Degenerate dwarfs in binary systems

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A model of the set of white dwarfs that have been formed in binary systems in the Galaxy is constructed, based on a stellar evolution scenario. The number of helium, carbon–oxygen, and oxygen–neon white dwarfs is estimated under various assumptions about the initial distribution of binaries with respect to the mass ratio of the components and the efficiency of conversion of orbital energy into the energy of ejection of matter in the common-envelope stage. It is shown that up to ~24% of all stars that begin life as binaries end it as single objects. It is found that ~80% of all white dwarfs more massive than 0.65 $M_\odot$ should be single products of the merging of components. Only ~10% of degenerate dwarfs have radial velocities with semiamplitudes of variation $K_1 \geq 100$ km/sec and less than 1% have orbital periods $P \leq 3$ h.

1. INTRODUCTION

Degenerate dwarfs are one of the most important components of the Galaxy's stellar population, so they yield irreplacable information about stellar evolution. The evolution of single stars and the components of binary systems with an initial mass less than ~10 $M_\odot$ ends with the formation of degenerate dwarfs, as a rule. On the other hand, in addition to single white dwarfs, two close binary degenerate dwarfs[1,2] and six astrometric pairs of white dwarfs[3] have been observed. Detection of the companions of white dwarfs is a complex observational task, so the true degree of their duplicity is unknown. The set of white dwarfs that have been formed in binary systems will be analyzed in the present paper.

Theory and some observations indicate the possibility that three classes of degenerate dwarfs exist, differing in the chemical composition of their interior and their average mass: helium, carbon–oxygen, and oxygen–neon dwarfs. Most dwarfs with a mass less than ~0.45 $M_\odot$ must consist of helium, since helium burning is impossible in lower-mass objects, according to our present understanding. The average mass of carbon–oxygen dwarfs is ~0.6 $M_\odot$, and the mass of oxygen–neon dwarfs exceeds ~1.1 $M_\odot$ (Ref. 4). The existence of carbon–oxygen and possibly of oxygen–neon dwarfs is confirmed by an analysis of the chemical composition of the shells of novae, which reveal considerable excesses of C, N, O, Ne, and Mg (see, e.g., the summary in Ref. 5).

Details of the numerical investigation of the evolution of binaries in the scenario approach have been given in Ref. 6. Here we confine ourselves to giving the most general outlines of the essence of that method. The rate of formation of binary systems in the Galaxy is given by the star-formation function, which we take in the form[7]

$$d^3\nu = 0.2d\log a\, M_1^{-2.5}dM_1\, f(q)dq,\ \mathrm{yr}^{-1}, \tag{1}$$

where $10 \leq a/R_\odot \leq 10^6$ is the semimajor axis of the orbit, $0.8 \leq M_1/M_\odot \leq 100$ is the mass of the primary component, and $0 < q \leq 1$ is the mass ratio of the components. The function (1) was obtained through a detailed statistical study of eclipsing, spectroscopic, and visual binaries in star catalogs. The most uncertain component of this function is the initial distribution of binaries with respect to the masses of the components, which we take in the form $f(q) = Cq\alpha$ $[\int_0^1 f(q)dq = 1]$. The quantity α was varied from −1 to +1 to investigate its influence on the parameters of the models. (For brevity, we shall characterize specific models by the quantity α below.)

In essence, the algorithm entails a systematic examination of the evolutionary status of the components of a system, the changes in their masses M_1 and M_2 and the semimajor axis of the orbit a, with allowance for mass transfer between the components and the loss of mass and orbital angular momentum by the system. The evolution of the system is monitored from a pair of main sequence stars to the formation of a pair of "final" objects that are noninteracting on the cosmic time scale – neutron stars and/or white dwarfs – or destruction of the components in supernova explosions. The possibility that the components merge and the possibility that the system decays as it loses mass are taken into account. The number of objects in each stage, assuming steady-state star formation, is the product of the star-formation rate and the lifetime of a system in the given stage.

Equation (1) predicts that approximately one binary system with a primary of mass >0.8 $M_\odot$ is born per year in the Galaxy, which is essentially the same as the birth rate of stars of such mass in the Galaxy. In the steady state the birth rate of potentially detectable white dwarfs is somewhat lower, since some dwarfs are considerably fainter than their companions – main sequence stars of mass 0.3-0.8 $M_\odot$.

We assumed, in accordance with evolutionary calculations, that degenerate dwarfs are formed in the evolution of stars with initial masses 0.8-10 $M_\odot$ in wide systems or 0.8-11.4 $M_\odot$ in close systems. The boundary between close and wide systems in terms of distance between components depends on mass, and is close to 1000 $R_\odot$.

The description of the evolution of binaries depends on a number of parameters, the most important of which is the common-envelope parameter α_{CE}, characterizing the efficiency of conversion of the orbital energy of the components into the energy of ejection of the common envelope in accordance with the equation[8]

$$\frac{G(M_1 + M_2)\Delta M}{a_0} = \alpha_{\mathrm{CE}} G M_{1\mathrm{R}} M_2 \left(\frac{1}{a_f} - \frac{1}{a_0}\right), \tag{2}$$

TABLE I. Classification of Binary Stars

2 1	He	CO	ONe	MS	N	NS	BH	RG	RG(He)	AGB	Hes	H don	He don
He	+	+	x	+	+	x	x	—	—	x	—	—	x
CO	+	+	+	+	+	—	x	—	—	—	—	—	—
ONe	+	+	+	+	+	—	x	—	—	—	—	—	—
MS	x	x	x	—	—	x	x	x	x	x	x	x	x
NS	—	—	—	—	—	—	—	—	—	—	—	—	—
BH	—	—	—	—	—	—	—	—	x	—	—	x	—
RG	x	x	x	—	—	x	x	—	—	x	x	x	x
RG(He)	x	x	x	—	—	x	x	—	—	x	x	x	x
AGB	x	x	x	—	—	x	x	—	—	—	x	x	x
Hes	x	—	—	—	—	x	x	—	—	—	—	—	—
H don	x	x	x	—	x	x	x	x	x	x	x	—	x
He don	x	x	x	—	x	x	x	x	x	x	—	x	—

