, respectively. For pairs of CO (or ONe) dwarfs, the oldest merging ones have $M_1+M_2\approx 1.4$ $\rm M_\odot$, while the youngest have total masses exceeding ~ 2 $\rm M_\odot$.

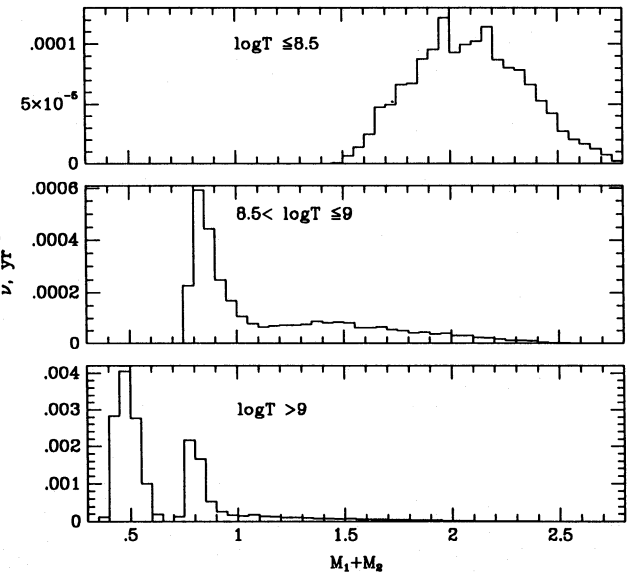

Figure 4. The distribution of merging double degenerates over the sum of the masses of their components for three age intervals: $\log T\leq 8.5$, $8.5<\log T\leq 9$ and $\log T>9$. The three groups of objects are, respectively, CO + CO ($M_{\rm t}\gtrsim 1$ $\rm M_\odot$), CO + He ($M_{\rm t}\approx 0.7$–1.0 $\rm M_\odot$) and He + He ($M_{\rm t}\lesssim 0.7$ $\rm M_\odot$) pairs.

1994MNRAS.268..871T

The total mass of an exploding object can control the mass of burned material, and in this way can influence the observed $v_{\rm exp}$. At present, however, the models of exploding accreting dwarfs do not indicate how (and if at all) the difference in masses of exploding objects can result in a difference of expansion velocities.

We cannot at present exclude the possibility that progenitors of SNeIa in elliptical and spiral galaxies have different chemical compositions. Fig. 4 shows that, with the transition from young to old stellar populations, in merging pairs first CO + He pairs and then He + He pairs begin to dominate. At present, however, the model calculations of explosions of accreting He dwarfs result in expansion velocities of $\sim 18\,000$ km s^{-1} (Woosley, Taam & Weaver 1986), much higher than observed in SNeIa exploding in elliptical galaxies (~ 8500 km s^{-1}).

The four upper panels of Fig. 5 show the contribution to the present rate of mergers of double degenerates by systems with different ages and different chemical compositions [for completeness, we duplicate Fig. 1(a) in the upper panel]. Since the main portion of the time interval between the formation of a binary and the merger of the degenerate remnants of the components is due to the main-sequence lifetime of the components, Fig. 5 shows the systematic growth of this interval with the decrease of the total mass of the merging pairs. This provides an explanation of the dependence shown in Fig. 4.

