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# Formation of neutron stars in binary systems

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We use the scenario approach to analyze the evolution of binary systems of different masses that results in the formation of neutron stars. We construct a numerical model of the Galactic population of neutron stars. We estimate the frequency of formation and the numbers of single neutron stars and those with companions that are main sequence stars, supergiants, helium stars, white dwarfs, and black holes. We analyze in detail the theoretical distributions of neutron stars in binary systems of various types with respect to orbital period, the masses of the components, orbital eccentricity, and spatial velocity. We investigate the dependence of the main results on the parameters of the problem: the efficiency of angular momentum loss at phases with a common envelope and the distribution of close binaries with respect to the initial mass ratio of the components.

## 1. INTRODUCTION

The existence of something altogether uncommon — neutron stars — was predicted 35 years before the discovery of radio pulsars and x-ray pulsars, which are identified with neutron stars. From the time of their discovery, neutron stars became one of the central objects of astrophysics. Their relationship to supernova explosions, which complete the evolution of massive stars, their unusually high space velocities, and the presence of companions of some neutron stars that fill their Roche lobe and transform the binary system into a bright x-ray source have attracted the interest of many investigators.

The relationship between neutron stars and binary systems has been investigated essentially since the discovery of the first radio pulsars.[1-6] The purpose of the present work is a systematic investigation of the possibilities of the formation of neutron stars in the course of evolution of binary systems in terms of the scenario approach. The essence of the scenario method of investigation consists in numerically modeling the successive changes in the main parameters of a binary system, starting from main sequence stars and ending with the formation of white dwarfs, neutron stars, and black holes and the possible disruption of the system. We have used the numerical scenario approach to investigate Wolf–Rayet stars,[7,8] degenerate dwarfs,[9] and supernovae in binary systems.[10]

## 2. CALCULATION METHOD

We assume that the rate of formation of binaries in the Galaxy can be represented in the form of the function[11,12]

$$d^3\nu = 0.2 d\log aM_1^{-2.5}dM_1 f(q)dq \ \mathrm{yr}^{-1}, \qquad (1)$$

where $a$ is the initial semimajor axis of the orbit in $R_\odot$, $M_1$ is

**TABLE I.** Populations and Frequencies of Creation (per Year) of Neutron Stars of Different Types in the Galaxy

| | NS | NS + NS | NS + MS | NS + COd | COd + NS | NS + ONed | ONed + NS | NS + Hed | BH + NS | NS + Hes | NS + SG | NS + AGB |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Close systems ($\alpha = -1$, $\alpha_{ce} = 1$) | | | | | | | | | | | | |
| $N$ | $0,28\cdot 10^{9}$ | $0,12\cdot 10^{7}$ | $0,12\cdot 10^{7}$ | $0,39\cdot 10^{7}$ | $0,75\cdot 10^{6}$ | $0,26\cdot 10^{6}$ | $0,91\cdot 10^{5}$ | $0,97\cdot 10^{6}$ | $0,22\cdot 10^{7}$ | $0,65\cdot 10^{5}$ | $0,90\cdot 10^{4}$ | 0 |
| $\nu$ | $0,19\cdot 10^{-1}$ | $0,28\cdot 10^{-3}$ | $0,32\cdot 10^{-2}$ | $0,86\cdot 10^{-3}$ | $0,15\cdot 10^{-3}$ | $0,24\cdot 10^{-3}$ | $0,76\cdot 10^{-4}$ | $0,21\cdot 10^{-3}$ | $0,16\cdot 10^{-3}$ | $0,22\cdot 10^{-2}$ | $0,29\cdot 10^{-3}$ | 0 |
| Close systems ($\alpha = 0$, $\alpha_{ce} = 1$) | | | | | | | | | | | | |
| $N$ | $0,18\cdot 10^{9}$ | $0,16\ 10^{7}$ | $0,25\cdot 10^{6}$ | $0,22\cdot 10^{7}$ | $0,16\cdot 10^{7}$ | $0,20\cdot 10^{6}$ | $0,18\cdot 10^{6}$ | $0,28\cdot 10^{6}$ | $0,18\cdot 10^{7}$ | $0,34\cdot 10^{5}$ | $0,41\cdot 10^{4}$ | 0 |
| $\nu$ | $0,13\cdot 10^{-1}$ | $0,39\cdot 10^{-3}$ | $0,35\cdot 10^{-2}$ | $0,62\cdot 10^{-3}$ | $0,31\cdot 10^{-3}$ | $0,28\cdot 10^{-3}$ | $0,14\cdot 10^{-3}$ | $0,67\cdot 10^{-4}$ | $0,13\cdot 10^{-3}$ | $0,27\cdot 10^{-2}$ | $0,29\cdot 10^{-3}$ | 0 |
| Close systems ($\alpha = 1$, $\alpha_{ce} = 1$) | | | | | | | | | | | | |
| $N$ | $0,23\cdot 10^{9}$ | $0,22\cdot 10^{7}$ | $0,97\cdot 10^{5}$ | $0,12\cdot 10^{7}$ | $0,30\cdot 10^{7}$ | $0,16\cdot 10^{6}$ | $0,31\cdot 10^{6}$ | $0,72\cdot 10^{5}$ | $0,13\cdot 10^{7}$ | $0,18\cdot 10^{5}$ | $0,18\cdot 10^{4}$ | 0 |
| $\nu$ | $0,16\cdot 10^{-1}$ | $0,51\cdot 10^{-3}$ | $0,43\cdot 10^{-2}$ | $0,45\cdot 10^{-3}$ | $0,57\cdot 10^{-3}$ | $0,32\cdot 10^{-3}$ | $0,24\cdot 10^{-3}$ | $0,19\cdot 10^{-4}$ | $0,97\cdot 10^{-4}$ | $0,35\cdot 10^{-2}$ | $0,39\cdot 10^{-3}$ | 0 |
| Close systems ($\alpha = 0$, $\alpha_{ce} = 0.5$) | | | | | | | | | | | | |
| $N$ | $0,17\ 10^{9}$ | $0,25\cdot 10^{5}$ | $0,13\cdot 10^{6}$ | $0,31\cdot 10^{5}$ | $0,14\cdot 10^{6}$ | $0,91\cdot 10^{3}$ | $0,99\cdot 10^{4}$ | $0,24\cdot 10^{4}$ | $0,15\cdot 10^{7}$ | $0,77\cdot 10^{4}$ | $0,49\cdot 10^{3}$ | 0 |
| $\nu$ | $0,12\cdot 10^{-1}$ | $0,26\cdot 10^{-3}$ | $0,29\cdot 10^{-2}$ | $0,22\cdot 10^{-3}$ | $0,25\cdot 10^{-3}$ | $0,14\cdot 10^{-3}$ | $0,59\cdot 10^{-4}$ | $0,25\cdot 10^{-4}$ | $0,12\cdot 10^{-3}$ | $0,18\cdot 10^{-2}$ | $0,18\cdot 10^{-3}$ | 0 |
| Wide systems ($\alpha = -2.5$) | | | | | | | | | | | | |
| $N$ | $0,16\cdot 10^{9}$ | $0,62\cdot 10^{6}$ | $0,76\cdot 10^{6}$ | $0,26\cdot 10^{8}$ | 0 | 0 | 0 | 0 | $0,85\cdot 10^{6}$ | 0 | $0,25\cdot 10^{5}$ | $0,77\cdot 10^{3}$ |
| $\nu$ | $0,11\cdot 10^{-1}$ | $0,41\cdot 10^{-4}$ | $0,27\cdot 10^{-2}$ | $0,18\cdot 10^{-2}$ | 0 | 0 | 0 | 0 | $0,57\cdot 10^{-4}$ | 0 | $0,27\cdot 10^{-2}$ | $0,21\cdot 10^{-2}$ |

the initial mass of the primary component in $M_\odot$, and $q$ is the zero-age mass ratio of the components. Equation (1) is valid for $1 \le \log(a/R_\odot) \le 6$, $0.8 \le M_1/M_\odot \le 100$, $0 < q \le 1$, and $\int_0^1 f(q)dq = 1$. The normalization factor in Eq. (1) is determined by the frequency of formation of white dwarfs. It follows from Eq. (1) that essentially all stars are formed in binary systems and that one binary with $M_1 \ge 0.8\ M_\odot$ is created per year in the Galaxy. The distribution of close binaries ($a \lesssim 3\cdot 10^3\ R_\odot$) with respect to the initial mass ratio $q$ of the components has not yet been established. To investigate the dependence of the results on $f(q) = cq^\alpha$, we therefore carried out calculations for three initial distributions: $f(q) = 1$, $f(q) = 2q$, and $f(q) \propto 1/q$ for $0.1 \le q \le 1$. For the basic version, however, with allowance for the results of Ref. 13, we took that with $f(q) = 1$, for which all of the main graphs are given. For wide systems, according to Ref. 14, $f(q) \propto (1/q)^{2.5}$ for $0.3 \le q \le 1$, which corresponds, for a fixed mass of the primary component, to a Salpeter mass distribution for the secondary component if the zero-age mass function is $dN/dM = M^{-2.5}$. For wide systems with $q < 0.3$, the distribution with respect to $q$ is essentially unknown. We therefore assumed, to first order, that $f(q) = \text{const}$ in the range $0 < q < 0.3$.

The evolution of a binary system is completely determined by the initial values of $M_1$, $a$, and $q$. For each initial combination $M_{10}$, $a_0$, $q_0$, we traced the sequence of states through which a system with the given parameters passes in the course of evolution. We investigated the space of initial parameters with step sizes 0.0125 in $\log M_1$, 0.05 in $\log a$, and 0.025 in $q$. The choice of the evolutionary scenario was based on the results of calculations of the evolution of single and close binary stars and observational data. The components can pass through the following main evolutionary phases: a main sequence star, a giant with a degenerate or a nondegenerate helium core, a supergiant with a carbon–oxygen core, a helium star, a (helium, carbon–oxygen, or oxygen–neon) white dwarf, a neutron star, and a black hole. The transitions of components between states can be due either to nuclear evolution, or to mass transfer between components, or to their merging due to the loss of orbital angular momentum by the binary system as a whole. The changes in the masses of the components in the course of mass transfer and the lifetimes of the stars in individual stages were determined by analytic equations approximating the results of numerical calculations of stellar evolution.

The loss of orbital angular momentum plays an important role in the evolution of close binary systems. We allowed for three loss mechanisms: braking in common envelopes, the emission of gravitational waves, and ordinary and magnetic stellar wind.

In the case of common envelopes, we estimated the change in the semimajor axis of the orbit on the basis of the law of conservation of orbital energy. The latter can be written schematically in the form[15]

$$(M_1 + M_2)(M_1 - M_{1R})/a_0 = \alpha_{ce} M_{1R} M_2 (1/a_f - 1/a_0), \tag{2}$$

where $\alpha_{ce}$ is a parameter of order unity, indicating what fraction of the orbital energy is expended on the loss of the envelope, $M_1$ and $M_2$ are the masses of the components, $M_{1R}$ is the mass of the remnant of the primary component, and $a_0$ and $a_f$ are the initial and final values of the semimajor axis of the orbit. The value of the parameter $\alpha$ is still unknown. To estimate its influence on the results of the calculations, we carried out the latter for $\alpha_{ce} = 0.5$ and 1. If after the end of the stage of a common envelope, the radius of one of the components was larger than the radius of the Roche lobe for that component, we assumed that the components merge into a single star. We determined the evolutionary status of the latter from the evolutionary status of the merging objects.

