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The merger rate of neutron star and black hole binaries

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ABSTRACT

We estimate the frequency of mergers of binary neutron stars and black holes by using a numerical scenario computer program to model the binary star population of the Galaxy. An extrapolation of the estimates shows that, with the gravitational wave detectors now under development, one may hope to detect up to ~100 neutron star mergers per year at distances ≤ 200 Mpc. The calculated merger rates based on binary pulsar statistics probably underestimate this detection rate, because of the short lifetimes of most new-born neutron star binaries. The rate of detection of black hole mergers may be comparable to that of neutron star mergers because of the strong dependence of the power of outbursts on the masses of merging objects.

Key words: black hole physics – radiation mechanisms: gravitational – binaries: close – stars: neutron.

1 INTRODUCTION

Several gravitational wave observatory projects are now under development (e.g. Vogt 1991). Among the main objects to be observed are binaries with neutron star and/or black hole components, merging because of the orbital energy and angular momentum loss due to gravitational wave radiation (GWR, Clark & Eardley 1977). The merger events are of outstanding importance because their detection allows the determination, with extreme accuracy, of parameters such as the Hubble constant, the masses of the merging components and the distance to the merging stars (Schutz 1986). They also provide important information about the late stages of binary star evolution. In this paper we show, by means of a model of the binary star population of the Galaxy based on the scenario approach to the description of stellar evolution, that one may hope to detect ~100 mergers of binary neutron stars per year from our own and nearby galaxies ($r \leq 200$ Mpc). At the same level of detector sensitivity, the rate of detection of binary black hole mergers may be even higher.

2 METHOD OF CALCULATION

The rate of merger of neutron star (NS) binaries is usually estimated by means of single and binary pulsar statistics and supernova rates (e.g. Clark, van den Heuvel & Sutantyo 1979; Phinney 1991; Narayan, Piran & Shemi 1991) or by making certain assumptions about the distribution of new-born compact binaries over orbital periods (Hils, Bender & Webbink 1990). For our Galaxy, estimates of merger rates thus obtained range from 10^{-6} to 10^{-4} yr^{-1}.

The essence of our approach is as follows. The present birthrate of binaries may be written as

$$\mathrm{d}^3\nu = 0.2\,\mathrm{d}\log(A/\mathrm{R}_\odot)\, M_1^{-2.5}\,\mathrm{d}M_1 f(q)\,\mathrm{d}q \ \mathrm{yr}^{-1} \qquad (1)$$

(Popova, Tutukov & Yungelson 1982), where M_1 is the mass of the primary in $\mathrm{M}_\odot$, q is the mass ratio of the components, A is the semimajor axis of the orbit, $f(q) = Cq^{\alpha}$ is the initial distribution of binaries over q, and $\int_0^1 f(q)\,\mathrm{d}q = 1$. The function $f(q)$ is still uncertain and we treat α as one of the free parameters. Equation (1) implies that all stars are born in binaries with $10 \leq A/\mathrm{R}_\odot \leq 10^6$. The integral of equation (1) corresponds to the formation of one binary with $M_1 \geq 0.8\ \mathrm{M}_\odot$ per year in the Galaxy. We assume that the star formation rate has been constant throughout the Galactic lifetime. This is probably true within a factor of about 2 for the last 5×10^9 years at least (Miller & Scalo 1978; see, however, Gallagher, Hunter & Tutukov 1984). One may take a set of initial parameters (M_{10}, A_0, q_0) and follow all evolutionary transformations of M_1, A and q by means of analytical approximations to the results of full-scale evolutionary computations, accounting for all mass and angular momentum loss/exchange episodes. Thus one obtains the parameters of stars of various types produced by this 'initial binary'. Evolutionary scenarios considered by us include, in certain cases, up to 12 stages and/or transformations. By exploring the whole permitted space of M_{10}, A_0 and q_0, and combining ν from equation (1) and the lifetimes of stars in particular stages, one obtains the numbers of different stars in the Galaxy and their distributions over observed parameters. This method has already been successfully applied by us to supernovae (Tutukov, Yungelson & Iben 1992), Wolf–Rayet (WR) stars (Tutukov & Yungelson

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1991; Vrancken et al. 1991), white dwarfs (Tutukov & Yungelson 1992a,b) and neutron stars (Tutukov & Yungelson 1993). It also allows one to obtain quantitative evolutionary scenarios for particular systems with well-known orbits and masses of the components. The construction of scenarios makes it possible to place limits on certain free parameters that inevitably enter the modelling.