Note. 1) Primary, initially more massive component; 2) secondary, initially less massive component. Notation: He) degenerate helium dwarf; CO) degenerate carbon–oxygen dwarf; ONe) degenerate oxygen–neon dwarf; MS) main sequence star with a mass less than 0.3 $M_\odot$; N) no companion; NS) neutron star; BH) black hole; RG) red giant with a degenerate helium core; RG(He) red giant in the stage of helium burning in the core; AGB) star on the asymptotic giant branch; Hes) nondegenerate helium star; H don, He don) cataclysmic system with a hydrogen or helium donor. Symbols: +) systems discussed in this paper; –) systems proposed to be considered in subsequent papers; ×) systems that cannot originate in the ordinary evolution of the components of binary systems.

where M_1 and M_2 are the initial masses of the components, M_{1R} is the mass of the remnant of the primary, ΔM is the mass of the common envelope ($\Delta M = M_1 - M_{1R}$), and a_0 and a_f are the initial and final values of the semimajor axis of the system's orbit. Reliable theoretical arguments for estimating α_{CE} do not yet exist because of the complexity of the problem; existing calculations indicate that the conversion efficiency is high.[9,10] The most promising way to estimate it now is to construct evolutionary scenarios for well-studied systems that have passed through one or several common-envelope phases. Outstanding among these are the binary nucleus of the planetary nebula V 651 Mon and the close binary white dwarf L 870–2. Those systems will be discussed in more detail in Sec. 3.

We note a drawback of our method of estimating the frequencies of formation and the populations of degenerate dwarfs of different types – the assumption that the rate of star formation in the Galaxy is constant over its entire life. In reality, that rate, as in other massive spiral galaxies, was probably several times higher than the average at early phases of evolution.[11] Our approach thus probably underestimates the current frequency of formation of degenerate dwarfs that are products of the evolution of systems with components of initial mass ~0.8 $M_\odot$, since the lifetime of such stars is comparable to the age of the Galaxy itself. In later papers we shall consider the influence of the rate of star formation in the Galaxy on the frequency of birth of degenerate dwarfs.

2. RESULTS

More than 200 qualitatively different scenarios for the evolution of close binaries are generated in our program. The basic combinations of components encountered in the scenarios are given in Table I. In the present paper we were confined to the consideration of single white dwarfs that are the products of merging of close binary systems consisting of two white dwarfs and of detached binaries in which the white dwarf is brighter than its companion, a main sequence star. The following criterion was used here. The brightest dwarfs in the sky have luminosity ~10^{-2} $L_\odot$ (Ref. 4), so they are the ones that are represented most completely in catalogs and statistics. Main sequence stars with mass ~0.3 $M_\odot$ have such a luminosity. Dwarfs are therefore detectable in systems in which the companion is a main sequence star of mass $\lesssim 0.3$ $M_\odot$. Such pairs were included in our statistics.

Almost any combination of two degenerate dwarfs is an allowed, "legal" product of the evolution of binaries. The only exceptions are systems in which the dwarf that was formed first (a helium dwarf) might be the companion of an oxygen–neon dwarf. For a helium dwarf to be formed, the mass of the primary must be $\lesssim 2.8$ $M_\odot$, and for an oxygen–neon dwarf to be formed, it must be $\gtrsim 9$ $M_\odot$. It is obvious that even evolution that is conservative with respect to mass is incapable of producing such a pair. We assume that in systems of two white dwarfs one can, as a rule, observe only one of them, the younger and second to be formed.

Besides the eleven varieties of systems of binary white dwarfs and white dwarfs with low-mass companions, and the three varieties of single objects, white dwarfs can appear in pairs with other stars, in which they are the fainter members of the system (see Table I). The combined population of such systems is relatively low, but their investigation, which we shall pursue in later papers, is also of considerable interest. In close systems consisting of a dwarf and a main sequence star, for example, the latter may accumulate in its envelope some fraction of the matter from the initially more massive component, which may appreciably alter the chemical composition of the envelope. This may be the mechanism that produces barium stars.[12-14] Systems containing (super)giants and white dwarfs may, under certain conditions, appear as symbiotic stars, etc.

Most of the scenarios generated in the program contain degenerate dwarfs, either as intermediate stages or as end products. Some of those scenarios are complicated, and include up to 13 phases. The most productive in numbers of dwarfs formed are the following:

TABLE II. Formation Rates (upper row) and Numbers of Degenerate Dwarfs of Different Types (lower row) in Our Galaxy

He + He	CO + CO	ONe + ONe	He + CO	CO + He	ONe + He	CO + ONe	ONe + CO	WD + MS $M_{MS} < 03M_\odot$	Single stars
				$\alpha = 0$	$\alpha_{CE} = 1$				
0,029	0,20	$0{,}85\cdot10^{-4}$	$0{,}44\cdot10^{-2}$	0,036	$0{,}14\cdot10^{-3}$	$0{,}47\cdot10^{-3}$	$0{,}42\cdot10^{-3}$	0,178	0,19
$0{,}13\cdot10^{9}$	$0{,}2\cdot10^{10}$	$0{,}42\cdot10^{4}$	$0{,}55\cdot10^{8}$	$0{,}24\cdot10^{9}$	$0{,}12\cdot10^{7}$	$0{,}12\cdot10^{7}$	$0{,}58\cdot10^{6}$	$0{,}15\cdot10^{9}$	$0{,}2\cdot10^{10}$
				$\alpha = -1$	$\alpha_{CE} = 1$				
0,016	0,14	$0{,}43\cdot10^{-4}$	$0{,}25\cdot10^{-2}$	0,028	$0{,}33\cdot10^{-3}$	$0{,}23\cdot10^{-3}$	$0{,}31\cdot10^{-3}$	0,27	0,19
$0{,}67\cdot10^{8}$	0,13·1010	$0{,}20\cdot10^{4}$	$0{,}30\cdot10^{8}$	$0{,}17\cdot10^{9}$	$0{,}12\cdot10^{7}$	$0{,}57\cdot10^{6}$	$0{,}6\cdot10^{6}$	$0{,}21\cdot10^{9}$	$0{,}20\cdot10^{10}$
				$\alpha = 0$	$\alpha_{CE} = 0{,}5$				
0,021	0,20	$0{,}11\cdot10^{-4}$	$0{,}44\cdot10^{-2}$	0,031	$0{,}61\cdot10^{-4}$	$0{,}27\cdot10^{-3}$	$0{,}69\cdot10^{-4}$	0,17	0,24
$0{,}43\cdot10^{8}$	0,19·1010	$0{,}27\cdot10^{1}$	$0{,}22\cdot10^{8}$	$0{,}13\cdot10^{9}$	$0{,}66\cdot10^{5}$	$0{,}39\cdot10^{6}$	$0{,}72\cdot10^{3}$	$0{,}14\cdot10^{9}$	$0{,}24\cdot10^{10}$
				$\alpha = 1$	$\alpha_{CE} = 1$				
0,049	0,29	$0{,}15\cdot10^{-3}$	$0{,}73\cdot10^{-2}$	0,046	$0{,}53\cdot10^{-4}$	$0{,}86\cdot10^{-3}$	$0{,}55\cdot10^{-3}$	0,05	0,19
$0{,}22\cdot10^{9}$	$0{,}29\cdot10^{10}$	$0{,}8\cdot10^{4}$	$0{,}93\cdot10^{8}$	$0{,}32\cdot10^{9}$	$0{,}12\cdot10^{6}$	$0{,}24\cdot10^{7}$	$0{,}56\cdot10^{6}$	$0{,}36\cdot10^{9}$	$0{,}2\cdot10^{10}$