We determined the change in the semimajor axis of the orbit in the emission of gravitational waves in accordance with the standard equation of general relativity (see, e.g., Ref. 16). To allow for the change in momentum due to magnetic stellar wind, we used a semiempirical approach based on extrapolation of a law of braking of stellar rotation of the Skumanich solar type.[17]

For the most massive hydrogen and helium stars, we allowed for mass and momentum loss due to ordinary stellar wind using dependences of $\dot{M}$ on stellar mass suggested by Vanbeveren[18] and Langer.[19]

We assumed that the initial eccentricities of the orbits of close binaries are zero. For wide systems we assumed that the initial orbital eccentricity is a function of the orbital period:[20] $e = 0.1 \cdot \log P$, where $P$ is the period in days. We took into account the change in eccentricity in a supernova explosion in accordance with Hills.[21] We also assumed that the orbital eccentricity can change upon the ejection of a planetary nebula by one of the components if the system's orbital period exceeds the characteristic ejection time ($\sim 10^4$ years). The latter imparts the characteristics of an explosion to the process of loss of the envelope.

The use of the function (1) for the formation of binary systems and the systematic investigation of the entire space of the initial parameters of binary stars distinguish our approach to modeling the population of neutron stars from that of Refs. 5, 6, 22, and 23, for example, in which the initial parameters and the lifetime of individual systems were modeled by the Monte Carlo method. An advantage of our approach is that it does not require the investigation of many millions of original systems to obtain statistically reliable results on rare but nevertheless observed systems (e.g., x-ray sources).

## 3. MAIN SCENARIOS OF THE FORMATION OF NEUTRON STARS IN BINARY SYSTEMS

In accordance with our present understanding of the evolution of single stars and close binaries (see, e.g., Refs. 24 and 25), we can schematize its final stages as follows.

**1.** Single stars (and the components of wide pairs that are evolving independently) are converted into carbon–oxygen (CO) white dwarfs if the initial stellar mass is $0.8 \le M/M_{\odot} \le 10$.

**2.** Single stars (and the components of wide pairs) with an initial mass $10 < M/M_{\odot} \le 40$ produce neutron stars, whose formation is accompanied by a supernova explosion.

**3.** Stars with an initial mass $M > 40\ M_{\odot}$ are converted into black holes, also with a supernova explosion. The choice of the boundary value of the mass between the progenitors of neutron stars and black holes is fairly arbitrary. The ratio between the numbers of massive x-ray sources with components that are neutron stars and with components that are black hole candidates observed in the Galaxy can serve as justification for $M = 40\ M_{\odot}$. We ascribed a mass $1.4\ M_{\odot}$ to neutron stars at the time of their formation and a mass $5\ M_{\odot}$ to black holes. Those values are based on the estimated masses of compact stars in x-ray sources.[26]

**4.** The components of close binary systems with masses $0.8 \le M/M_{\odot} \le 2.8$, if they fill their Roche lobe before helium ignition in their interior, are converted into helium white dwarfs as a result of mass loss. Roche lobe filling by stars in this mass range at later stages of evolution leads to the formation of CO dwarfs.

**5.** Components of close binary systems with masses $2.8 < M/M_{\odot} \le 9$ that fill their Roche lobe before He ignition in the core are converted first into helium stars and then into CO dwarfs as a result of mass loss. Roche lobe filling by these stars at later stages of evolution leads directly to the formation of CO dwarfs.

**6.** Components of close binary systems with masses $9 < M/M_{\odot} \le 11.4$ are converted first into helium stars and then into oxygen–neon (ONe) white dwarfs after mass transfer ends. The value of the mass separating the progenitors of CO and ONe dwarfs is not yet known reliably from evolutionary calculations. But the considerable fraction (30-50%) of ONe dwarfs among the exploding components of classical novae[27] indicates that the mass range of their progenitors must be 1-2 $M_{\odot}$.

**7.** Components of close binary systems with masses $11.4 \le M/M_{\odot} \le 40$ are also transformed into helium stars as a

**TABLE II.** Populations and Frequencies of Creation (per Year) of Single Neutron Stars for the Main Channels of Their Formation

| Mechanism of formation | Close systems | | | | Wide systems |
|---|---|---|---|---|---|
| | $\alpha = -1$<br>$\alpha_{ce} = 1$ | $\alpha = 0$<br>$\alpha_{ce} = 1$ | $\alpha = 1$<br>$\alpha_{ce} = 1$ | $\alpha = 0$<br>$\alpha_{ce} = 0{,}5$ | $\alpha = -2{,}5$ |
| Disruption of binary systems in the first SN explosion | $0{,}20 \cdot 10^{-2}$<br>$0{,}29 \cdot 10^{8}$ | $0{,}22 \cdot 10^{-3}$<br>$0{,}33 \cdot 10^{7}$ | $0{,}41 \cdot 10^{-4}$<br>$0{,}61 \cdot 10^{6}$ | $0{,}14 \cdot 10^{-3}$<br>$0{,}22 \cdot 10^{7}$ | $0{,}17 \cdot 10^{-2}$<br>$0{,}25 \cdot 10^{8}$ |
| Disruption of binary systems in the second SN explosion | $0{,}20 \cdot 10^{-2}$<br>$0{,}30 \cdot 10^{8}$ | $0{,}35 \cdot 10^{-2}$<br>$0{,}52 \cdot 10^{8}$ | $0{,}57 \cdot 10^{-2}$<br>$0{,}85 \cdot 10^{8}$ | $0{,}27 \cdot 10^{-2}$<br>$0{,}41 \cdot 10^{8}$ | $0{,}84 \cdot 10^{-2}$<br>$0{,}13 \cdot 10^{9}$ |
| Merging of the components in a common envelope | $0{,}10 \cdot 10^{-1}$<br>$0{,}15 \cdot 10^{9}$ | $0{,}30 \cdot 10^{-2}$<br>$0{,}45 \cdot 10^{8}$ | $0{,}22 \cdot 10^{-2}$<br>$0{,}32 \cdot 10^{8}$ | $0{,}44 \cdot 10^{-2}$<br>$0{,}66 \cdot 10^{8}$ | —<br>— |
| Merging of neutron stars with their companions to form Thorne–Zytkow objects | $0{,}16 \cdot 10^{-2}$<br>$0{,}22 \cdot 10^{8}$ | $0{,}16 \cdot 10^{-2}$<br>$0{,}23 \cdot 10^{8}$ | $0{,}19 \cdot 10^{-2}$<br>$0{,}26 \cdot 10^{8}$ | $0{,}16 \cdot 10^{-2}$<br>$0{,}26 \cdot 10^{8}$ | —<br>— |
| Merging of white dwarfs | $0{,}19 \cdot 10^{-2}$<br>$0{,}25 \cdot 10^{8}$ | $0{,}33 \cdot 10^{-2}$<br>$0{,}44 \cdot 10^{8}$ | $0{,}54 \cdot 10^{-2}$<br>$0{,}72 \cdot 10^{8}$ | $0{,}14 \cdot 10^{-2}$<br>$0{,}19 \cdot 10^{8}$ | —<br>— |

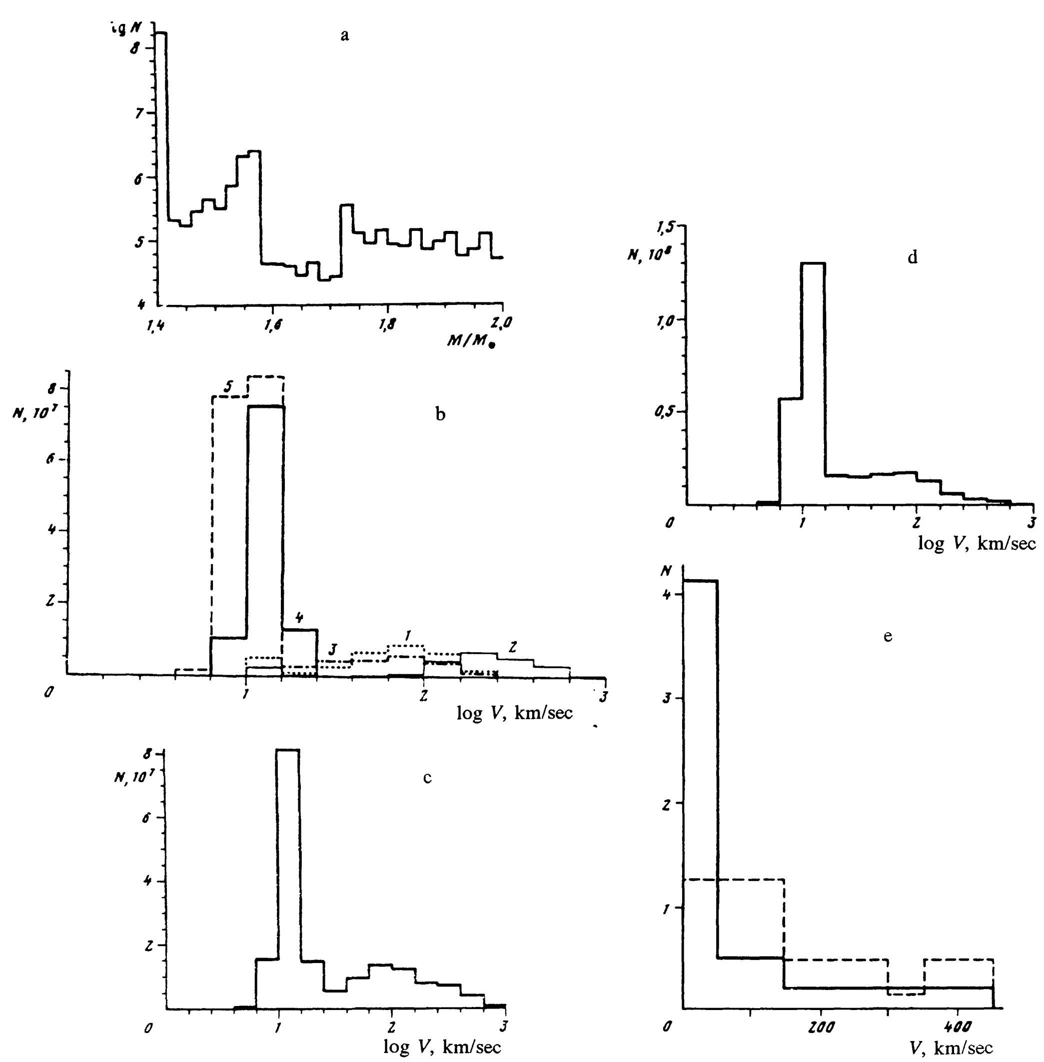

FIG. 1. Single neutron stars: a) distribution with respect to mass; b) distribution with respect to rms spatial velocity (for $\alpha = 0$ and $\alpha_{ce} = 1$): 1) "young" neutron stars from close systems, as a result of explosions of whose progenitors, the systems were disrupted; 2) "old" neutron stars from close systems; 3) neutron stars that have passed through the stage of a Thorne–Zytkow object; 4) neutron stars formed from products of merging of the components of close systems; 5) neutron stars formed in wide systems; c) total distribution with respect to rms spatial velocity for $\alpha = 0$ and $\alpha_{ce} = 0.5$; d) total distribution with respect to rms spatial velocity for $\alpha = -1$ and $\alpha_{ce} = 1$; e) distribution of 26 observed radio pulsars with respect to transverse velocity; dashed line) observational data;[45] solid line) observational data after allowance for selection effects.[41]

result of mass loss, and they then explode as supernovae, forming neutron stars. Heavier objects produce black holes.

This information about the final states of single and binary stars, along with data on their evolutionary paths as a function of mass (and distance between components in the case of close systems), serve as the basis for constructing the population model of binary stars in the Galaxy.

An analysis of the evolution of close binaries shows that about 200 qualitatively different scenarios in which neutron stars appear are possible. The probabilities of occurrence of individual scenarios can differ by several orders of magnitude, on the one hand, while many scenarios differ only in slight details, on the other.