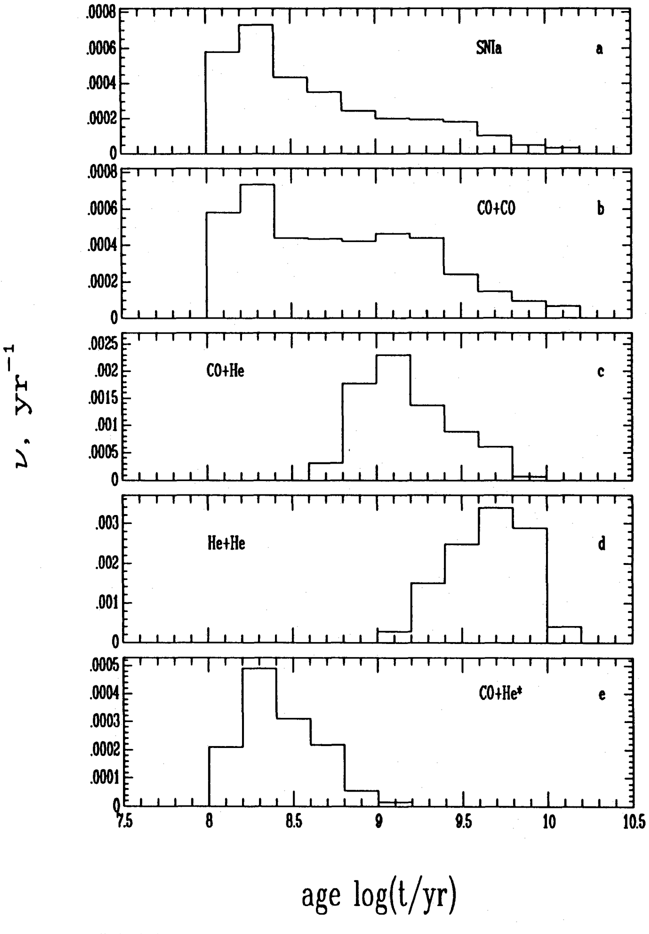

Figure 5. (a)–(d) The contribution to the present annual rate of mergers as the function of the age of the precursors for systems with different chemical compositions of the components. (a) CO + CO (or ONe) pairs with $M_1 + M_2 \geq M_{\rm Ch}$, (b) all CO + CO pairs, (c) CO + He pairs, (d) He + He pairs. (e) The contribution–age relation for He-shell explosions in CO + He-star pairs. The scaling is as in Fig. 1(a). For the case of an instantaneous star formation burst, the figure gives the history of mergers.

The lowest panel of Fig. 5 shows the contribution of stars of different ages to the model results for the present rate of explosions of degenerate helium shells on the surfaces of accreting CO (ONe) dwarfs in close semidetached binaries with non-degenerate helium donors, evolving under the influence of angular momentum loss via GWR. In this case the supernova-scale events are possible if a helium shell of ~ 0.15 M$_\odot$ is accumulated on the surface of a degenerate dwarf with mass exceeding ~ 0.8 M$_\odot$ (Iben & Tutukov 1991). Helium detonation may result in a high $v_{\rm exp}$, observed for SNeIa in spiral galaxies. However, (i) it is still unclear if explosions of this kind can be identified with SNeIa, and (ii) their rate (~ 0.001 yr^{-1}) comprises only one-third of the rate of CO + CO (or ONe) mergers.

Thus the explanation of the systematic dependence of $v_{\rm exp}$ on the age of the progenitor population represents a very important problem. Its solution can provide a very useful discriminator of theoretical models of SNe.

2.3 Merging double degenerates as emitters of GWR

The ensemble of merging CO and He double degenerates represents the most powerful source of continuous gravitational wave radiation in the Galaxy at frequencies $\nu \gtrsim 0.001$ Hz (at lower frequencies, the spectrum of GWR is dominated by W UMa stars and cataclysmic variables). The mergers of pairs of He dwarfs occur about three times more frequently than SNIa events (in the standard model). The average mass of merging He dwarfs, however, is about a factor of 4 lower than the mass of merging CO dwarfs. This makes the total radiated energy about a factor of 16 lower than in the CO + CO case. CO and He dwarfs (0.6 + 0.25 M$_\odot$) merge about twice as frequently as CO + CO dwarfs, but the total radiated energy is about a factor of 6 lower than in the CO + CO case. Thus pre-SNIa pairs are the most powerful sources of GWR.