1. MS + MS → RLF1 + MS → Hed + MS → Hed + RLF2 → CE → → Hed + Hed (COd),
2. MS + MS → CE → Hed + MS → CE → Hed + Hed(COd),
3. MS + MS → RLF1 + MS → Hes + MS → COd + MS → CE → → COd + Hed(COd),
4. MS + MS → CE → CO(ONed)+MS → CE → COd(ONed) + Hed(COd),
5. MS + MS → CE → MS → COd,
6. MS + MS → COd + MS → COd + COd.

Here MS is a main sequence star, Hed, COd, and ONed are the respective degenerate dwarfs, Hes is a nondegenerate helium star, CE is a common envelope, RLF is Roche lobe filling. Scenarios 1-5 are typical of close binary systems and 6 of wide ones. Scenarios 1-4 for the closest systems often have a continuation leading to merging of the components as a result of loss of angular momentum by the system due to gravitational wave emission. Some of the systems merge in stages with a common envelope.

Degenerate dwarfs are usually subdivided into types DA, DB, DC, etc. based on the appearance of their spectra. That division indicates mainly the differences in chemical composition of the dwarfs' atmospheres. We, however, divide dwarfs based on the chemical composition of their interiors, which reflects differences in their origin. This complicates the comparison of our results with observations, and probably only the masses of the observed dwarfs can serve to some extent as a guide in differentiating them by internal chemical composition. The distribution of the systems by chemical types is given in Table II. We are considering systems that follow different sequences in the formation of dwarfs of different types, since we assume, as mentioned above, that the brighter dwarf will always be the later to be formed, and only it will be detectable. In reality, the question of the relative brightness of the degenerate components is complicated, and its answer depends entirely on the still unstudied interaction of an old dwarf with a common envelope.

Table II implies the rather unexpected conclusion that 30-36% of all degenerate dwarfs turn out to be single or, in other words, are formed by the merging of components. Recall that our original star-formation function (1) assumes all of the stars to be binaries. The following are the main evolutionary channels leading to the formation of single objects: convergence of two main sequence stars as a result of the loss of orbital angular momentum by magnetic stellar wind, merging in common envelopes that originate in accretion onto main sequence stars and white dwarfs, and convergence of white dwarfs as a result of momentum loss by gravitational wave emission. It is noteworthy that for a fixed α_{CE}, the fraction of merging systems does not depend on the initial q distribution of the stars. A decrease in α_{CE} naturally leads to an increase in the number of systems that merge in common envelopes.

The most common of the potentially detectable binary objects are binary carbon–oxygen dwarfs and dwarfs paired with $\lesssim 0.3\ M_\odot$ main sequence stars. Dwarfs are born in pairs with $\gtrsim 0.3\ M_\odot$ main sequence stars at a rate of about 0.4 per year. Degenerate dwarfs in such systems are normally undetectable, because of their low luminosity; Sirius B is a rare exception.

From Table II it also follows that the birthrates and populations of different dwarfs depend relatively little on the basic parameters of the scenario program. The overall birthrate of detectable dwarfs is 0.63-0.76 per year, in satisfactory agreement with their observed birthrate,[4] ~0.5 per year.

Let us consider further the mass function for degenerate dwarfs. Recall that in each pair of dwarfs we choose the younger, since it is the brighter. In Figs. 1a-4a we give the mass distribution of degenerate dwarfs – single ones and components of binary systems – from our calculations. We can identify three main components of the mass function. The first is $\lesssim 0.5\ M_\odot$ dwarfs, which are mainly helium dwarfs in close systems whose components have not merged on the cosmological time scale due to gravitational wave emission. The second component, the main one, is $\sim 0.6\ M_\odot$ carbon–oxygen dwarfs, in accordance with the relationship between the initial and final masses of the stars, which we chose from Ref. 15:

$$\lg(M_{co}/M_\odot) = \begin{cases} -0.22 + 0.36\,[\lg(M_i/M_\odot)]^{2.5}, & \lg(M_i/M_\odot) > 0 \\ -0.22 - 0.36\,|\,[\lg(M_i/M_\odot)]\,|^{2.5}, & \lg(M_i/M_\odot) \le 0. \end{cases}$$

Finally, there is a small local maximum in the mass function at $\sim 0.8\ M_\odot$ (Figs. 1a-4a), due mainly to single

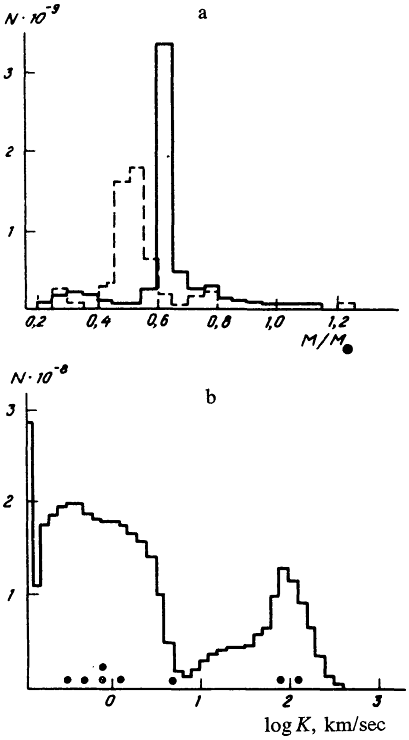

FIG. 1. Histograms of the distribution of white dwarfs. a) Mass spectrum of white dwarfs. Solid curve: theoretical model; dashed curve: observational data[16]; b) theoretical spectrum of the maximum semiamplitudes of the radial velocities of white dwarfs. Points: positions of observed binary white dwarfs.[1-3] Case of $\alpha = 0$ and $\alpha_{CE} = 1$.