Omitting unimportant stages, we consider the most probable scenarios in which neutron stars appear. We introduce the following notation: MS) main sequence star; RLOF) stage of mass transfer; CE) common envelope, in which merging of the components occurs; Hes) helium star; SN) supernova explosion; Hed, COd, ONed) white dwarfs; AGB) stars with a degenerate CO or ONe core and a hydrogen envelope; RCrB) stars with a degenerate CO or ONe core and a helium envelope; NS) neutron star; S(He)) star with a helium core and a hydrogen envelope; COs) carbon–oxygen star; ThZ) Thorne–Zytkow object; CV) cataclysmic variable with a hydrogen or helium donor; RSG) red supergiant.

We denote states and events pertaining to the primary and secondary components by the indices 1 and 2, respectively; if the star is single, the index is omitted.

Using this notation, we can describe evolutionary scenarios schematically. We begin with close systems.

**1.** In the logically simplest scenario, the two components successively fill their Roche lobes, form helium stars, explode,

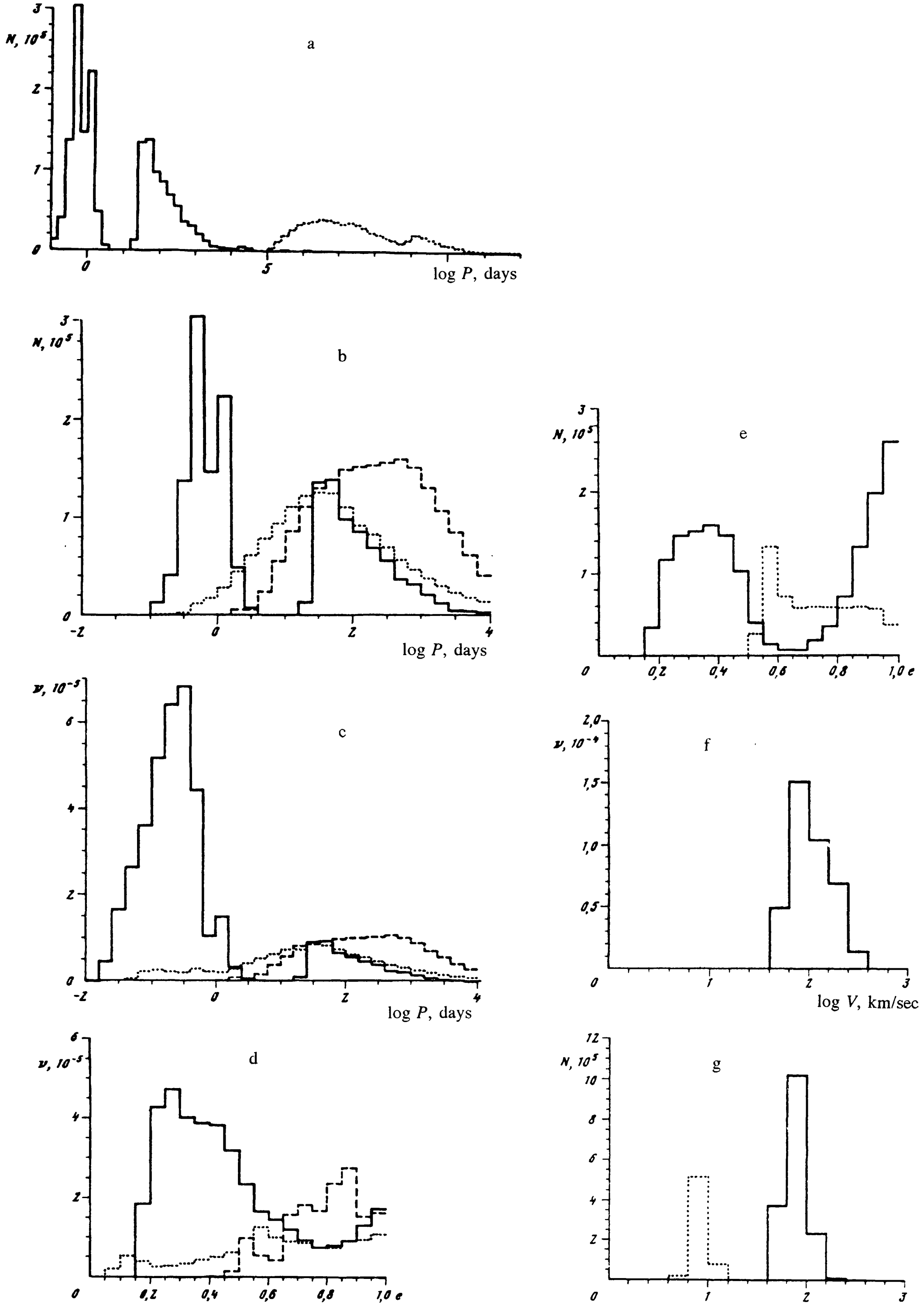

**FIG. 2.** Binary neutron stars: a) distribution of all binary neutron stars with respect to orbital period; b) distribution of close NS + NS, BH + NS, and BH + BH systems with respect to orbital period; solid line) NS + NS systems; dotted line) BH + NS systems; dashed line) BH + BH systems; c) relationship between frequency of formation of close systems containing neutron stars and black holes and their orbital periods; notation same as in Fig. 2b; d) relationship between frequencies of formation of close NS + NS, BH + NS, and BH + BH systems and their orbital eccentricities; notation same as in Fig. 2b; e) distribution of NS + NS systems with respect to orbital eccentricity; solid line) close systems; dotted line) wide systems; e) relationship between frequency of formation of close binary neutron stars and their rms spatial velocity; solid line) close systems; dotted line) wide systems; g) distribution of binary neutron stars with respect to rms spatial velocity; solid line) close systems; dotted line) wide systems.

and produce neutron stars:

**1.1: MS1 + MS2→ RLOF1 → Hes1 + MS2 → SN1 → NS1 + MS2 → RLOF2 → NS1 + Hes2 → SN2 → NS1 + NS2.**

The important role of this scenario is that it "supplies" binary neutron stars. The system can be disrupted in one of the two supernova explosions (the second, as a rule). If the pair is disrupted in the first explosion, then the secondary component, if it is sufficiently massive, completes its evolution "independently," forming a neutron star. The system can also be disrupted due to mass loss by one of the components on the asymptotic giant branch if the characteristic time of ejection ($\sim 10^4$ years) is loss than the system's orbital period. The possibility of disruption of the system and the change in orbital eccentricity for undisrupted systems, both in supernova explosions and in rapid envelope ejection, depend on the fraction of ejected matter with respect to the system's total mass and the initial orbital eccentricity.[21] The secondary component in the (1,1) scenario can have an initial mass less than 11.4 $M_\odot$, but can increase it by accretion to the mass needed to form a neutron star. If a helium star's mass is $2.8 < M_{\rm He}/M_\odot \lesssim 3.6$, then after He depletion in the core, the star expands and, if the system is sufficiently close, it may pass through one more stage of a common envelope.

On the other hand, scenarios are possible in which the primary component of a close system produces a white dwarf while the secondary acquires the mass needed to form a neutron star, and vice versa, the primary component is converted into a neutron star and the secondary into a white dwarf. These scenarios can be written in our notation as follows:

**1.2: MS1 + MS2→ RLOF1 → Hes1 + MS2 → COd1 + MS2 → RLOF2 → COd1 + Hes2 → COd1 + SN2 → COd1 + NS2.**

**1.3: MS1 + MS2→ RLOF1 → Hes1 + MS2 → SN1 + MS2 → → NS1 + MS2 → RLOF2 → NS1 + Hes2 → NS1 + COd2.**

If Roche lobe filling occurs after helium has been depleted up in the stellar core, then the stage of a helium star is absent and the scenario may take the form

**1.4: MS1 + MS2 → RLOF1 → COd1 + MS2 → RLOF2 → COd1 + Hes2 → COd1 + SN2 → COd1 + NS2.**

The place of the CO dwarf in scenarios 1.2-1.4 may be filled by an ONe dwarf.

**2.** In a considerable fraction of binary systems, in one of the evolutionary stages with a common envelope, the components come so close together that the radii of their Roche lobes become less than the radii of the components. We assume that the components merge in that case. A single object is formed, whose evolution may end with the formation of a single neutron star. Merging upon the formation of a common envelope occurs most often when the primary component in a system with $q$ considerably different from unity fills its Roche lobe. Such a scenario can be written formally, for example, as

**2.1: MS1 + MS2→ CE → S(He) → SN → NS.**

Merging can also occur in a common envelope originating in Roche lobe filling by helium stars, the envelopes of which expand after helium depletion in their cores (for $0.78 < M_{\rm He}/M_\odot \leq 3.6$). The corresponding scenario may be as follows, for example:

2.2: MS1 + MS2→ RLOF1 → Hes1 + MS2 → CE → S(He) → SN → NS.

**3.** In the scenarios considered above, the appearance of a neutron star is associated with the evolution of the initially fairly massive components of binaries ($M > 11.4\ M_\odot$) or massive single stars that owe their origin to merging of the components of binary systems of moderate mass (for $M_1 + M_2 > 10\ M_\odot$). We have also considered several more hypothetical channels for the formation of neutron stars, however.

**a)** The evolution of each component of a close binary with a mass less than 9 $M_\odot$ ends with the formation of a CO dwarf. If the pair of degenerate dwarfs is close enough, then in a time less than the Hubble time, the dwarfs may come into contact due to momentum loss from the system in the emission of gravitational waves and may merge. The outcome of the merging of dwarfs is unclear. It is possible that in the process of merging, explosive carbon ignition occurs that completely destroys the star. This is the standard model of type Ia supernovae.[12] According to Saio and Nomoto,[28] however, a nondegenerate CO star may be formed as a result of the merging. After the carbon is exhausted, a core of O, Ne, and Mg is formed which, if its mass reaches 1.37 $M_\odot$, collapses due to electron captures, forming a neutron star. We assume that the latter occurs. Some of the single neutron stars may be formed in this way. The corresponding scenario may be, for example,

3.1: MS1 + MS2→ RLOF1 → COd1+MS2 → RLOF2 → COd1+COd2 → COs → NS.

In a scenario of type 3.1, the place of one or both CO dwarfs may be taken by an ONe dwarf. Collapse with the formation of a neutron star seems inevitable in the merging of ONe dwarfs.

**b)** The evolution of a component of a close binary system ends with the formation of a CO dwarf for $0.8 \leq M/M_\odot \leq 9$ or an ONe dwarf for $9 \leq M/M_\odot \leq 11.4$. Roche lobe filling by its companion results in the formation of a common envelope and merging of the components. We assume that an object similar to a star on the asymptotic giant branch (AGB) is formed in this case: a star with a degenerate CO (or ONe) core and an extended hydrogen envelope. For "normal" AGB stars, it is probably impossible for the mass of the core to increase to the Chandrasekhar limit because of intense mass loss by stellar wind. They therefore produce white dwarfs. For the products of merging that we are considering, however, the masses of the hydrogen envelope and the core can be considerably greater than for ordinary AGB stars, so the cores of some of them may reach the Chandrasekhar mass, as shown by a comparison of the rate of mass loss based on Nugis's semi-empirical equations[29] and the rate of growth of the core. If the core consists of a mixture of carbon and oxygen, then we assume that it explodes without forming a bound remnant, but if the core consists of oxygen, neon, and magnesium, it collapses to form a neutron star. The system can thus evolve by the following scenario, for example:

3.2: MS1 + MS2→ RLOF1 → ONe1 + MS2 → CE → AGB → SN → NS.