The merging time of a pair of WDs with equal masses of components M and semimajor axis of the orbit A is

$$T_{\rm GRW} \approx 8 \times 10^7 \left(\frac{A}{{\rm R}_\odot}\right)^4 \left(\frac{{\rm M}_\odot}{M}\right)^3 \ {\rm yr}. \qquad (10)$$

Since the average time between two subsequent SNIa events in our Galaxy is ~ 300 yr and the average mass of a merging dwarf is ~ 1 M$_\odot$, the present orbital period of the next pre-SNIa pair is ~ 1 min, and the semimajor axis of its orbit is $A \approx 0.044$ R$_\odot$. Curiously enough, the power of GWR from such a pair, $\sim 3 \times 10^5$ L$_\odot$, is comparable to the optical luminosities of the brightest stars of the Galaxy and much higher than the luminosities of the brightest stars of elliptical galaxies. The almost constant frequency of GWR (with a timescale of variation of ~ 100 yr) raises the possibility of detection of such systems by resonant detectors of GWR.

3 AGE AND SPACE DISTRIBUTIONS OF MERGING NEUTRON STAR AND BLACK HOLE BINARIES

The existence of short-orbital-period binary pulsars like PSR 1913 + 16 ($P_{\rm orb} \approx 0.32$ d) and PSR 2127 + 11C ($P_{\rm orb} \approx 0.34$ d) clearly shows the possibility of the merger of binary neutron stars in a Hubble time. The merger events may possibly produce bursts of GWR and γ-ray bursts. The rate of occurrence of mergers of NS + NS binaries in the Galaxy in the 'standard' model is 3×10^{-4} yr^{-1}, that of BH + NS binaries is 1.5×10^{-5} yr^{-1}, and that of BH + BH binaries is 1.4×10^{-6} yr^{-1}. The merger of two NSs results in the emission of $\sim 3 \times 10^{53}$ erg in a few milliseconds (equation 10). The system 'next' to merge now has an orbital period of ~ 3 min and an emission power of $\sim 3 \times 10^4$ L$_\odot$. Within binary star population synthesis, it is possible to study the dependence of the merger rate of neutron star and black hole (NS/BH) binaries on time. Fig. 6 gives the dependences of the merger rates of NS + NS, BH + NS and BH + BH binaries on the age of the system in cases of continuous star formation (Fig. 6a) and of an instantaneous burst of star formation (Fig. 6b). In the latter case, for a galaxy with a total mass of stars M, the rates given by the figure, as in the case of merging WDs, have to be scaled by a factor $\frac{1}{2}(M/{\rm M}_\odot)(M_{\rm min}/{\rm M}_\odot)^{1/2}$. The 'age of the system' we define here as the time interval between the formation of the younger NS or BH in the system and the merger event, Δt, because in most cases Δt is much longer than the pre-SN lifetimes of the precursors of NSs and BHs, which are lower than $\sim 2 \times 10^7$ yr. In the case of continuous star formation with a constant rate, Fig. 6(a) gives the ages of the binaries currently merging. For an instantaneous burst of star formation, Fig. 6(b) gives the number of events at different times after the burst. The maximal frequency of mergers of NS + NS pairs is reached in $\sim 2 \times 10^7$ yr after their formation. For BH + NS and BH + BH pairs the time intervals prior to the beginning of intense mergers are respectively $\sim 4 \times 10^7$ and $\sim 2 \times 10^9$ yr after formation. Later on, for NS + NS and BH + NS systems the merger rate falls slightly slower than t^{-1}, while for BH + BH systems it is almost constant. The rate of decrease of merger rate reflects the post-formation distribution of pairs over $P_{\rm orb}$ (see fig. 1 of Tutukov & Yungelson 1993a).