dwarfs that are the products of merging of carbon–oxygen and helium dwarfs. Our analysis showed that ~80% of all $\gtrsim 0.65\ M_\odot$ dwarfs are single, i.e., were either produced by the merging of degenerate dwarfs or dwarfs and ordinary stars, or are the products of the evolution of objects formed by merging before the white dwarf stage was reached. The latter channel (in the scenario with α_0, $\alpha_{CE} = 1$) generates ~75% of the single objects; among the rest, ~6% are white dwarfs from cataclysmic systems in which the donors were completely destroyed, ~5% are dwarfs formed in systems that decayed due to supernova explosions or rapid mass ejection in the concluding stage of evolution on the asymptotic giant branch. The remaining ~14% result from mergers of dwarfs. For other combinations of α and α_{CE}, those proportions differ, since they depend on the mass ratio of the components in the original systems and the degree of decrease in the semimajor axis of the orbit in common-envelope stages.

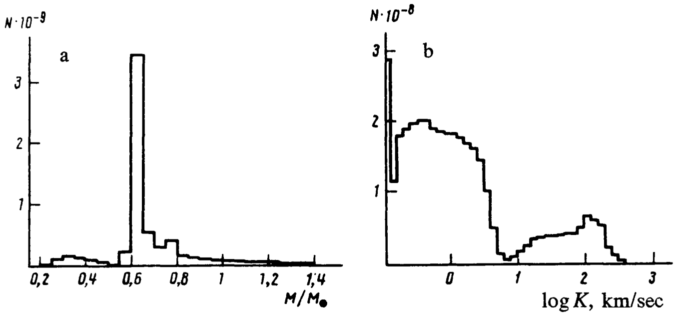

FIG. 2. Same as in Fig. 1, for $\alpha = 0$ and $\alpha_{CE} = 0.5$.

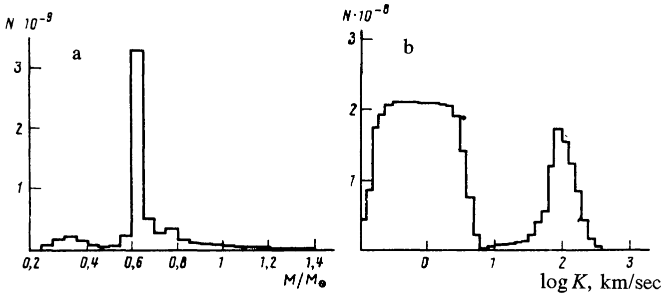

FIG. 3. Same as in Fig. 1, for $\alpha = 1$ and $\alpha_{CE} = 1$.

Our calculations yield the total number of dwarfs formed in the Galaxy over $15 \cdot 10^9$ years, which is given in the figures. Calculations of the cooling of white dwarfs show, however, that they are observable only for the first ~10^8 years of life.[4] That factor, together with other selection effects, must be taken into account when comparing with observations.

The mass distribution of the observable degenerate dwarfs has yet to be accurately determined. Even estimates of the average mass of dwarfs are still uncertain and range from $0.53\ M_\odot$ (Ref. 16) to $0.60\ M_\odot$ (Ref. 17) and even $0.75\ M_\odot$ (Ref. 18). As an example, in Fig. 1a we plot the observed mass function for 120 DA dwarfs,[6] which comprise 90% of all dwarfs and are therefore fairly representative. The observed mass function was normalized so that the areas under the histograms are equal. In the observed distribution, as in the theoretical one, a group of stars with masses 0.45-0.55 $M_\odot$ stands out; it probably corresponds to the group of ~0.6 $M_\odot$ carbon–oxygen dwarfs in the theoretical mass function. Two other groups of dwarfs with average masses ~0.3 $M_\odot$ and ~0.8 $M_\odot$ are also notable. The first group can be identified preliminarily with degenerate helium dwarfs and the second with carbon–oxygen dwarfs that are merger products of the components of close binaries in previous phases of evolution. Additional study is needed to decide conclusively whether the latter two maxima are present in the mass distribution of observed dwarfs. Differences in the lifetimes of degenerate dwarfs at different phases of cooling as a function of mass, which can also alter the model of their observed mass distribution, must be taken into account, in particular. As for the observations, estimates of the masses of dwarfs with $M \lesssim 0.5\ M_\odot$ from the gravitational acceleration g at their surface may be underestimates, since when the apparent magnitude of a dwarf reaches the usual sample limit $V \approx 15^m$-16^m, its radius is still considerably greater than the theoretical radius of a cold white dwarf, which is normally used

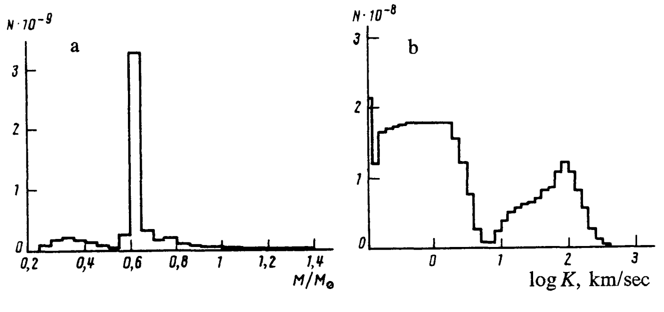

FIG. 4. Same as in Fig. 1, for $\alpha = -1$ and $\alpha_{CE} = 1$.