**c)** A neutron star can be formed similarly, according to our hypothesis, in the course of evolution of an object formed in the merging of a degenerate ONe dwarf and a helium star. In this case, the intermediate object with an extended helium envelope is probably an RCrB star:

**3.3: MS1 + MS2→ RLOF1 → ONe1 + MS2 →RLOF2 → ONe1 + Hes2 → CE → RCrB → SN → NS.**

d) A version in which one of the components produces a CO or ONe dwarf and the other produces a He dwarf, and the dwarfs merge as a result of momentum loss by the system in the emission of gravitational waves, is also possible; for example,

**3.4: MS1 + MS2→ RLOF1 → ONe1 + MS2 →RLOF2 → ONe1 + Hed2 → RCrB → SN → NS.**

e) The next channel for the formation of single neutron stars is associated with the formation of Thorne–Zytkow objects[30] – stars with neutron cores and hydrogen, helium, or carbon–oxygen envelopes. The evolution of such stars is determined by two processes: mass accretion by the neutron core with the burnup of nuclear fuel in very thin layered sources adjacent to it, and mass loss from the surface, inevitable in high-luminosity objects with extended envelopes. The latter, in particular, determines the lifetime of Thorne–Zytkow objects, as a comparison of the rates of those processes shows. Following the loss of the envelope, a neutron star remains, whose mass may grow somewhat due to accretion in comparison with the initial mass that we adopted for neutron stars, 1.4 $M_\odot$. The scenario leading to the formation of single neutron stars can then be, for example,

**3.5: MS1 + MS2 → RLOF1 → Hes1 + MS2 →SN1 + MS2 → NS1 + MS2 → RLOF2 → NS1 + Hed2 → ThZ → NS.**

The scenarios in which a neutron star merges with a CO or ONe dwarf look similar.

f) Scenarios are possible in which the companion of the neutron star turns out to be a low-mass hydrogen or helium star. As a result of convergence of the components due to momentum loss by the system by magnetic stellar wind and (or) in the emission of gravitational waves, the companion fills its Roche lobe. If the conditions for stable mass transfer are satisfied,[31] a system similar to a cataclysmic binary arises, which may be a source of x rays. The evolution of cataclysmic binaries probably ends with complete disruption of the donor or its conversion into one or several planets when its mass has decreased to $\approx 0.01\ M_\odot$ (Ref. 32). (We do not distinguish between these two possibilities, assuming that a single neutron star is formed.) Such a scenario can be written in the form

**3.6: MS1 + MS2→ RLOF1 → Hes1 + MS2 →SN1 → NS1 + MS2 → RLOF2 → NS1 + Hes2 → CV → NS.**

In the latter scenario, the neutron star is able to accrete all of its companion's mass, since the characteristic rates of mass transfer in systems with a low-mass helium donor are less than the critical rate[33-36] and we assumed the evolution to be conservative at the CV stage. (We neglect mass loss due to stellar

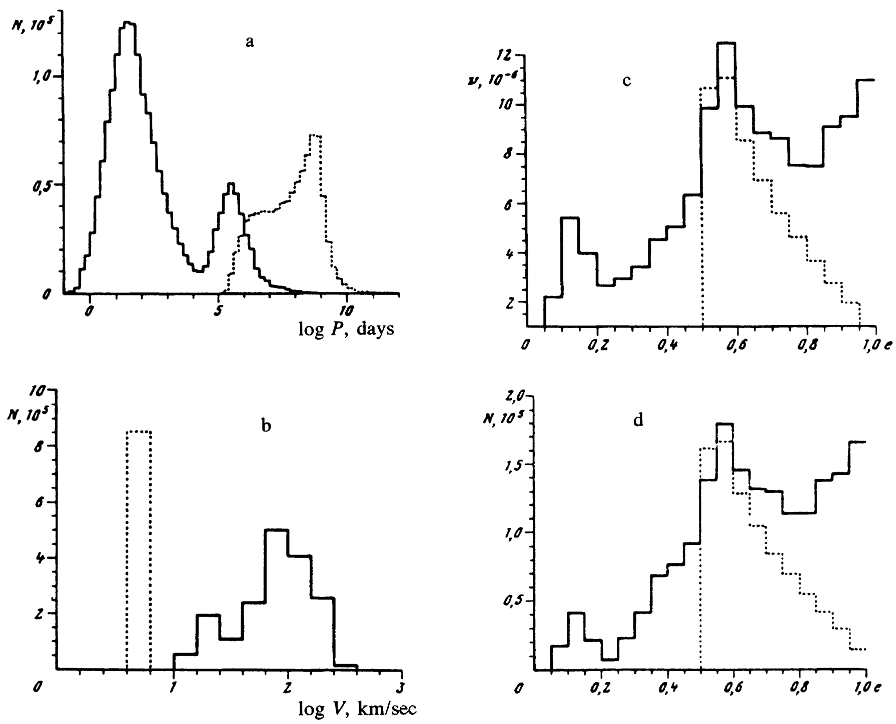

FIG. 3. Neutron stars with black holes as companions: a) distribution of BH + NS systems with respect to orbital period; solid line) close systems; dotted line) wide systems; b) distribution of BH + NS systems with respect to rms spatial velocity; solid line) close systems; dotted line) wide systems; c) relationship between frequency of formation and orbital eccentricity of BH + NS systems; solid line) close systems; dotted line) wide systems; d) distribution of BH + NS systems with respect to orbital eccentricity; solid line) close systems; dotted line) wide systems.

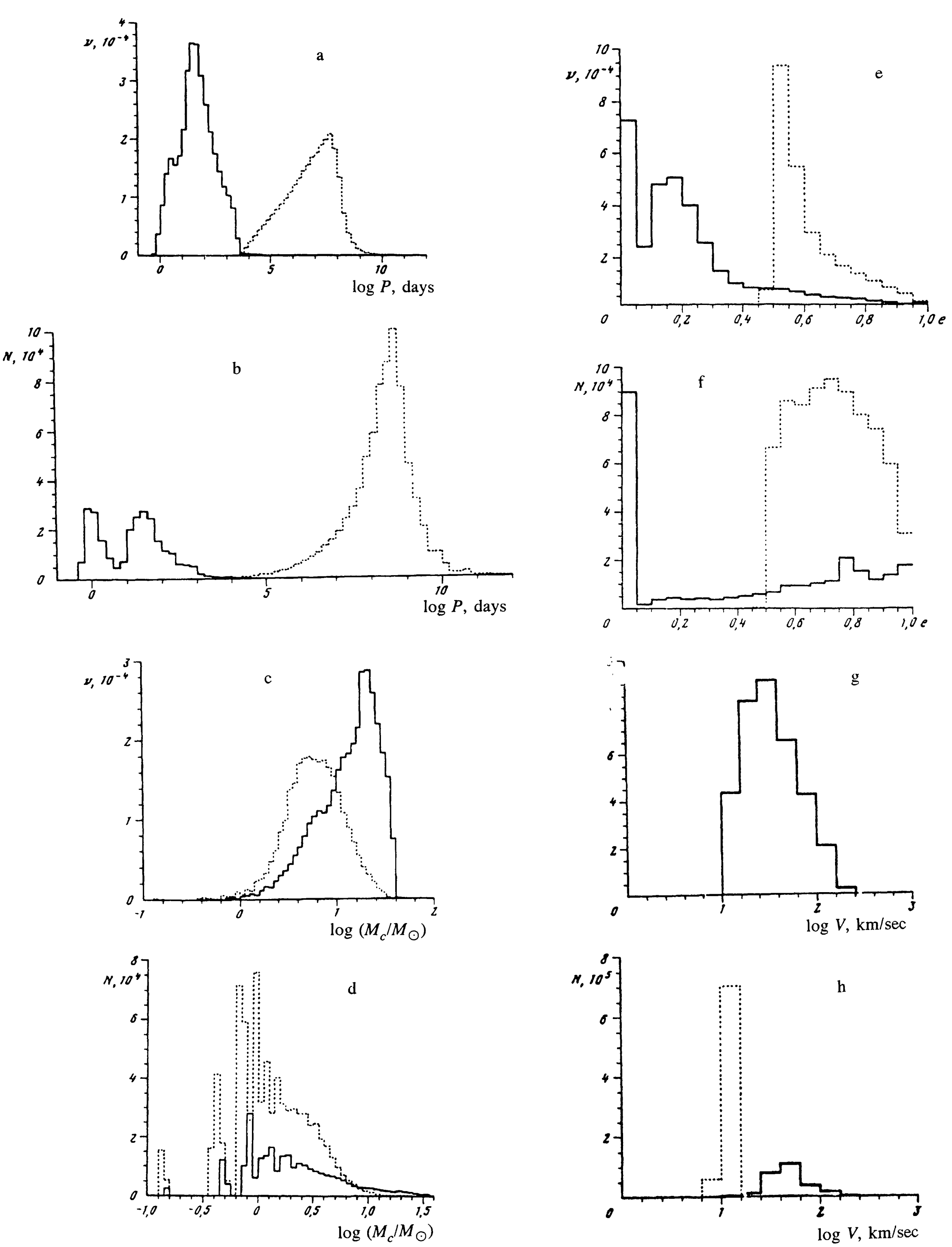

FIG. 4. Neutron stars with main sequence stars as companions: a) relationship between frequency of formation of NS + MS systems and their orbital periods; solid line) close systems; dashed line) wide systems; b) distribution of NS + MS systems with respect to orbital period; solid line) close systems; dotted line) wide systems; c) relationship between frequency of formation of NS + MS systems and the mass of the neutron star's companion; solid line) close systems; dotted line) wide systems; d) distribution of NS + MS systems with respect to the mass of the neutron star's companion; solid line) close systems; dotted line) wide systems; e) relationship between frequency of formation of NS + MS systems and their orbital eccentricity; solid line) close systems; dotted line) wide systems; f) distribution of NS + MS systems with respect to their orbital eccentricity; solid line) close systems; dotted line) wide systems; g) relationship between frequency of formation of close NS + MS systems and their rms spatial velocity; solid line) close systems; dotted line) wide systems; h) distribution of NS + MS systems with respect to their rms spatial velocity; solid line) close systems; dotted line) wide systems.

wind induced by x rays in the present paper.) The neutron star's mass can reach 2 $M_\odot$ as a result of accretion. We assumed that this is the limiting value for neutron stars, and that upon reaching 2 $M_\odot$, neutron stars collapse to form black holes. Under our assumptions, thus, only some "postcataclysmic" systems produce single neutron stars.

**4.** The components in wide binary systems evolve independently, each component following the same evolutionary path as a single star of the corresponding mass. The total number of scenarios for wide systems generated by our program is about 25. We describe the most common ones.

The case in which both components have a mass $10 < M/M_\odot < 40$:

**4.1:MS1 + MS2→ RSG1 + MS2 → SN1 + MS2 → NS1 + MS2 →NS1 + RSG2 → NS1 + SN2 → NS1 + NS2.**

The case in which the primary component has a mass $10 < M_1/M_\odot \lesssim 40$ and the secondary has $0.8 \leq M_2/M_\odot \leq 10$:

**4.2: MS1 + MS2→ RSG1 + MS2 → SN1 + MS2 → NS1 + MS2 → NS1 + RSG2 → NS1 + COd2.**

The case in which the primary component has a mass $10 < M_1/M_\odot \lesssim 40$ and the secondary has $M_2/M_\odot < 0.8$:

**4.3: MS1 + MS2→ RSG1 + MS2 → SN1 + MS2 → NS1 + MS2.**

If the mass of one or both components exceeds 40 $M_\odot$, a black hole takes the place of the neutron star.

We note at once that the periods of wide systems can only increase in the course of evolution due to momentum and mass loss by stellar wind and the ejection of an envelope in a supernova explosion or the formation of a planetary nebula.

In scenarios 4.1-4.3, we have omitted relatively brief phases of evolution such as the stage of hydrogen burning in a layered source and the entire phase of nuclear burning following He depletion. However, systems in which the companion of the neutron star is a star on the asymptotic giant branch, for example, must exist, of course, which is reflected below in Table I.

## 4. RESULTS

Neutron stars show up observationally as sources of radio and x-ray emission. In the present paper we confine ourselves mainly to the consideration of single neutron stars and neutron stars that are components of detached binary systems. Such neutron stars can be detected as radio pulsars if they have a strong enough magnetic field and rotate rapidly. A direct comparison of the calculated results with observations is hindered, however, by the fact we do not yet know sufficiently well just what conditions must be satisfied by neutron stars for radio emission to appear, since both the rotation and the field vary with time.