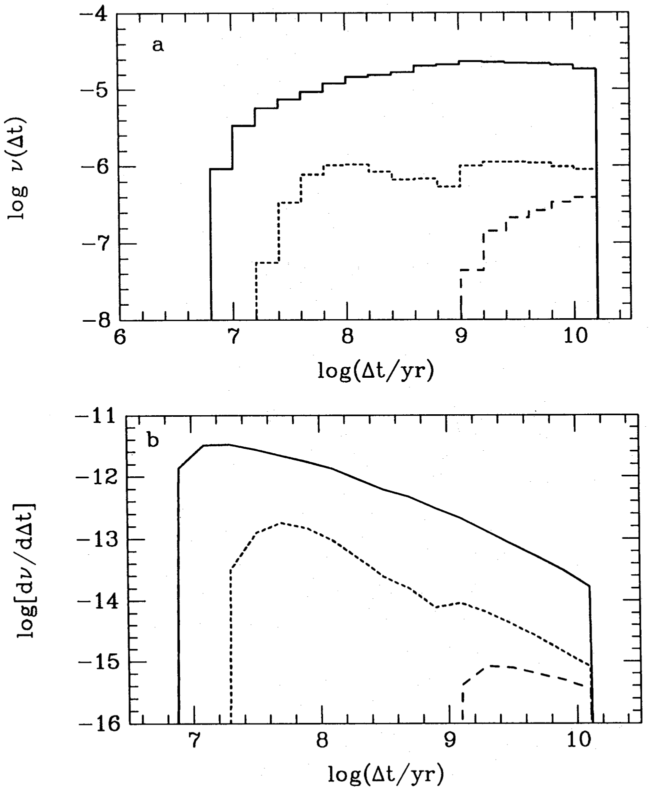

Figure 6. (a) The contributions to the present annual rate of mergers of NS + NS pairs (continuous line), BH + NS pairs (short-dashed line) and BH + BH pairs (long-dashed line) as functions of the delay between the formation of the pair and the merger event for the case of continuous star formation and $\alpha_{\rm ce} = 1$. (b) The dependence of the rate of occurrence of mergers of NS/BH pairs on the delay time for the case of an instantaneous star formation burst and $\alpha_{\rm ce} = 1$. The rate is scaled as in Fig. 1.

The dependences in Fig. 6 are more sensitive to the star formation history than those for pre-SNeIa. If, as above, we assume that in the first 5×10^9 yr of the galactic history the star formation rate was about ten times higher than presently, the rate of mergers of NS + NS pairs will increase by about a factor of 2, and for BH + NS systems by an even higher factor.

The dependence of the occurrence rate of mergers of NS/BH binaries is also rather sensitive to variations of $\alpha_{\rm ce}$. For example, a change of the common-envelope parameter to $\alpha_{\rm ce} = 0.5$ significantly reduces the separations of components of NS/BH binaries. This results in a significant reduction of time delay between the formation of the system and the merger of components (Fig. 7): for example, the mergers of NS + NS pairs start less than $\sim 2 \times 10^6$ yr after formation. The time behaviour of the merger rate is almost independent of $\alpha_{\rm ce}$.

Phinney (1991) and Narayan, Piran & Shemi (1991), using data on several observed binary radio pulsars able to merge in a Hubble time, have estimated the rate of their merger as $\sim 10^{-6}$–10^{-5} yr^{-1}. This rate formally agrees with the rate of merger of young ($\Delta t \lesssim 3 \times 10^7$ yr) binary neutron stars in our 'standard' model (Fig. 6). More detailed discussion of this point at present is inhibited by insufficient knowledge of the genesis of pulsars and their evolution. The commonly accepted assumption that the age of a radio pulsar may be estimated by $p/2\dot{p}$ is probably not always correct, as is shown by the example of J0437 − 4715 (Danziger, Baade & Della Valle 1993): the characteristic age of the pulsar is ~ 0.8 Gyr, while the cooling time of its companion is ~ 6 Gyr.