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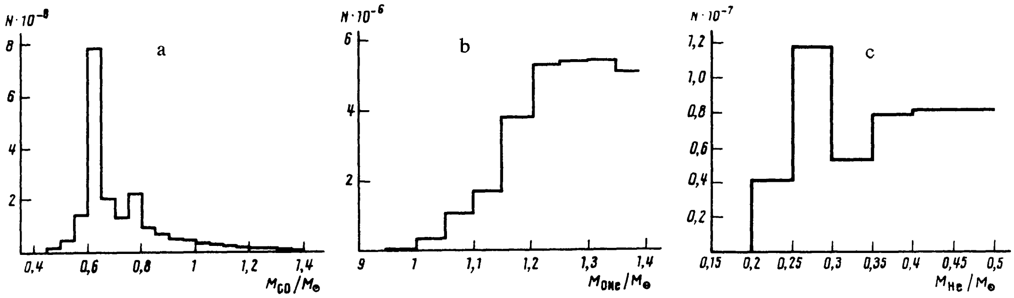

FIG. 5. Theoretical mass spectra of single carbon–oxygen (a), oxygen–neon (b), and helium (c) white dwarfs. Case of $\alpha = 0$ and $\alpha_{CE} = 1$.

to estimate M from g.[4] It is also possible that the maximum in the distribution of observed dwarfs near 0.8 $M_\odot$ originates from the fact that when the core of a star on the asymptotic giant branch reaches a mass close to 0.8 $M_\odot$, the "superwind" mechanism is turned on and the envelope is rapidly ejected. More massive dwarfs in this case are the remnants of stars for which the mass of the CO core already exceeded ~0.8 $M_\odot$ at the time of formation.[19]

From Table II it follows (as noted above) that a considerable number of binaries (30-36%) become single stars and ultimately produce single degenerate dwarfs in the course of evolution. The mass distribution of single dwarfs is shown in Fig. 5. Overall, it coincides with the mass distribution of all dwarfs. The increase in the number of oxygen–neon dwarfs as their mass approaches the Chandrasekhar limit (Fig. 5b) is noteworthy; the number of single helium dwarfs also increases as their mass increases (Fig. 5c). Stars with mass 0.25-0.30 $M_\odot$ stand out among helium dwarfs. Most of them are former helium accretors in cataclysmic biniaries, in which the donors, in accordance with our adopted formalism, "dissipated" as a result of transfer to the accretor and subsequent layered explosions on the dwarf. The more massive single helium dwarfs are merger products of helium dwarfs that are remnants of primary components with low-mass secondaries; in this case, it is possible that the core of the product does not reach the helium ignition mass (~0.5 $M_\odot$). It is also important to note the very fact that a considerable number of single helium white dwarfs are formed by the coalescence of components during the evolution of close binaries.

Detection of duplicity in degenerate dwarfs is a very complex problem because of their low brightness and small size relative to the orbital size. The most promising method of searching for unseen close companions of degenerate dwarfs is normally to search for time variation in the radial velocities of dwarfs. We have therefore paired all of our numerical models with the distribution of binary dwarfs with respect to maximum semiamplitude of radial velocity (Figs. 1b-4b). Each of these distributions has two main components: one associated with close systems with $K_1 \gtrsim 100$ km/sec, and one associated with wide systems with $K_1 \lesssim 3$ km/sec. Because of the faintness of even the brightest dwarfs and the large width of their spectral lines, only those with $K_1 \gtrsim 100$ km/sec can be detected as spectroscopic binaries, in practice. Two dwarfs of that type have been found: L 870–2 (Ref. 1) and WD 0957–666 (Refs. 1 and 2). Their K_1 positions are marked by dots in Fig. 1b. Moreover, six wide pairs of dwarfs are known to be visual binaries.[3] Both close and wide binary systems of degenerate dwarfs have thus been observed. These statistics on binary degenerate dwarfs are as yet insufficient for a serious comparison of theoretical predictions with observations, of course. But the detection of close binary degenerate dwarfs is an important argument favoring convergence of the components of evolving close binaries, thereby confirming the possible coalescence of the components of such systems.

Robinson and Shafter,[20] who investigated 40 dwarfs for radial-velocity variation with a period shorter than 3 h, found no binary systems. According to our theoretical distributions of dwarfs with respect to orbital period, only one system out of 150 (for $\alpha = 0$ and $\alpha_{CE} = 1$), or one out of 200 (for $\alpha = -1$ and $\alpha_{CE} = 1$), or one out of 460 (for $\alpha = 0$ and $\alpha_{CE} = 5$) could be detected by the method used in Ref. 20. The number of such short-period systems is small because, once formed, they merge rapidly due to gravitational wave emission.

Bragaglia et al.,[2] who investigated 50 white dwarfs for duplicity, found one close system of two degenerate dwarfs, two systems consisting of a degenerate and a red dwarf, and suspected the duplicity of two other white dwarfs at the level $K_1 > 60$ km/sec. Averaging K_1 over the angle of inclination of the orbital plane reduces K_1 by a factor $\pi/4$. The limit on K_1 of the investigation in Ref. 2 is therefore equivalent to a limit $\log K_1 > 1.9$ in our distributions (Figs. 1b-4b). Only ~10% of all systems satisfy this condition (see Fig. 1b). This limit is consistent with the estimated fraction of close binaries among observed systems found in Ref. 2.

The most promising way to search for systems of degenerate dwarfs and low-mass, main sequence stars is to investigate the infrared excesses in the spectra of dwarfs. Probst,[21] who studied 120 white dwarfs, found that ~7% of them have red dwarf companions. According to Table I, the number of such systems can reach 30%. But only companions that are main sequence stars with a luminosity comparable to that of the white dwarf (~10^{-2} $L_\odot$, on the average) can be detected by Probst's method. Stars with lower masses and luminosities are not detectable by such a method. We therefore believe that our models yield estimates of duplicity consistent with Probst's observations.[21]

Some fraction of main sequence stars have low-luminosity degenerate dwarfs as companions. Such systems can be detected through variations in the primary's radial velocity as single-line spectroscopic binaries. In Fig. 6 we give the theoretical distribution, with respect to the maximum radial-velocity semiamplitude, of main sequence stars paired with low-

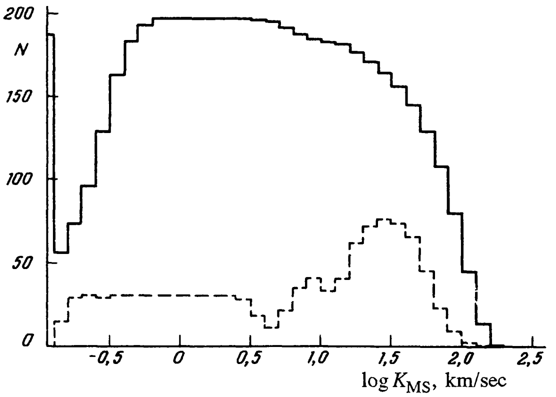

FIG. 6. Theoretical spectrum of maximum semiamplitudes of radial velocities of single-line spectroscopic binaries in which the companion is a main sequence star (solid curve) and a white dwarf (dashed curve). Case of $\alpha = 0$ and $\alpha_{CE} = 1$.