In analyzing the results of our calculations, we shall refer mainly to the version in which the original distribution of close binary systems with respect to the mass ratio of the components is $f(q) = 1$, (i.e., $\alpha = 0$) and the parameter of the common envelope is $\alpha_{ce} = 1$, since those values of the model parameters seem to correspond best to the observations, in light of our earlier population models[7-10] and the results of modeling the evolutionary paths leading to the formation of certain binary systems with well-defined characteristics.

**4.1. Frequency of formation and total population of neutron stars.** The frequencies of formation and the populations of neutron stars in binary systems with different components for a number of values of $\alpha$ and $\alpha_{ce}$ are given in Table I. For the most populous single neutron stars, in Table II we give the frequencies of formation and the populations of stars formed in different scenarios. Here the scenarios are combined for events that determine the appearance of a single neutron star. Row 1 corresponds to scenarios 1.1, 1.3, 4.1, 4.2, and 4.3, row 2 to scenarios 1.1, 1.2, 1.4, and 4.1, row 3 to scenarios 2.1, 2.2, 3.2, and 3.3, row 4 to scenario 3.5, and row 5 to scenarios 3.1 and 3.4.

We determined the populations of neutrons stars in systems of different types as follows:

$$N = \nu\cdot\tau \text{ for } T + \tau \leq T_H,$$
$$N = \nu\cdot(T_H - T) \text{ for } T + \tau > T_H,$$

where $\nu$ is the frequency of creation of systems that produce neutron stars, $T_H = 15\cdot10^9$ years is the Hubble time, $T$ is the system's lifetime before the neutron star is formed, and $\tau$ is the neutron star's lifetime in that system. The time $\tau$ is limited either to the time before merging of the components or to the time $T_H - T$.

The total frequency of formation of neutron stars in the Galaxy, according to our model, is 0.024 $\text{yr}^{-1}$ (for $\alpha = 0$ and $\alpha_{ce} = 1$), which is comparable to the estimates of Narayan and Ostriker,[37] obtained by numerical modeling of the observed population of radio pulsars: (0.017-0.055) $\text{yr}^{-1}$. It is interesting the frequency $\nu$ of creation of neutron stars increases both with increasing and with decreasing $\alpha$. An increase in the relative population of stars with components of comparable masses ($\alpha = 1$) increases $\nu$ due to the increase in the population of secondary components that produce neutron stars. An increase in the relative population of systems with components having a large initial mass ratio ($\alpha = -1$) leads to an increase in the frequency of mergings of components, thereby expanding the range of masses of systems in which the formation of neutron stars is possible. A decrease in the common envelope parameter $\alpha_{ce}$ hardly changes $\nu$, since the population of systems that merge at stages with a common envelope increases, but the population of systems that produce binary CO dwarfs decreases (Table II).

Variation of the distribution of the original binary systems with respect to the masses of the components has relatively little effect on either the frequencies of formation of systems of different types or their numbers in the Galaxy. Only $\nu$ for NS + Hed systems changes more than tenfold with a change in $\alpha$ from $-1$ to 1, since those systems originate in the evolution of stars with large initial mass ratios.

The total number of neutron stars formed over the Hubble time in the Galaxy, assuming the present rate of star formation over its entire history, is $(3\text{-}4)\cdot10^8$ (Table I). It may actually turn out to be several times larger if we allow for the accelerated star formation at early stages of evolution of the Galaxy. The present rate of star formation in the galaxy is[38] $(6 \pm 3)$ $M_\odot$/yr. Consequently, over the Hubble time with steady star formation, a stellar population with a mass of about $9\cdot10^{10}$ $M_\odot$ could have been formed, with most of that matter being in long-lived, low-mass stars. Since the mass of the Galaxy ex-

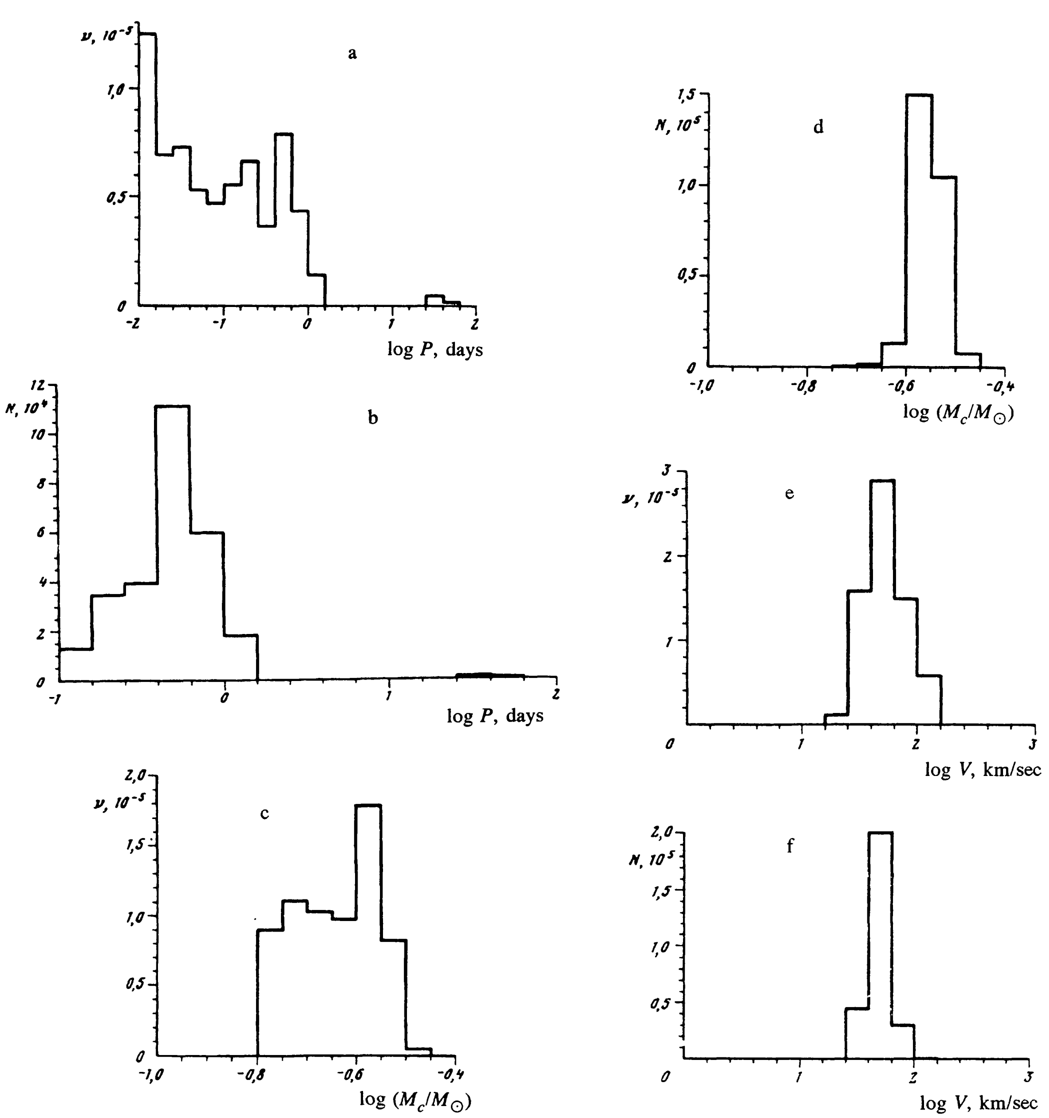

FIG. 5. Neutron stars with helium dwarfs as companions: a) relationship between frequency of formation of NS + Hed systems and their orbital periods; b) distribution of NS + Hed systems with respect to orbital period; c) relationship between frequency of formation of NS + Hed systems and the masses of the dwarfs; d) distribution of NS + Hed systems with respect to the mass of the dwarf; e) relationship between frequency of formation of NS + Hed systems and their rms spatial velocity; f) distribution of NS + Hed systems with respect to rms spatial velocity.

ceeds $\sim 2 \cdot 10^{11}\ M_\odot$, it is obvious that most of the stars were formed at early stages of its evolution. Since the lifetimes of the progenitors of neutron stars are short, most of these "additional" neutron stars are old and, as a rule, should not be radio pulsars. The nearest of them might be detected, however, from the ultraviolet and soft x-ray emission originating in mass accretion from the interstellar medium.[39]

It turns out that 2-3% of the neutron stars are components of close binary systems and 6-7% are components of wide systems (Table I). Among the companions of neutron stars, CO and ONe dwarfs, black holes, and neutron stars predominate. About ten radio pulsars that are components of binary systems are known outside of globular clusters (we do not consider globular clusters, in which systems containing neutron stars are formed by inelastic binary collisions). At the same time, the period derivatives of 265 Galactic pulsars have been measured, making it possible to detect duplicity.[37] The observed degree of duplicity is therefore formally explained by our model. It is noteworthy, however, that wide pairs ($P_{\rm orb} \approx 10^2$-$10^6$ yr) predominate among "theoretical" binary systems, while essentially all observed systems, except for PSR 0820+02 ($P_{\rm orb}$ = 1200 days), are close. According to Ref. 40, for a typical time of observation of radio pulsars of about ten years, companions of comparable mass can be detected for $P_{\rm orb} \leq 2 \cdot 10^4$ yr. The distribution of binary neutron stars with respect to orbital period shows that about 1/100 of all young neutron stars (i.e., four radio pulsars) may be components of binary systems with orbital periods from 300 to 20,000 years (see Figs. 2-10). It is not yet clear why no such systems are known among the 400 investigated pulsars. It is possible that