There have been several attempts to model the population of massive evolved stars in the Galaxy and their descendants, including neutron stars, with special attention given to the spatial velocities of the latter (e.g. Kornilov & Lipunov 1983a,b; Dewey & Cordes 1987; Bailes 1989). In these studies, the parameters of 'initial binaries' and the ages of stars were obtained by Monte Carlo simulations. In contrast to these studies, our method does not require simulations of millions of systems in order to obtain statistically reliable results for rare but nevertheless observed systems, e.g. massive X-ray sources. Meurs & van den Heuvel (1989) performed integrations over a range of initial masses of stars and mass ratios, thus obtaining the numbers of different binaries, but they did not consider the distribution of stars as a function of the separation of the components.

The most complex and unexplored process involved in the evolutionary scenarios of compact binary formation is the evolution inside the common envelope. Its outcome depends on the efficiency $\alpha_{\rm ce}$ of the orbital energy expenditure on the dispersal of the common envelope. In the limit of completely non-conservative evolution (Tutukov & Yungelson 1979),

$$(M_1+M_2)(M_1-M_{1R})/A_0=\alpha_{\rm ce}M_{1R}M_2(1/A_{\rm f}-1/A_0), \qquad (2)$$

where M_{1R} is the final mass of the donor, and A_0 and $A_{\rm f}$ are the initial and final semi-axes of the orbit. One can limit the possible range of $\alpha_{\rm ce}$ by trying to reproduce the parameters of well-studied binaries. Our attempts to construct quantitative scenarios of the origin of close binaries that have certainly passed through several common-envelope stages [such as the double-line binary white dwarf L870-2 (Saffer, Liebert & Olszewski 1988), V651 Mon – binary core of a planetary nebula with a known period and main-sequence component mass (Mendez & Niemela 1981), and binary pulsars] have resulted in $0.6 \leq \alpha_{\rm ce} \leq 1$. High values of $\alpha_{\rm ce}$ are also suggested by results of the modelling of the population of planetary nebulae with short-period binary cores (de Kool 1990), as well as by numerical modelling of common-envelope evolution (Livio & Soker 1988; Taam & Bodenheimer 1989). For the semidetached stages occurring on nuclear time-scales of the donor, we assume that mass is exchanged conservatively, while for semidetached stages occurring on thermal time-scales we assume that the fraction of accreted matter is proportional to the ratio of the Kelvin time-scales of the components. The excess matter is assumed to be lost, with A changing according to equation (2).

We consider the set of binaries with $0.8 \leq M_1/{\rm M}_\odot \leq 100$, $10 \leq A/{\rm R}_\odot \leq 10^6$ and $0<q<1$, and explore it with steps of 0.0125 in $\log(M_1)$, 0.025 in q and 0.05 in $\log(A)$. In close binaries (CB), components exchange matter, while in wide ones they evolve independently. The limit of initial major semi-axes of orbits for CB with main-sequence components depends on mass and lies at several $\times$ 1000 ${\rm R}_\odot$. In the CB that are of interest for the present study, NS are assumed to descend from stars with initial masses of $11.4 \leq M/{\rm M}_\odot \leq 40$. This choice of M for NS precursors provides a reasonable agreement between predicted and observed numbers of pulsars and massive X-ray binaries. Stars with $M > 40$ ${\rm M}_\odot$ are assumed to produce black holes (BH). This latter limit is also suggested by the relative numbers of Galactic massive X-ray binaries and WR stars with suspected NS and BH components, as well as by results of attempts to model the evolutionary scenarios of binary systems that are suspected to contain black holes: LMC X-3 (van den Heuvel & Habets 1984) and A0620 − 00 (de Kool, van den Heuvel & Pylyser 1987). Unfortunately, purely theoretical considerations still do not allow one to determine this important limit. If the mass of the precursor of a BH is assumed to be higher than ≈ 50 ${\rm M}_\odot$, merging BH + BH or BH + NS binaries cannot form: the heavy stellar wind mass-loss removal of orbital momentum in the Jeans mode dominates Roche lobe overflow (RLOF) and the separation between components only increases in the course of evolution. For the mass of a BH we take 5 ${\rm M}_\odot$ to be a minimum value, as suggested by the masses of unseen components in observed X-ray binaries (see table 1 of Cherepashchuk & Tutukov 1992). This value is one of the major sources of uncertainty in the estimate of the frequency of BH + NS and BH + BH mergers, because it influences the probabilities of disruption by supernova explosions and the number of detectable events (see below). Additional details of the modelling are given elsewhere (Tutukov & Yungelson 1991, 1992a,b, 1993; Vrancken et al. 1991; Tutukov et al. 1992).