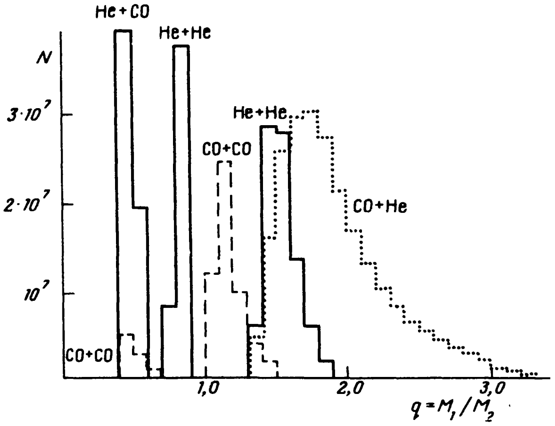

FIG. 8. Distribution of close binary degenerate dwarfs ($K_1 \geq 6$ km/sec, $\alpha = 0$, $\alpha_{CE} = 1$) with respect to the masses of the components. For each combination of components, the remnant of the primary is given first and then the remnant of its companion.

mass companions that are main sequence stars and with degenerate dwarfs (for $\alpha = 0$ and $\alpha_{CE} = 1$, a "flat" initial distribution of stars with respect to q seems preferable, as investigations of different types of stars show[22]). Most of the latter systems have $K_{MS} \geq 10$ km/sec. Since for binaries with $\sim 2\ M_\odot$ companions, which predominate among observable spectroscopic binaries, radial-velocity variation with a semiamplitude $K_{MS} \geq 3$ km/sec is detectable,[7] a considerable fraction of the known single-line spectroscopic binaries should have a degenerate dwarf as a companion. This pertains especially to systems with $K_1 \approx 30$ km/sec. These systems cannot be identified among observable single-line binaries, however, since the inclination of the orbit is normally unknown. The semimajor axis of the orbit can be estimated more accurately from Kepler's law for these systems. Models of the distributions of bright binaries ($m_V \leq 7^m$) with respect to the semimajor axis of their orbit for a flat initial distribution of binary systems with respect to the mass ratios of the components ($\alpha = 0$, $\alpha_{CE} = 1$) are plotted in Fig. 7. About a third of all such close binaries with $a \leq 100\ R_\odot$ obviously have companions that are degenerate dwarfs. The companions should be degenerate in almost all of the observed single-line binary systems with $a \leq 10\ R_\odot$ (see Fig. 7), since such systems do not appear in the initial distribution. The components converge as a result of the loss of orbital angular momentum in the common-envelope phase.

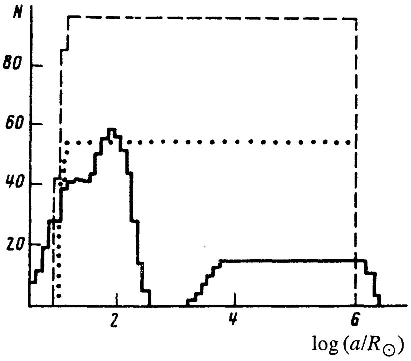

FIG. 7. Theoretical distribution of semimajor axes of the orbits of bright ($M_V \leq 7^m$) spectroscopic binaries. Dotted curve: double-line systems; dashed curve: single-line systems in which the companion is a main sequence star; solid curve) single-line systems in which the companion is a white dwarf. Case of $\alpha = 0$ and $\alpha_{CE} = 1$.

As the components in systems of degenerate dwarfs cool, the difference in their luminosities decreases with time. This increases the chance of detecting lines of the fainter component in the spectrum, which leads to the appearance of detectable systems of double-line spectrocopic binaries. Such was the first detected system of this type,[2] L 870−2. The presence of two spectra makes it possible to measure the mass ratio q of the components directly, and it then becomes possible to use the theoretical q distribution for evolutionary diagnostics of the systems. Those distributions for close pairs of dwarfs according to our calculations for $\alpha = 0$ and $\alpha_{CE} = 1$ are plotted in Fig. 8. Recall that q is the ratio of the mass of the fainter remnant of the primary to the mass of the remnant of the secondary, presumed to be brighter.

Binary degenerate helium dwarfs form two groups, with $q \approx 0.85$ and 1.4-1.7. The first includes systems that have passed through the Algol stage,[23] and the second includes systems that have passed through two common-envelope phases. All of the systems in which the remnant of the primary is a degenerate helium dwarf and the remnant of the secondary is a carbon−oxygen dwarf also have a fairly "specific" mass ratio, $q = 0.4$-0.6. Such pairs result from quasi-conservative evolution in the first transfer phase and nonconservative evolution in the second transfer phase in systems with components of initial masses $\sim 2.5\ M_\odot$ and an initial mass ratio close to unity.

Systems in which the remnant of the primary is a carbon−oxygen dwarf and that of the secondary is a helium dwarf should have an average mass ratio ~ 2 in accordance with the ratio of the typical masses of those dwarfs. The considerable spread reflects the initial mass function of the components. Systems of this type are most common among close pairs of degenerate dwarfs.

The distribution of close pairs of carbon−oxygen dwarfs turned out to be bimodal: $q \approx 0.5$ and 1.2. Systems in the first group are predominantly produced by evolution of binaries with components of similar initial mass ~ 3-$4\ M_\odot$. A low-mass helium star is formed in the first mass transfer in this case; it then gives rise to a white dwarf with $M \approx 0.3$-$0.4\ M_\odot$. The second mass transfer is nonconservative. Sys-

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TABLE III. Scenario for the Formation of L 870−2 for $\alpha_{CE} = 1$