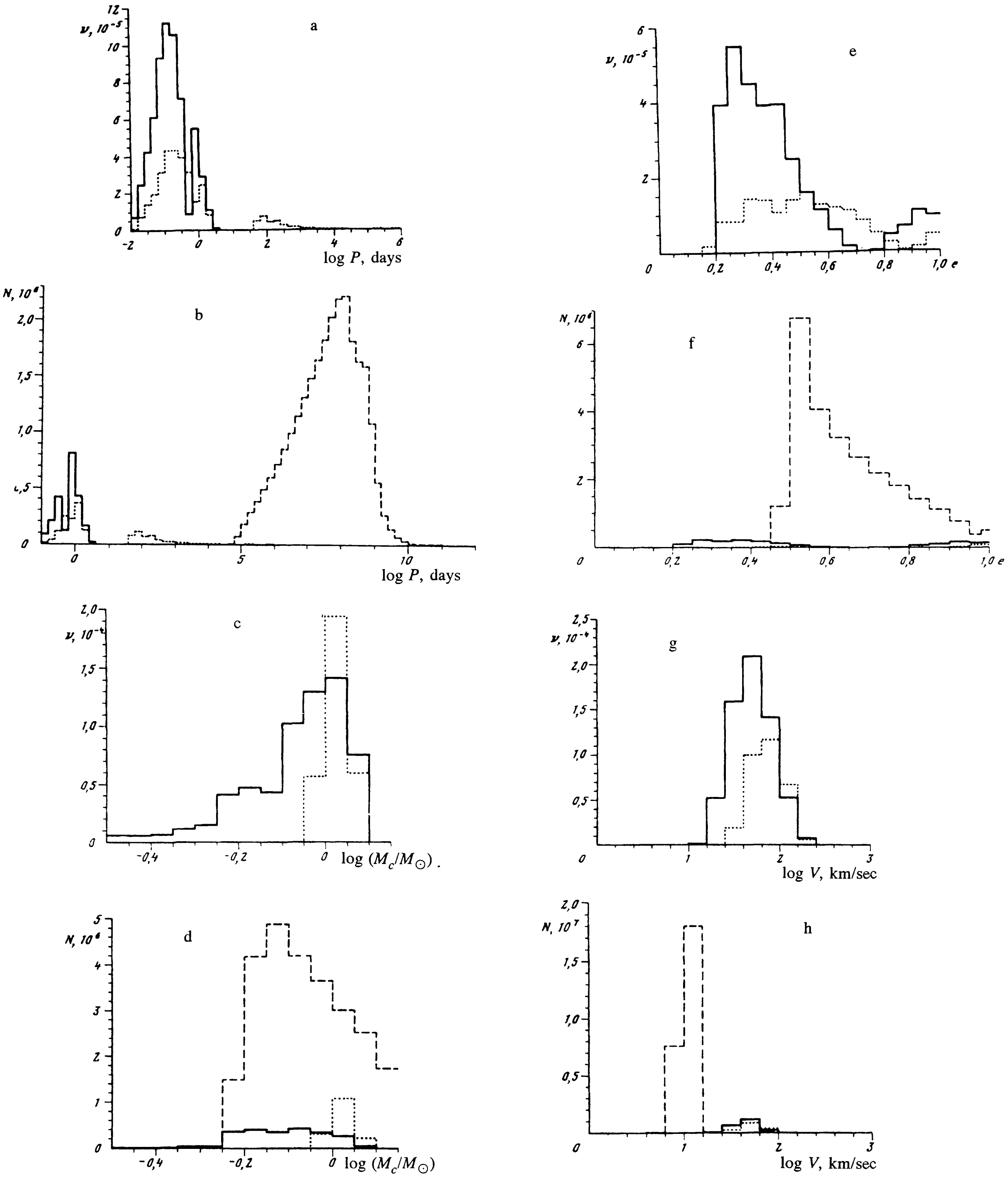

FIG. 6. Systems with carbon–oxygen dwarfs as companions: a) relationship between frequency of formation of close systems and their orbital periods; solid line) NS + COd systems; dotted line) COd + NS systems; b) distribution of systems with carbon–oxygen dwarfs as companions with respect to orbital period; solid line) close NS + COd systems; dotted line) COd + NS systems; dashed line) wide NS + COd systems; c) relationship between frequency of formation of close NS + COd and COd + NS systems and the masses of the dwarfs; solid line) NS + COd systems; dotted line) COd + NS systems; d) distribution of systems containing CO dwarfs with respect to the dwarf's mass; solid line) close NS + COd systems; dotted line) close COd + NS systems; dashed line) wide NS + COd systems; e) relationship between frequency of formation and orbital eccentricities of close COd + NS (solid line) and ONed + NS (dotted line) systems; f) distribution of systems with white dwarfs as companions with respect to orbital eccentricity; solid line) COd + NS systems; dotted line) ONed + NS systems; dashed line) wide NS + COd systems; g) relationship between the frequency of formation and the rms spatial velocity of NS + COd systems; solid line) close systems; dotted line) wide systems; h) distribution of systems with carbon–oxygen dwarfs as companions with respect to rms spatial velocity; solid line) close NS + COd systems; dotted line) close COd + NS systems; dashed line) wide COd + NS systems.

neutron stars whose progenitors were components of wide systems or were single stars are not strong radio sources because of their slow rotation.[41] The pulsar PSR 0820+02 in a binary system is well known; it is not close in the evolutionary sense, but has a relatively short period for wide systems, 1200 days. Because of the latter fact, the appearance of a radio pulsar in this system can be explained by acceleration of the neutron star's rotation in the accretion of stellar wind from its companion when that star was a supergiant. The small eccentricity of the orbit of the secondary component in this system ($e = 0.1$) is evidence that it passed through the supergiant stage.

The most well-represented close binary systems with neutron stars as components are binary neutron stars and neutron stars paired with carbon−oxygen or oxygen−neon dwarfs and black holes. A considerably smaller number of neutron stars have main sequence stars, supergiants, and nondegenerate helium stars as companions, although the frequencies of creation of all of those systems considerably exceed the frequency of creation of neutron stars in pairs with CO and ONe dwarfs. The latter is easily explained by the short lifetime of main sequence and helium stars compared to the lifetime of neutron stars and dwarfs, which turns out to be close to the Hubble time because of the short life of their progenitors.

The relatively low ratio of the population of close systems with neutron stars as components to that of single neutron stars ($\approx 0.03$) in comparison to the ratio of the frequencies of their formation ($\approx 0.14$) is explained by the merging of the components of close systems due to the emission of gravitational waves. The fraction of merging systems among close NS + COd stars is also large. The difference between the relative frequencies of formation and the populations of NS + ONed and ONed + NS systems is especially pronounced. The frequency of formation of single neutron stars is only 50 times higher than the frequency of formation of systems of each of these types, while the populations differ by a factor of more than 1000. The components of essentially all NS + ONed and ONed + NS systems merge due to the emission of gravitational waves, since their orbital periods are short owing to the three or four stages of a common envelope that their progenitors pass through.

About 1% of neutron stars may have black holes as companions (Table I).

The frequencies of formation of single and binary neutron stars in close and wide systems are comparable. Some classes of objects containing neutron stars in wide systems are not formed at all in our model, however. For example, COd + NS systems cannot be formed in principle, since this requires that the secondary component initially be more massive than the primary. No NS + Hed, NS + ONed, or ONed + NS systems develop, since we have assumed, in accordance with our present understanding, that helium and oxygen−neon dwarfs are formed only in the course of evolution of the components of close binaries.

It is interesting that most of the close binaries that could have formed COd + NS systems and all of the close systems that are potential progenitors of NS + BH systems are disrupted in the second supernova explosion because of the large mass ratio of the components at that time. About 0.4% of all wide systems in which the mass of the primary component is more than 10 $M_{\odot}$ are converted into wide binary neutron stars, i.e., they are not disrupted in the second supernova explosion. About 40% of them have orbital periods that are detectable,[40] but no such pulsars have yet been found (see the discussion in Sec. 4.2.b for more detail).

**4.2. Main characteristics of binary and single neutron stars.** We now analyze the frequencies of formation and the populations of systems containing neutron stars as a function of their main parameters: the masses of the components and the semimajor axes and eccentricities of their orbits. For the reasons described above, we chose the model with $\alpha = 0$ and $\alpha_{ce} = 1$ for graphic representation and a detailed analysis. The need to analyze the frequencies of formation is related to the possibility of comparing the properties of our models of the population of neutron stars with the characteristics of the set of observed radio pulsars. The typical lifetime of radio pulsars is $\sim 10^7$ years,[42] which is far less than either the cosmological time or the lifetime of systems containing neutron stars. The theoretical distributions of systems containing neutron stars with respect to the frequency of their formation are therefore close to the distributions of the observed objects.

For an adequate comparison of the observed set of radio pulsars with theoretical models of the population of neutron stars, we must isolate the radio pulsars among the latter. For this we must model the time variations in the radio power, which must, in turn, allow for the evolution of the rotation rate and magnetic field of neutron stars. We must also allow for effects of observational selection, including, for example, a test of the conditions of passage of the radiation through the circumstellar matter, whose density may be considerable if there is a strong stellar wind from the neutron star's companion. In a study of the rotation of a neutron star, it is of fundamental importance to allow for its accretion of matter from its companion, which is capable of either accelerating or slowing the rotation.[43] Without modeling of the evolution of radio emission from neutron stars and allowance for selection, any comparison of the properties of young neutron stars with radio pulsars can only be very tentative and preliminary.

**a) Single neutron stars.** Most neutron stars that are products of the evolution of binary systems end up as single stars. The total frequency of their formation and their population under various assumptions about the model parameters are given in Table I. The efficiencies of the most productive channels of formation of single neutron stars are characterized by the data in Table II. We must again stipulate the hypothetical nature of the possibility of the formation of neutron stars as a result of the merging of carbon−oxygen dwarfs, since the merging may result in explosive ignition of carbon without the formation of a bound remnant (see the discussion of scenario 3.1).

In our model, single neutron stars are characterized by only two parameters: mass and spatial velocity. We assumed that a neutron star's initial mass is 1.4 $M_{\odot}$. The masses of neutron stars can grow in the course of mass transfer in close systems; they remain constant in wide ones. The rate of mass increase is limited, in our adopted model, by the critical accretion rate corresponding to the Eddington luminosity. In cases in which the estimated rate of mass transfer at the time of Roche lobe filling by the neutron star's companion exceeded the critical value, we assumed that the components merge with the formation of a Thorne−Zytkow object.

The distribution of single neutron stars with respect to

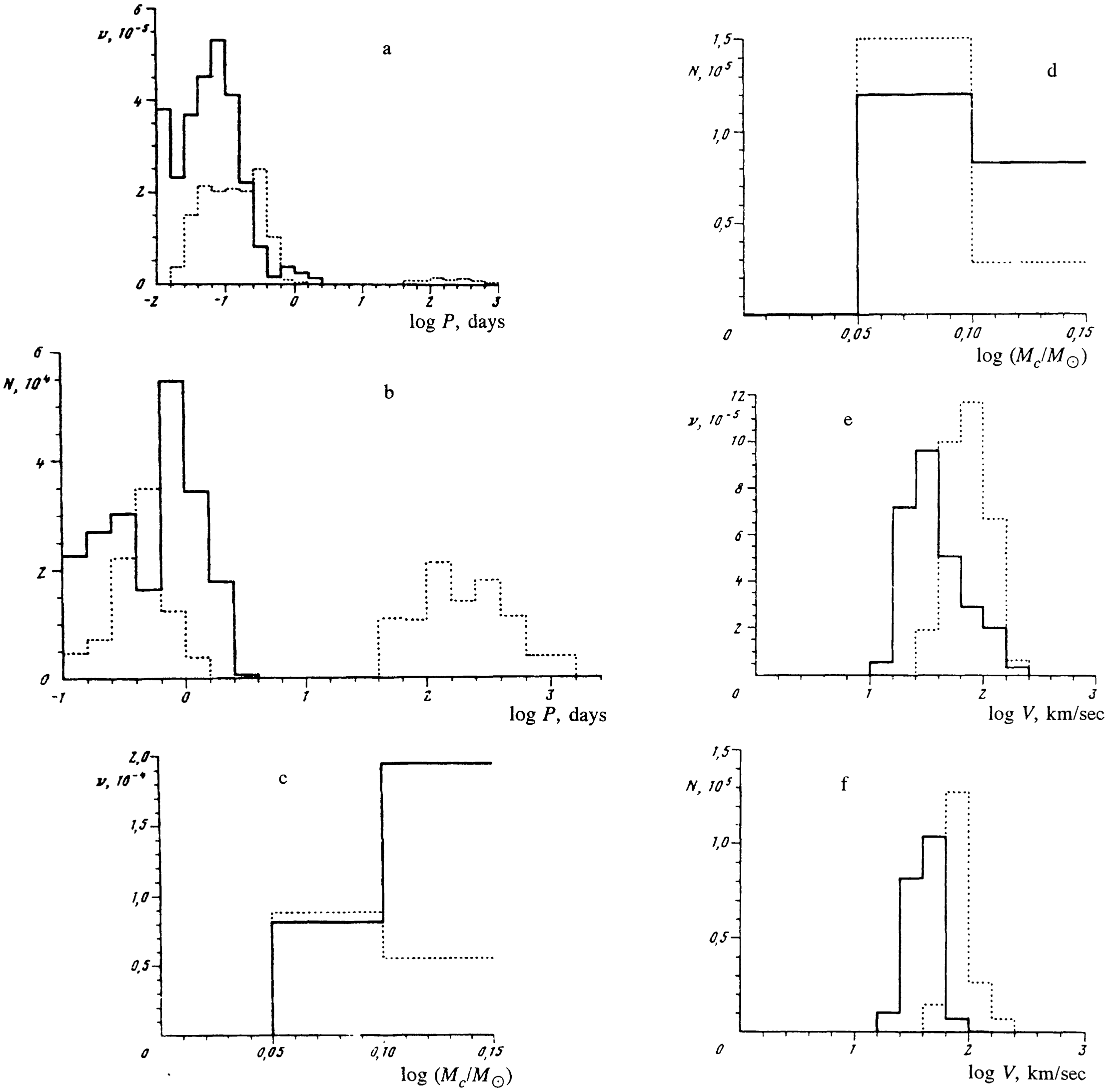

FIG. 7. Systems with oxygen–neon dwarfs as companions: a) relationship between frequency of formation and orbital periods of NS + ONed systems (solid line) and ONed + NS systems (dotted line); b) distributions of NS + ONed and ONed + NS systems with respect to orbital period; notation same as in Fig. 7a; c) relationship between frequency of formation and masses of the neutron stars' companions in NS + ONed and ONed + NS systems; notation same as in Fig. 7a; d) distribution of NS + ONed and ONed + NS systems with respect to the mass of the neutron star's companion; notation same as in Fig. 7a; e) relationship between frequency of formation and rms spatial velocity of NS + ONed and ONed + NS systems; notation same as in Fig. 7a; f) distributions of NS + ONed and ONed + NS systems with respect to rms spatial velocity; notation same as in Fig. 7a.

mass is given in Fig. 1a. The vast majority of the stars retain their initial masses. Several percent of single stars have large masses, up to our chosen limit of 2 $M_\odot$. The maxima in the distribution near 1.57 and 1.74 $M_\odot$ originate from accretion in Thorne–Zytkow objects with hydrogen envelopes and in low-mass x-ray sources with helium donors, respectively.