The evolutionary scenarios of the formation of NS and/or BH binaries have been described many times, starting with the papers by van den Heuvel & Heise (1972) and Tutukov & Yungelson (1973). Without going into detail, we illustrate them in Table 1 with the scenario for formation of a binary pulsar with parameters reasonably close to those of PSR 1913 + 16 (see line 11 of Table 1).

3 RESULTS AND DISCUSSION

Our basic model with $\alpha = 0$ in equation (1) and $\alpha_{\rm ce} = 1$ in equation (2) gives a relative frequency of formation of young

Table 1. A scenario of formation of a pulsar such as PSR 1913 + 16.

N	*Stage*	M_1	M_2	$\log(A/R_\odot)$	*P, day*	*e*	V_s, *km/s*	*T*, 10^6 *yrs*
1	MS+MS	13.1	9.8	2.14	38.8	0.	11	13.0
2	RLOF	6.7	12.5	2.02	28.0	0.	11	0.015
3	He+MS	3.3	15.4	1.92	20.0	0.	11	1.4
4	SN (Ib?)							
5	NS+MS	1.4	15.4	1.97	25.3	0.12	23	8.3
6	X-Ray bin.	1.4	15.4	1.96	24.8	0.	23	0.004
7	RLOF	1.4	8.3	1.22	2.5	0.	23	0.0003
8	MS+He	1.4	4.2	0.50	0.31	0.	23	1.0
9	RLOF	1.4	3.6	0.31	0.15	0.	23	0.0016
10	SN (Ib?)							
11	**NS+NS**	**1.4**	**1.4**	**0.44**	**0.31**	**0.62**	**157**	**740.0**
12	Merger	2.7					2710	

Notes. Masses are in ${\rm M}_\odot$. MS: main-sequence star; RLOF: Roche lobe overflow stage; SN: supernova explosion; He: helium core star with thin hydrogen envelope; $V_{\rm s}$: spatial velocity; e: eccentricity. For stages 2, 3, 7, 8, 9 and 11 the time-averaged values of masses, semi-axes and period are given. During stage 6, circularization of the orbit occurs. During the merger event, the 0.1-${\rm M}_\odot$ remnant of the NS completely disrupts (Blinnikov et al. 1984, 1990); the orbital velocity of the merger product (BH?) transforms into spatial velocity.

Table 2. The merger rates of NS and BH binaries (yr^{-1}).

Type of the system	NS+NS	BH+NS	BH+BH
Merger rate in the Galaxy	$3.2\cdot10^{-4}$	$1.5\cdot10^{-5}$	$1.4\cdot10^{-6}$
Merger rate in the Universe	$1.2\cdot10^{6}$	$6.0\cdot10^{4}$	$6.0\cdot10^{3}$
Rate of events from $r \le 200 Mpc$	100	5	0.5

close NS + NS systems of ~ 1/100. Under the assumption of equal lifetimes of young single and binary pulsars, one can then expect to observe ~ 5 binary radio pulsars with strong ($B\approx10^{12}$ G) magnetic fields, while only one binary pulsar with $B\approx6\times10^{11}$ G and suspected NS companion is presently known (PSR 2303 + 46). The reason for this discrepancy is unclear. It may be due to our overestimate of the number of NS that remain bound in binaries, because we assume that during the supernova explosion the NS does not receive a 'kick'. This assumption enables us to fit the theoretical distribution of young single NS over spatial velocities to the observed distribution of pulsars (Tutukov & Yungelson 1993). Another reason for the deficit of strong-field binary pulsars may be the systematically weakened magnetic fields of NS that are born in binaries. Finally, one must also bear in mind that the statistics of binary pulsars are still very poor.