Stage	$M_1/M_\odot$	$M_2/M_\odot$	$\log(a/R_\odot)$	P, days	M_1/M_2	Note
			Formation of two degenerate helium dwarfs			
MS + MS	0,83	0,72	1,50	16,23	1,14	
Algol	0,48	0,96	1,70	33,42	0,50	Average parameters
Hed + MS	0,28	1,27	1,90	63,97	0,22	
CE	0,28	0,65	1,29	10,16	0,43	Average parameters
Hed + Hed	0,28	0,33	0,68	1,54	0,85	
			Formation of two carbon−oxygen dwarfs			
MS + MS	2,86	2,64	2,92	1182,5	1,08	
AGB + MS	2,86	2,64	2,92	1182,5	1,08	
CE	1,39	2,64	2,49	312,9	0,53	Average parameters
COd + MS	0,68	2,64	2,05	75,4	0,26	
COd + AGB	0,68	2,64	2,05	75,4	0,26	
CE	0,68	1,25	1,42	11,2	0,54	Average parameters
COd + COd	0,68	0,59	0,79	1,57	1,15	

TABLE IV. Scenario for the Formation of V 651 Mon for $\alpha_{CE} = 1$

Stage	$M_1/M_\odot$	$M_2/M_\odot$	$\log(a/R_\odot)$	P, days	K_1, km/sec	Note
MS + MS	1,80	1,76	2,42	266	25,66	
CE	0,87	1,76	1,98	68	23,98	Average parameters
Hed + MS	0,42	1,76	1,54	16,21	21.16	

tems in the second group (with $q \approx 1.2$) have passed through two common-envelope stages, and their mass ratio reflects the q distribution of the unevolved systems. The mass ratio of the components in close binary degenerate dwarfs can be a good classification criterion, on the whole, and for almost any q (except $1.3 \leq q \leq 1.6$) it usually enables one to determine the "chemical" type of the system uniquely. For an initial q distribution different from the model with $\alpha = 0$ and $\alpha_{CE} = 1$, for which Fig. 8 was constructed, the location of the peaks in the $(q-N)$ diagram will be the same, but the relative numbers of stars of different types will change (see Table II).

3. EVOLUTIONARY SCENARIOS FOR SELECTED SYSTEMS

Among the several hundred scenarios calculated in our statistical analysis program on the properties of a set of binary systems, specific scenarios can be identified that best describe the parameters of certain well-studied stars. In this section we discuss in more detail the evolution of four detached systems with degenerate dwarfs as components: L 870−2, V 651 Mon, Sirius (α CMa), and 40 Eri. The first two systems are especially important for estimating the general evolutionary parameter α_{CE}, since they have passed through the common-envelope phase.

L 870−2 is one of two known close binary systems consisting of two degenerate dwarfs.[1] The internal chemical composition of the components is as yet unclear. The most accurate two parameters of the system are the mass ratio of the components, equal to 0.85 (or 1.15, depending on which of the dwarfs is brighter), and the orbital period, $P = 1.6$ days. We took these parameters as the criteria for selecting scenarios that could lead to the formation of such a system. Our scenario program generates only two such scenarios. The main stages of the scenarios are given in Table III.

We have previously considered a scenario that includes an Algol phase.[23] The second scenario was suggested by Iben and Webbink.[24] The scenario leading to the formation of two helium dwarfs is preferrable, since two helium dwarfs have a larger total surface area than two carbon−oxygen dwarfs, and the L 870−2 system has a $\sim 1^m$ luminosity excess.[1] Additional research is needed for final conclusions; the ratio of the mass of the fainter dwarf to that of the brighter one has to be clarified, in particular. According to our calculations, the first scenario is almost an order of magnitude more likely than the second. Changing α_{CE} to 0.6 results in the disappearance of the first scenario and a decrease in the probability of the second by almost an order of magnitude. For $\alpha_{CE} = 0.5$ it becomes impossible to explain the origin of the L 870−2 system: we must take $\alpha_{CE} \geq 0.5$.

The V 651 Mon system is the binary nucleus of the planetary nebula NGC 2346 (Ref. 25). The dwarf in this system is probably a helium dwarf with mass $\sim 0.4\ M_\odot$, and is the companion of a star of spectral type A. The only possible scenario for forming such a system is given in Table IV.

For $\alpha_{CE} < 0.8$, this scenario no longer occurs, for the reason that we have explained earlier.[24] The presence of a helium dwarf in the system implies that the mass of its progenitor was less than $\sim 2.3\ M_\odot$. On the other hand, the initial mass of its progenitor must have been higher than the mass of the A star (about 1.8 $M_\odot$). The initial mass of the primary thus turns out to be bounded fairly reliably, and the only scenario parameter that can be varied is α_{CE} As a result, the V 651 Mon system now provides a unique opportunity for estimating α_{CE}.

The companion of Sirius − α CMaB − was probably the first degenerate dwarf discovered. Its mass (1.05 $M_\odot$) is fairly well known[17]; its large value is notable. This dwarf may, of course, simply be the product of the evolution of a fairly massive star with an initial mass $\sim$5-7 $M_\odot$ that has passed through the asymptotic giant branch. But such massive stars are rare, and the orbit of the system that includes Sirius B, is also quite eccentric. It is not yet entirely clear whether such a large eccentricity is retained if the dwarf's progenitor has spent some time in the supergiant phase with an extended envelope ($\sim 10^3\ R_\odot$), given an initial orbital semimajor axis $\sim 3000\ R_\odot$.

It is well known, however, that a considerable fraction of close binary systems are in triple systems, i.e., they have

TABLE V. Fate of Merging Degenerate Components of Close Binary Systems. Merging Frequencies in units of yr^{-1} in the Galaxy

Result of merging	$\alpha = 0$ $\alpha_{CE} = 1$	$\alpha = 0$ $\alpha_{CE} = 0{,}5$	$\alpha = -1$ $\alpha_{CE} = 1$	Comments
SN Ia	0,0032	0,0013	0,0018	Merging of CO dwarfs
SN Ib	0,0015	0,0012	0,0012	Layered helium explosions in CO dwarfs
SN Ib	0,0001	0,0001	0,0001	Growth of an ONe dwarf in a cataclysmic binary with a helium donor of up to 1.39 $M_{\odot}$
SN Ib?	0,000045	0,00002	0,00003	Explosion of an R CrB star
sdB, sdO	0,01	0,015	0,006	Merging of He dwarfs
R CrB	0,01	0,017	0,009	Merging of He and CO dwarfs

distant companions. It is even possible that the fractions of double and triple systems are comparable. Since most massive white dwarfs are probably formed by merging, as mentioned above, this enables us to suggest the following generalized scenario for the formation of Sirius B: the system originally consisted of a close binary system with components of several $M_{\odot}$ and a distant (presently observed) companion. The components in the close binary system have merged, forming a massive white dwarf. The obvious advantage of explaining the origin of the massive degenerate dwarf in the Sirius system by merging is that there is no need for a massive progenitor of the dwarf. In this case we can also avoid a prolonged phase in which the progenitor had an extended envelope, and the orbit can therefore retain a large eccentricity.