The distribution of single neutron stars with respect to mass is not yet known from observations. An analysis of the rotation rates of radio pulsars, however, enables us to identify millisecond pulsars, which may have masses that differ considerably from the initial mass 1.4 $M_\odot$. The acceleration is due to the disk nature of accretion onto very compact objects in binary systems, since a very small angular momentum is sufficient for a disk to develop in this case. The rotation period of a neutron star with a weak magnetic field decreases as a result of accretion of a mass $M$ to $P_f \approx 2.6 \cdot 10^{-4} (M/M_\odot)^{-1}$ (Ref. 44). From the expression for $P_f$, it follows that the accretion of 0.1 $M_\odot$ is capable of accelerating the neutron star's rotation to a period 0.0026 sec, typical of millisecond radio pulsars. This means that all neutron stars with masses greater than 1.5 $M_\odot$, which comprise about 2.5% of the total population of single neutron stars, may become millisecond pulsars after a certain time, according to our model. Consequently, if the envelope matter in Thorne–Zytkow objects retains a considerable fraction of the system's original orbital angular momentum, their cores become millisecond radio pulsars after dissipation of the envelope. We can thus point to another mechanism of origin of millisecond pulsars in addition to that usually suggested, which is associated with acceleration of the rotation of neutron stars in semidetached systems, with the collapse of accreting oxygen–neon white dwarfs, and (or) the disruption of the donors in low-mass x-ray sources.

The distribution of single neutron stars with respect to their rms spatial velocities, which characterize the velocity dispersion, has two components (Figs. 1b-d). We ascribed $v_{sp} = (1500/M_1)^{1/2}$ km/sec to the original systems. The products of the evolution of the components of wide systems and the objects formed in the merging of the components of close systems in common envelopes and the merging of degenerate dwarfs retain their low velocities ($\approx 10$ km/sec). The remnants of components whose explosion destroyed the system have velocities from 10 to 250 km/sec. Neutron stars that are the products of the evolution of primary components that were in bound systems before the second supernova explosion ("old" neutron stars) acquire velocities up to 1000 km/sec. A comparison of Fig. 1b with Figs. 1c and d shows that the model distribution of neutron stars with respect to the dispersion of spatial velocities depends little on the parameters $\alpha$ and $\alpha_{ce}$ of the problem.

Allowance for observational selection effects associated with the brightness and lifetime of radio pulsars enabled us[41] to identify, among 26 observed pulsars with known transverse velocities,[45] two families of approximately equal populations: with velocity dispersions less than 50 km/sec and about 250 km/sec (Fig. 1e). Modeling of the distribution of observed radio pulsars with respect to spatial velocity with allowance for pulsar evolution and selection effects[37] confirmed the existence of "slow" and "fast" populations. A comparison of Figs. 1b and e shows satisfactory agreement between the theoretical model and observational data corrected for selection effects. A more detailed comparison of the calculated results and theory is difficult, since it is not known just which of the theoretically predicted neutron stars can be identified with radio pulsars.

The spatial velocities of the fastest radio pulsars (up to 1000 km/sec) are quite capable of resulting in their escape from their parent galaxy and the formation of a corona of neutron stars around it. The rate of loss of neutron stars by the Galaxy is $\sim 10^{-4}$ per year, according to our model.

An important result of the comparison of the observed velocities of radio pulsars with the theoretical values for neutron stars is the conclusion that our assumptions are sufficient to explain the observed velocity dispersion of pulsars. We did not find it necessary to introduce an initial "kick" or some substantial initial velocity, which the neutron stars acquire in a supernova explosion.[1,46] We cannot entirely rule out the possibility of a "kick" due to some asymmetry of the explosion, of course, but its absolute magnitude, if it occurs, probably does not exceed several dozen kilometers per second,[41,43] or else it would be complicated to explain the small velocity dispersion ($\leq 50$ km/sec) of the slow population of radio pulsars.

**b) Binary neutron stars.** The progenitors of close binary systems, both components of which are neutron stars, are mainly systems with components of initial masses 11.4-14.0 $M_\odot$. Most of the young NS + NS systems have orbital periods shorter than 0.3 days. The number of systems with short periods decreases rapidly, however, because of their merging due to the emission of gravitational waves: $N \propto P^{8/3}$ (Fig. 2b). The orbital periods of most of the potentially observable systems of this type must therefore be about 0.3-2 days (Figs. 2a, b). All four of the known radio pulsars with neutron stars as their presumed companions have orbital periods in the theoretically "predicted" range 7.8-12.34 h (Refs. 47 and 48). Among binary neutron stars that are products of the evolution of close binaries, there are systems with orbital periods up to several years. They originated from conservative evolution in the first mass transfer between the components. They cannot explain the appearance of the PSR 0820+26 system with a period 1232 days and a zero orbital eccentricity, however, since the orbits of all binary neutron stars should be noncircular (Figs. 2d, e). The companion of PSR 0820+26 is probably a degenerate dwarf – the product of the evolution of a red supergiant; this system's orbit could have become circular due to energy dissipation in the envelope of the supergiant – the progenitor of the dwarf.

Helium stars with masses 2.8-3.6 $M_\odot$ expand after a degenerate carbon–oxygen or oxygen–neon core has been formed in their interior. In the closest systems, they fill their Roche lobe, form a common envelope, and lose mass. The system's period and the mass of the progenitor of the supernova and hence the mass of the ejected shell decrease considerably as a result. The orbital eccentricity after the explosion consequently decreases as well, $e = \Delta M/M_f$, where $\Delta M$ is the change in the mass of the exploding component and $M_f$ is the system's final mass. Unfortunately, there are no detailed and sufficiently complete calculations of the evolution of mass-losing helium components of binary systems. The masses of the stellar remnants are not known, in particular. We assumed that expanding stars with carbon–oxygen cores lose 90% of the mass of their helium envelope. Under this assumption, most close NS + NS systems have an orbital eccentricity $e \approx$ 0.2-0.4. (It is possible, however, that a somewhat smaller fraction of the mass of the helium envelope is lost: about 70%.) An increase in the mass of the retained helium envelope, as a numerical experiment showed, leads to an increase in the fraction of systems with $e \geq 0.5$. The problem of mass loss by helium stars and its associated problem of the distribution of close binary neutron stars with respect to orbital eccentricity merit particular attention, since they can be tested directly by observations of binary radio pulsars.

Long-period systems ($P_{\rm orb} \geq 25$ days) have, as a rule, large orbital eccentricities ($e \geq 0.7$). Systems with large orbital eccentricities and relatively short periods were formed from close systems in which the masses of the helium supernova progenitors were in the range 3.6-4.2 $M_\odot$. Those progenitors did not expand after helium depletion in their core, and their explosion was accompanied by considerable mass loss. Systems in which one of the components is a neutron star and that had a circular orbit before the explosion are disrupted if the mass of the progenitor was more than 4.2 $M_\odot$. Since systems with small orbital eccentricities are among the closest ones, the emission of gravitational waves considerably shortens their lifetime and hence their population (cf. Figs. 2b and c with Figs. 2d and e). The emission of gravitational waves also affects the relative population of systems with large orbital eccentricities, however, since the time of merging of the components depends strongly on the initial orbital eccentricity $e_0$: $T \approx T_0(1 - e_0^2)^{3.42}$, where $T_0$ is the time of merging for $e_0 = 0$ (Ref. 49). Of the two candidates to be binary neutron stars observed outside of globular clusters[47] – PSR 1534+12 and PSR 1913+16 – the former has $e = 0.27$ and the latter has $e$

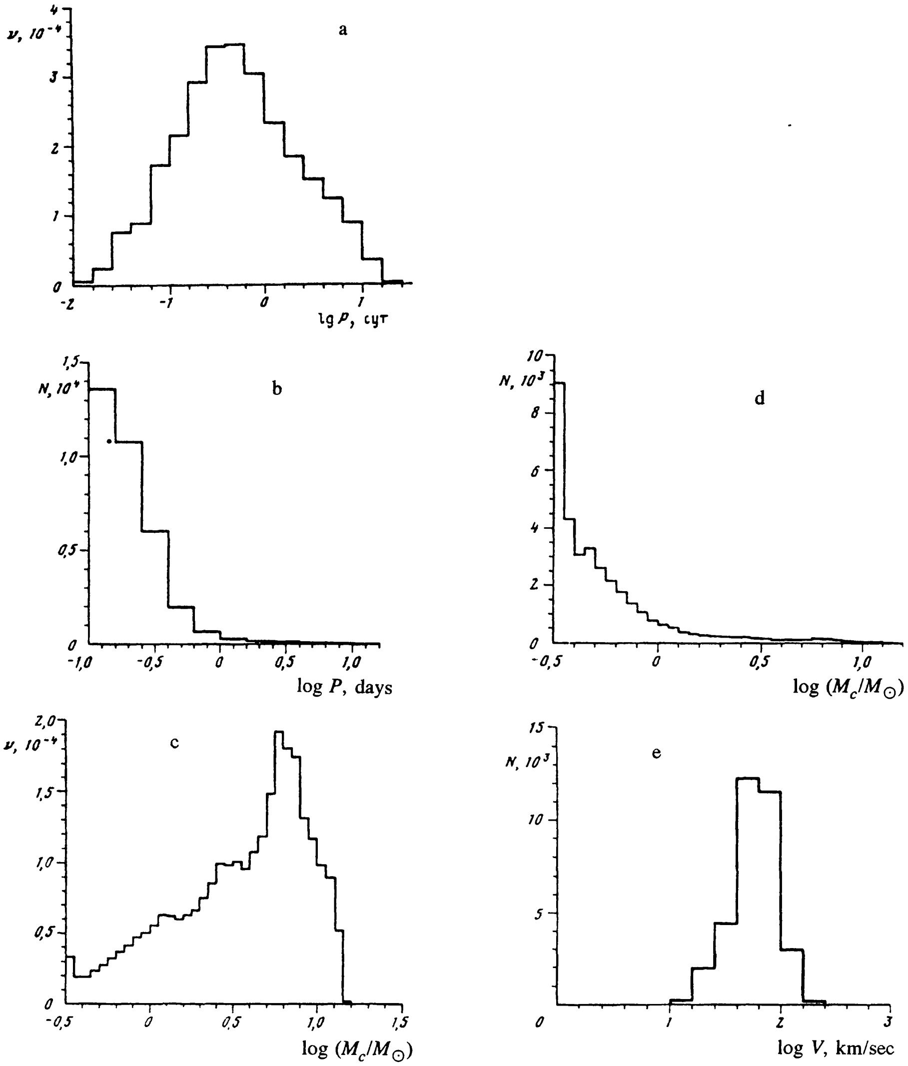

FIG. 8. Systems with helium stars as companions: a) relationship between frequency of formation and orbital periods of NS + Hes systems; b) distribution of NS + Hes systems with respect to orbital period; c) relationship between frequency of formation of NS + Hes systems and the mass of the neutron star's companion; d) distribution of NS + Hes systems with respect to the mass of the helium star; e) distribution of NS + Hes systems with respect to rms spatial velocity.