The computed rates of mergers in the Galaxy (for $T_{\mathrm{Hubble}}=1.5\times10^{10}$ yr) are given in the second row of Table 2. A variation of α from -1 to $+1$ and of α_{ce} from 1 to 0.5 changes these numbers by factors of 2 to 3. Our numbers for NS + NS systems are close to these determined by Clark et al. (1979). For BH + BH and BH + NS mergers, our present estimates are several times lower than earlier ones (Tutukov et al. 1992). This reflects the strong dependence of the formation rate of merging BH on the lower mass limit of their precursors and the fine details of the mass and orbital momentum loss from the systems.

Prior to coalescence, NS + NS binaries can travel up to $r\approx1000$ kpc from their birthplaces. The GWR bursts must be distributed uniformly over $\log(r/\mathrm{kpc})$ from 1 to 2.5.

Fig. 1 shows the birthrate–orbital period and number–orbital period relations for NS + NS, BH + NS and BH + BH systems (note that P here is averaged over the system lifetime; the periods of these systems immediately after formation are about 20 per cent higher). The fast decline of N below ≈ 1 d is due to mergers caused by GWR. This figure clearly shows that the overwhelming majority of NS + NS systems escape potential detection as binary pulsars because of their short lifetimes as binaries. The calculated values of merger rates based on observed pulsar statistics (e.g. Phinney 1991; Narayan et al. 1991) are therefore greatly underestimated.

The birthrate of binary pulsars should be proportional to the formation rate of massive stars. If, following Phinney (1991), we assume that throughout the whole Universe the rate of formation of stars of $M\ge10\,\mathrm{M}_\odot$ is proportional to the formation rate of 1–3 $\mathrm{M}_\odot$ stars and that the stellar initial mass function, proportion of binaries and evolution of stars are similar throughout the Universe, the total rate of mergers must be equal to the Galactic rate R_{Gal} multiplied by the ratio of the B-band luminosities of the Universe and the Galaxy: $R_{\mathrm{tot}}\approx10^{-2}\,R_{\mathrm{Gal}}H\,\mathrm{Mpc}^{-3}\,\mathrm{yr}^{-1}$, H being the Hubble constant in units of 100 km s^{-1} Mpc^{-1}. This results in the numbers given in the third row of Table 2. Note that our estimates are obtained under the assumption of a constant star formation rate. They may be higher if the latter was enhanced in the early history of galaxies.

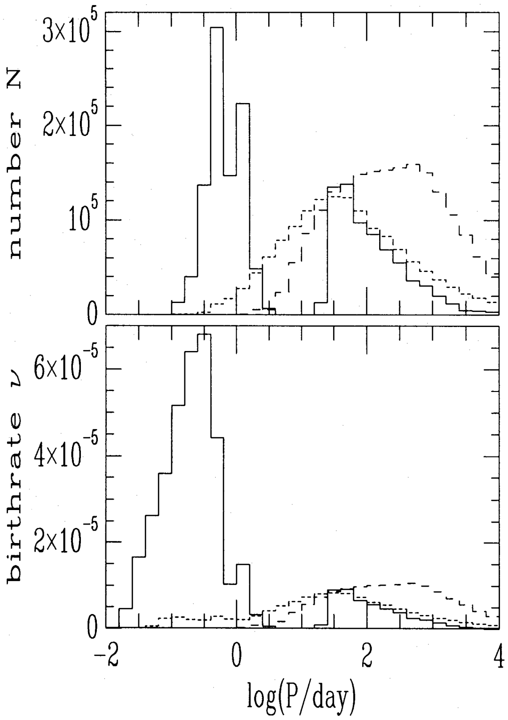

Figure 1. Birthrate–orbital period and number–orbital period distributions for NS + NS (solid line), BH + NS (short dashes) and BH + BH (long dashes) systems descending from close binaries.