The NS/BH binaries can have high spatial velocities as a result of two supernova explosions in the progenitor system. We consider the supernova explosions as spherically symmetric, because it is possible to explain the observed high spatial velocities of pulsars without assumption of a 'kick' accompanying their birth (Tutukov, Chugai & Yungelson 1984). The assumption of a 'kick' is not necessary for close systems, because the second SN explosion disrupts almost all of them. To disrupt wide binaries, a small kick would be sufficient. Radio pulsars in the latter systems may be absent because of the slow rotation of neutron stars with progenitors, which are components of wide systems. In the treatment of disruption probabilities of the systems due to SN explosions, we followed Hills (1983). Spatial velocities can be so

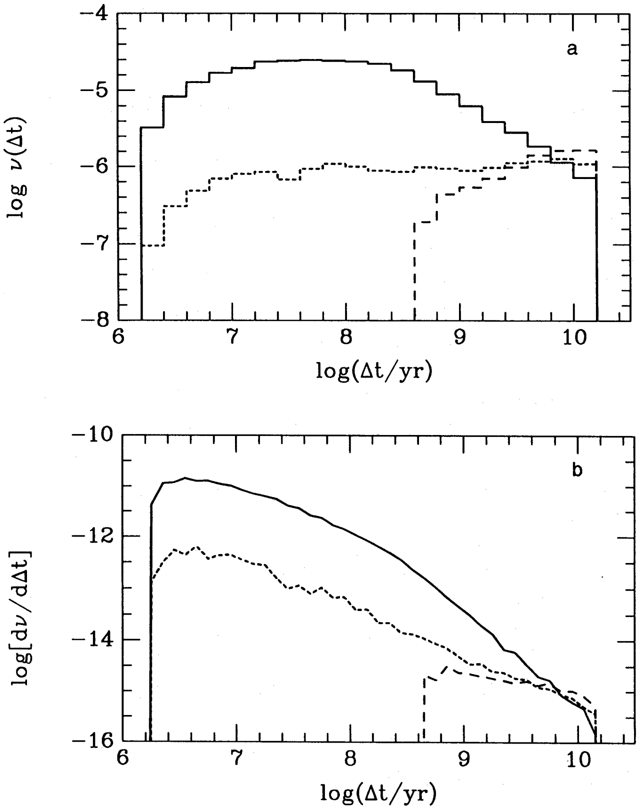

Figure 7. As in Fig. 6, but for $\alpha_{ce} = 0.5$.

high that merging NS + NS binaries can leave their parental galaxies. This problem is of particular interest in relation to the possible association of merging NS + NS binaries with γ-ray bursts (Paczyński 1986). For a simple estimate of the average distance R that NS/BH binaries can travel prior to merger, we assume that $R \approx \sqrt{(v_1\tau_1)^2 + (v_2\tau_2)^2}$, where v_1 and v_2 are the spatial velocities of the system after the first and the second SN explosions, respectively, τ_1 is the time interval between the first and the second SN explosions, and τ_2 is the lifetime of the NS/BH binary. In order to take into account the influence of the gravitational field of the parental galaxy, which can 'trap' the pair, we set equal to zero part of a track or the whole track if v_1 or v_2 are lower than 100 km s^{-1}. The maximal pre-merger travel distances of NS/BH binaries are shown in Fig. 8. This figure clearly shows that the overwhelming majority of all merging pairs remain confined to their parental galaxies with typical sizes of ~10 kpc. It is worth noting that the fastest moving systems have the shortest tracks. This is easily understandable, because a high spatial velocity implies both small pre-explosion and, especially, small post-explosion semimajor axes of the orbit of the binary. The latter implies a short pre-merger lifetime.

The value of the escape velocity limit of 100 km s^{-1} assumed by us is typical for dwarf blue galaxies with masses below 10^{10} M$_\odot$ (Gallagher et al. 1984). For giant galaxies like our own, the escape velocity is ~400 km s^{-1} (Allen 1973). Such high escape velocities almost exclude NS/BH mergers outside parental galaxies, and all positions of γ-ray bursts in this case, if they occur at all, have to be confined to the optical images of distant galaxies.