The total number of scenarios generated by the program for the formation of Sirius B, i.e., a degenerate dwarf with mass 1.05 $M_{\odot}$, is 29. The most likely of them are simple,

$$\mathrm{MS + MS \rightarrow CE \rightarrow MS \rightarrow G \rightarrow AGB \rightarrow COd},$$

while some scenarios are fairly complicated,

$$\mathrm{MS + MS \rightarrow CE \rightarrow COd + MS \rightarrow CE \rightarrow COd + Hes \rightarrow dSN + Hes}$$
$$\mathrm{\rightarrow COd + Hes \rightarrow Cod}$$

(G is a giant in the stage of helium burning in the core and dSN is a dwarf supernova due to explosive helium burning in a degenerate shell). The explosion of a dwarf supernova in the system is associated with its loss of ~0.15 $M_{\odot}$ (Ref. 26), which may result in the orbit becoming eccentric.

One of the most remarkable systems containing a degenerate dwarf is the visual binary 40 Eri. The mass of the degenerate dwarf, ~0.43 $M_{\odot}$, has been accurately determined.[27] That dwarf has a distant ($a = 7000\ R_{\odot}$) visual companion of low mass (~0.15 $M_{\odot}$), which could not have affected the evolution of the dwarf's progenitor. How could such a low-mass dwarf have originated? The evolution of single stars results in CO dwarfs with a mass exceeding ~0.50 $M_{\odot}$ (Fig. 1a). Therefore, bearing in mind the lack of a close companion of the dwarf in the 40 Eri system, it is natural to assume that it was formed by the coalescence of stars. The main scenarios for its formation can be represented schematically as

1. $\mathrm{MS + MS \rightarrow Ce \rightarrow Hed + MS \rightarrow CE \rightarrow Hed + Hed \rightarrow Hes \rightarrow COd}$;
2. $\mathrm{MS + MS \rightarrow CE \rightarrow Hed + MS \rightarrow CE \rightarrow G \rightarrow Hed}$;
3. $\mathrm{MS + MS \rightarrow CE \rightarrow Hed + MS \rightarrow Hed + Hdon \rightarrow Hed}$.

According to the calculations in our program, the second model is the most likely and the third is the least.

4. FATE OF CLOSE BINARY DEGENERATE DWARFS

The closest binary degenerate dwarfs can merge due to gravitational wave emission, producing type I supernovae,[28-30] hot sdO and sdB subdwarfs,[31,32] and R CrB stars.[30,33] The frequencies of some of these events are given in Table V.

The type of supernova is as yet very tentative, since there are no detailed numerical models either of coalescence itself or of the subsequent evolution ending in an explosion. If we assume that the coalescence of degenerate carbon–oxygen or oxygen–neon dwarfs can actually account for the appearance of type I supernovae, then the frequency of such events is close to that observed.[34] Some R CrB stars – the products of merging of massive CO dwarfs with helium stars or dwarfs – can also explode as supernovae.

The coalescence of helium dwarfs is the most efficient channel for producing hot sdB and sdO subdwarfs, which are probably low-mass helium stars. The space density of those stars, estimated from our model with $\alpha = 0$ and $\alpha_{CE} = 1$, is about $(4\text{-}5)\cdot 10^{-6}\ \mathrm{pc}^{-3}$. That value is almost independent of α and α_{CE} (see Table V). The theoretical space density almost coincides with the observed density of hot subdwarfs,[35,36] $(2\text{-}4)\cdot 10^{-6}\ \mathrm{pc}^{-3}$. The formation of helium subdwarfs in close binary systems as a result of mass loss occurs several times less efficiently. This is fully consistent with the results of Saffer and Liebert,[37] who found only one helium subdwarf in a close binary out of 39 studied. Although some binaries have remained undetected, of course, the low degree of duplicity of helium subdwarfs seems to be reliably established.

5. CONCLUSION

Calculations of statistical models of the population of degenerate dwarfs in our Galaxy indicate the fruitfulness of the scenario approach to describing stellar evolution, which synthesizes modern concepts about the evolution of single and binary stars, starting with their formation and ending in the formation of compact objects. Modeling of individual aspects of the evolution of certain classes of close binaries has been undertaken several times before (see, e.g., Refs. 13, 23, and 38-41). A distinctive feature of our model is the unified evolutionary approach for stars of all masses, based on an empirical star-formation function for binary systems.

Let us summarize the main results of the statistical modeling of the population of degenerate dwarfs in binary systems.

1. Our calculations enabled us to construct a mass spectrum for white dwarfs, both single objects and components of binary systems. We found a high frequency of processes leading to the coalescence of components as they evolved: 20-24% of all stars that begin their evolution as binaries end up as single stars. About 80% of all single dwarfs with a mass greater than $\sim 0.65\ M_\odot$ result from the coalescence of components. About 1% of single degenerate dwarfs are products of the decay of wider systems ($a > 10^5\ R_\odot$) following mass loss by their components in the process of forming a planetary nebula or the subsequent explosion of their primary component as a supernova.

2. A model of the distribution of degenerate dwarfs with respect to the semiamplitude of radial-velocity variation predicts that only a few per cent of all dwarfs can be detected as close spectroscopic binaries with $K_1 \geq 100$ km/sec. This agrees well with the results of searches for spectroscopic binary dwarfs.

3. The mass ratio of the components in close binary degenerate dwarfs can serve as a criterion for determining the chemical composition of the components.

4. The calculations provide examples of several specific scenarios for the formation of systems containing white dwarfs: L 870−2, V 651 Mon, α CMa, and 40 Eri. An investigation of the common-envelope parameter α_{CE} based on V 651 Mon and L 870−2 showed that $0.6 < \alpha_{CE} < 1$, with V 651 Mon making it possible to confine that parameter to an even narrower range, 0.8-1.

5. Modeling showed that $\sim 70\%$ of all single-line spectroscopic binaries ($m_V \leq 7^m$, $a \leq 100\ R_\odot$) have a main sequence star as a companion, and only about a third of such systems have a degenerate dwarf as a secondary.

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