The best estimates for first-generation Laser Interferometer Gravitational Wave Observatory (LIGO) projects give a detection limit for NS + NS merger events that is close to $r\approx200$ Mpc (Abramovici et al. 1992). For this value of r the rate of detectable events is given in the last row of Table 2. However, the measure of detectability of gravitational waves is the 'characteristic amplitude' of the wave, $h_{\mathrm{c}}=h\sqrt{n}$, where h is the dimensionless strain and n is the number of cycles that the waves spend near a frequency of 100 Hz. According to Thorne (1987) and Abramovici et al. (1992),

$$h_{\mathrm{c}}=5.8\times10^{-22}\,\nu_{100}^{-1/6}(\mu/\mathrm{M}_\odot)^{1/2}(M/\mathrm{M}_\odot)^{1/3}(100\,\mathrm{Mpc}/r), \tag{3}$$

where ν_{100} is the frequency of GW in units of 100 Hz, μ is the reduced mass and M is the total mass. Therefore, for binaries with components of equal mass and fixed h_{c}, the volume of space producing detectable events scales as $(M/\mathrm{M}_\odot)^{2.5}$. Thus, for a black hole mass $M_{\mathrm{BH}}=5\,\mathrm{M}_\odot$ and a neutron star mass $M_{\mathrm{NS}}=1.4\,\mathrm{M}_\odot$, BH + BH mergers are detectable in a volume that is a factor of about 24 larger than the corresponding volume for NS + NS mergers. However, the typical mass of black holes may be as high as 10 $\mathrm{M}_\odot$. For the same limiting h_{c}, therefore, the detection rate of BH + BH mergers may be comparable to that of NS + NS mergers, if BH + BH binaries are really formed as frequently as our model estimates.

4 CONCLUSION

Paczyński (1986) suggested that γ-ray bursts are located at cosmological distances, and that they may be caused, among other reasons, by NS + NS mergers. The first suggestion is strongly supported by the isotropic distribution of weak γ-ray bursts over the sky. Can γ-ray bursts be related to the stellar mergers discussed in the present paper? If one assumes that the decline of the slope of the number–flux relation at 10^{-5}–10^{-4} erg cm^{-2} (Cline 1984) is due to the attaining of the cosmological horizon at $r \approx 4600$ Mpc, then the frequency of γ-ray bursts with energy flux exceeding 10^{-5} erg cm^{-2} is about 30 yr^{-1}. Mao & Paczyński (1992) estimate the rate of γ-ray bursts in the same volume to be ~ 3000 yr^{-1}. Both estimates are much lower than our estimates of the frequency of NS + NS and BH + BH mergers (see Table 2). This discrepancy may imply that γ-ray radiation is strongly beamed or that γ-ray bursts are not associated with NS + NS (see Woosley & Baron 1992) or BH + BH mergers.

Apart from the production of bursts of GW and neutrinos (Clark & Eardley 1977), γ-rays (Paczyński 1986) and ejection of *r*-process elements into the interstellar medium (ISM) (Eichler et al. 1989), the coalescence of NS may have one more observational signature. When the mass of the NS, in the course of tidal stripping, is reduced to approximately 0.1 M$_\odot$, it explodes because of decompression (Blinnikov et al. 1984, 1990; Eichler et al. 1989). This explosion can produce a long-lived hot blob in the ISM (Igumenschev, Tutukov & Shustov 1992). However, it is presently unclear how to distinguish such blobs from ordinary supernova remnants.

The estimates of merger rates given in Table 2 are of course rather uncertain. They depend strongly on intricate details of the evolution of close binaries, especially inside common envelopes, and certain assumptions in our numerical code, as well as on the history of star formation in the Universe. Nevertheless, one may look with moderate optimism into the future of LIGOs.

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