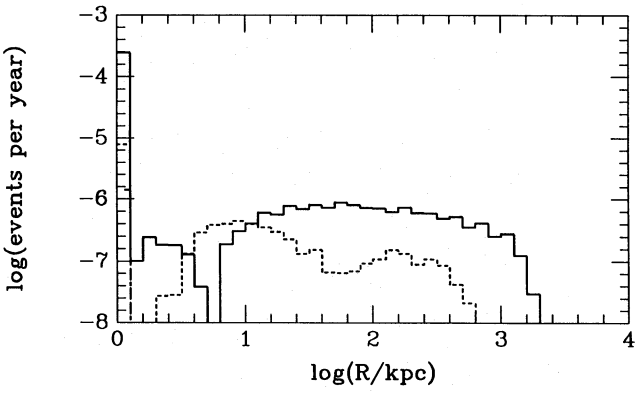

Figure 8. The relation between the rate of occurrence of mergers of NS/BH binaries and the average distance of travel prior to the merger, for $\alpha_{ce} = 1$. R for NS + NS pairs is shown by the continuous line, for BH + NS pairs by the short-dashed line and for BH + BH pairs by the long-dashed line.

Single neutron stars produced by close binaries that have been disrupted by SN explosions can have spatial velocities in excess of 100 km s^{-1} and sometimes even higher, up to ~1000 km s^{-1} (Tutukov & Yungelson 1993b). Such high velocities are actually observed for radio pulsars (Cordes, Romani & Lundgren 1993; Harrison, Lyne & Anderson 1993). As a result, single neutron stars can generate a certain type of evaporating 'wind' around the parental galaxies. Since even the fastest NSs have spatial velocities comparable to the escape velocities for giant galaxies, neutron stars will outflow preferentially in the equatorial direction due to 'rotational acceleration'. They can travel distances of up to ~10 Mpc. In this way, some of the fastest old neutron stars can form a common field in a cluster of galaxies. Most of the neutron stars, however, remain confined to the parental galaxies.

Apart from mergers of binary white dwarfs and binary neutron stars, mergers of NS + WD pairs are also possible. In our standard model, neutron stars merge with He white dwarfs at a rate of ~0.14×10^{-4} yr^{-1}, and with CO/ONe white dwarfs at a rate of ~0.1×10^{-2} yr^{-1}. The physics of such mergers is completely uninvestigated. One may only speculate that some kind of Thorne–Żytkow star (Thorne & Żytkow 1975) may form as an outcome of such a merger.

4 CONCLUSIONS

(1) We have studied the age dependence of the rate of occurrence of SNeIa. We have shown that, for the galaxies with continuing intense star formation, the latter only weakly depends on the history of star formation. About two-thirds of the SNeIa in galaxies like our own are produced by stars with ages of less than 10^9 yr, while the rest result from older systems. A rough two-component model of the population of pre-SNeIa (young and old) based on this finding is, for non-elliptical galaxies, in qualitative agreement with observational data on SNIa occurrence rates (Cappellaro et al. 1993b).

(2) The expansion velocity of SN shells may be an important selection factor for models attempting to explain the SNIa phenomenon. It is feasible that the class of possible pre-supernovae has to include objects with significant dispersion of masses. The creation of such objects can result from

mergers of double degenerates under the influence of angular momentum loss via GWR.

(3) We have studied the age dependence of the rate of mergers of pairs composed of neutron stars and/or black holes. This rate is much more sensitive to the history of star formation than is the SNIa rate. We have found that most mergers have to occur within the parental galaxies, because the spatial velocities acquired by NS/BH pairs due to the SN explosions of their progenitors are as a rule not high enough to overcome the potential barrier of the gravity.

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