= 0.62. Unfortunately, the statistics are still insufficient for a detailed discussion of the degree of correspondence between theory and observations with respect to orbital eccentricity.

Binary neutron stars can be divided into two groups with respect to rms space velocity (Figs. 2f, g). The products of the evolution of close systems acquire a velocity $v_{sp}$ = 40-250 km/sec after the second supernova explosion in the system, and the products of the evolution of wide systems remain slow with $v_{sp} \approx$ 10 km/sec, since we have assumed that supernova explosions are spherically symmetric and do not impart additional velocity to the products of the explosions.

Merging neutron stars are some of the main sources of pulsed gravitational radiation. According to our model, with the presently planned laser interferometric detectors of gravitational waves,[50] it is possible to detect up to ~100 pulses of gravitational waves per year at a frequency of about 100 Hz from a distance of up to 200 Mpc (Ref. 51).

Before merging, NS + NS systems may travel up to $r \approx$ 1000 kpc from the site of their creation. Their distribution with respect to log $r$ in the range $1 \leq \log r$ (kpc) $\leq 2.5$ is then essentially flat.

**c) Systems consisting of neutron stars and black holes.** No systems in which the primary component produces a neutron star and the secondary produces a black hole (NS + BH) arise under our assumptions, since with a mass 1.4 $M_\odot$ for neutron stars, $M \geq 40\ M_\odot$ for the progenitors of black holes, and $M = 5\ M_\odot$ for black holes, all pairs that are potential progenitors of NS + BH systems are disrupted in the second supernova explosion.

Fairly many BH + NS binary systems, both wide and close, arise, however, whose distribution with respect to orbital period is given in Fig. 3a. Since the formation of such systems is associated with the loss of a considerable part of the system's mass due to stellar wind, accompanied by divergence of the components, those systems are wider than NS + NS systems. The typical orbital periods of wide systems are $10^6$-$10^9$ days. The components of some of the closest BH + NS systems merge due to the emission of gravitational waves by the system (Figs. 2b, c).

The distributions of close and wide BH + NS systems with respect to rms spatial velocity differ (Fig. 3b). The products of the evolution of wide systems retain low velocities, even after two supernova explosions, whereas close BH + NS systems are accelerated by two supernova explosions to $v_{sp} \approx$ 400 km/sec.

The distributions of close and wide BH + NS systems with respect to orbital eccentricity differ considerably (Figs. 3c, d). The orbital eccentricities of essentially all wide systems range from 0.5 to 1, since two successive explosions eliminate the possibility of retaining small eccentricities (see Fig. 3 in the paper by Hills[21]). Close systems include objects with small $e$, since the supernova explosion is preceded by loss of its envelope by the helium star, which decreases the mass ejected in the explosion. Some of the close systems with small $e$ merge (Figs. 2b, c). Merging BH + NS systems may also be some of the main potentially detectable sources of pulsed emission of gravitational waves.[51]

It is not yet clear whether there are examples of BH + NS systems among the observed binary radio pulsars. The relative frequency of formation of BH + NS systems is only one third that of that for close NS + NS systems (Table I), so we can count on the detection soon of radio pulsars with black holes as companions.

**d) Neutron stars with main sequence stars as companions.** Most close binary systems remain bound following the first supernova explosion in the system (NS + MS, Fig. 4). Close NS + MS systems are separated into two groups. One is formed by initially relatively less massive systems that have passed through two stages of a common envelope: after Roche lobe filling by the hydrogen star and during expansion of the helium star after helium depletion in its core. Those systems have short periods and small orbital eccentricities (Figs. 4a, b, e, f). (Concerning the eccentricities after the first supernova explosion, we assumed that in systems for which $a/R \leq 10$ after the explosion, where $R$ is the radius of the companion, the orbits are circular, since the characteristic time of circularization of the orbits of such stars is less than the lifetime of the components on the main sequence.[52]) Since the mass of matter ejected in the explosions in small in these systems (less than 2 $M_\odot$), only systems with companions having initial masses no more than 0.4 $M_\odot$ are disrupted in the first explosion (Fig. 4c). Mass transfer was conservative in systems of the second group and there was no common helium envelope. Orbital periods longer than 10 days and a predominance of massive companions ($\geq 10\ M_\odot$) are typical of them after the supernova explosion (Figs. 4b-d). The lifetime of massive companions is short, however, so the populations of short-period and long-period binaries are comparable. The difference in the masses of the companions also disappears (Figs. 4c, d). Systems in the long-period group have initially small orbital eccentricities (0.1-0.5), since the mass of ejected matter is small relative to the system's total mass. Under our assumptions, however, those systems are wide enough after the explosion that the orbits do not rapidly become circular. They disappear in the population distributions of the systems (cf. Figs. 4e and f).

In wide systems, the initial distribution of the companions with respect to mass is determined, on the one hand, by their zero-age mass function, and on the other, by the disruption of systems with companions of small and medium mass. For circular orbits, all systems with $M_2 \leq 7.2\ M_\odot$ would be disrupted, but for elliptical orbits, some of those systems are preserved. It is they that predominate in the population distribution of companions, owing to their long lifetime (Figs. 4c, d). In the frequency distribution of wide systems with respect to orbital eccentricity, systems with $e \approx 0.5$-$0.6$ dominate (Fig. 4e). Binary systems following a supernova explosion are concentrated in this very range if their initial orbital eccentricities are $0.2 \leq e \leq 0.8$ and the ratio of the mass of ejected matter to the system's initial mass is $\Delta M/M \gtrsim 0.4$ (see Fig. 3 in Hills's paper[21]). Because of the short lifetimes of the companions, the $e-N$ distribution also becomes flat.

The observable objects corresponding to NS + MS systems are bright x-ray sources. They are subdivided into two classes. Some of them originate when the massive companion of the neutron star expands and almosts fills its Roche lobe. This enhances the stellar wind and increases the accretion rate, which results in the appearance of noticeable x rays. But when the massive star fills its Roche lobe, the x rays are completely absorbed by the dense matter in the vicinity of the neutron star. Sources in the other group are formed in systems with elliptical orbits, in which periodic Roche lobe filling is possible near periastron, which leads to the appearance of transient x-ray sources.

The orbital eccentricities of detached massive NS + MS systems, which are observed as transient x-ray sources, are in the range[53] 0.1-0.9, which agrees with the theoretical model (Figs. 4e, f). A comparison of the orbital periods of transient sources with the model (Fig. 4b) also reveals that essentially all of those sources in massive binaries belong to the group of systems with orbital periods 10-100 days, whereas steady massive x-ray systems (of the Cen X-3 type) have orbital periods 1.7-9 days, which correspond to the group of closest stars in Fig. 4b. The difference between transient and steady x-ray sources may thus be determined by their previous evolution: nonconservative mass transfer results in the appearance of steady sources, while transient sources arise in the course of conservative evolution.

At their creation, the main sequence stars in most close NS + MS systems have masses $M_c = 10$-$25\ M_\odot$ (Fig. 4c). This is completely confirmed by the observed distribution of massive transient and steady x-ray sources with respect to the mass of the visible component. According to data in the catalog of Aslanov et al.,[53] the masses of the visible components in 20 sources are in the range 2.2-30 $M_\odot$, with the masses of 18 of them exceeding 8 $M_\odot$. Since low-mass stars live longer than massive stars, most NS + MS systems should have companions of relatively low mass, 0.8-3 $M_\odot$ (Fig. 4d). This pertains to both close and wide systems.

Figure 4c demonstrates that in about a third of all close NS + MS systems, the mass of the companion at creation is $M \leq 10\ M_\odot$. Low-mass main-sequence stars in close systems do not have the intense stellar wind capable of screening the radio emission of the neutron star. Nor can the wind from massive companions of neutron stars in wide systems screen the radio emission. A natural question arises: why are no visible companions of radio pulsars with spectral types A, B, or O known, since they should be detectable at heliocentric distances up to several kiloparsecs? The results of a search for such companions will help to answer the question of whether radio pulsars are formed in the course of evolution of the components of wide systems and (or) single stars or if mass and momentum transfer between components are needed to spin up the neutron star, and whether the neutron star receives some "kick" of about 100 km/sec at the time of its formation.

The distribution of NS + MS systems with respect to rms spatial velocity is given in Figs. 4g, h. The products of the evolution of close systems have typical spatial velocities 25-100 km/sec, which coincide with the typical velocities of so-called runaway stars.[54,55]

**d) Neutron stars with helium white dwarfs as companions.** Some massive close binaries with secondary components having initial masses $\leq 2.8\ M_\odot$ evolve into systems in which the companion of the neutron star is a helium white dwarf (NS + Hed). Their distribution with respect to orbital period is given in Figs. 5a, b. The closest systems with periods shorter than 1 day passed through two stages of a common envelope – hydrogen and helium – in the course of evolution before the neutron star was formed. In those systems with orbital periods less than 1 day, the companion of the neutron star fills its Roche lobe and a common envelope is also formed. The components converge further, and the mass of the resulting dwarf does not exceed $0.25\ M_\odot$ (Fig. 5c). The components of those systems then merge due to the emission of gravitational waves (cf. Figs. 5a and b). Thorne–Zytkow objects with helium envelopes arise which, after losing the latter, form single neutron stars with masses not much greater than $1.4\ M_\odot$ (see scenario 3.5).

Some NS + Hed systems have periods 25-60 days. These stars originated somewhat differently. If the distance between the components of the system following the formation of the neutron star is $a \lesssim 30\ R_\odot$, and the ratio of the mass of the component with a degenerate helium core, which fills its Roche lobe, to the mass of the neutron star is less than 0.78, then steady mass transfer between components at a rate $\dot{M} \approx 10^{-10}(a/R_\odot)^{1.4}\ M_\odot$/yr is possible.[56,57] After the donor's hydrogen envelope is exhausted, there appears a helium dwarf paired with a neutron star. The population of these systems is small (Table I), which is easy to understand, since there are very stringent constraints on the initial mass of the secondary component: 0.8-1.09 $M_\odot$ (with the initial mass of the primary component exceeding $11.4\ M_\odot$). The mass of the degenerate helium dwarf in such systems is uniquely related to the system's orbital period:[56] $M_{\mathrm{He}}/M_\odot \approx 0.14(P/\mathrm{day})^{0.19}$.

Among the radio pulsars observed in close binary systems, at least one (PSR 1953+29), which is in a system with a circular orbit and a period 117 days, may have a helium dwarf as a companion. In this case, the dwarf's mass estimated from the orbital period is $0.32\ M_\odot$. Our model of the population of NS + Hed systems contains no such long-period stars, since we assumed that semidetached systems with a neutron star as the accretor evolve conservatively if $\dot{M} \leq \dot{M}_{Ed}$. At the end of the semidetached phase in long-period systems, therefore, the neutron star, having increased its mass by more than $0.6\ M_\odot$, collapses into a black hole. This removes such systems from our theoretical sample for NS + Hed stars. Partial evaporation of the donor by x rays from the accreting neutron star[58] may prevent the collapse of the neutron star and explain the observed orbital period of the PSR 1953+29 system.

The dispersion of the spatial velocities of NS + Hed sys-

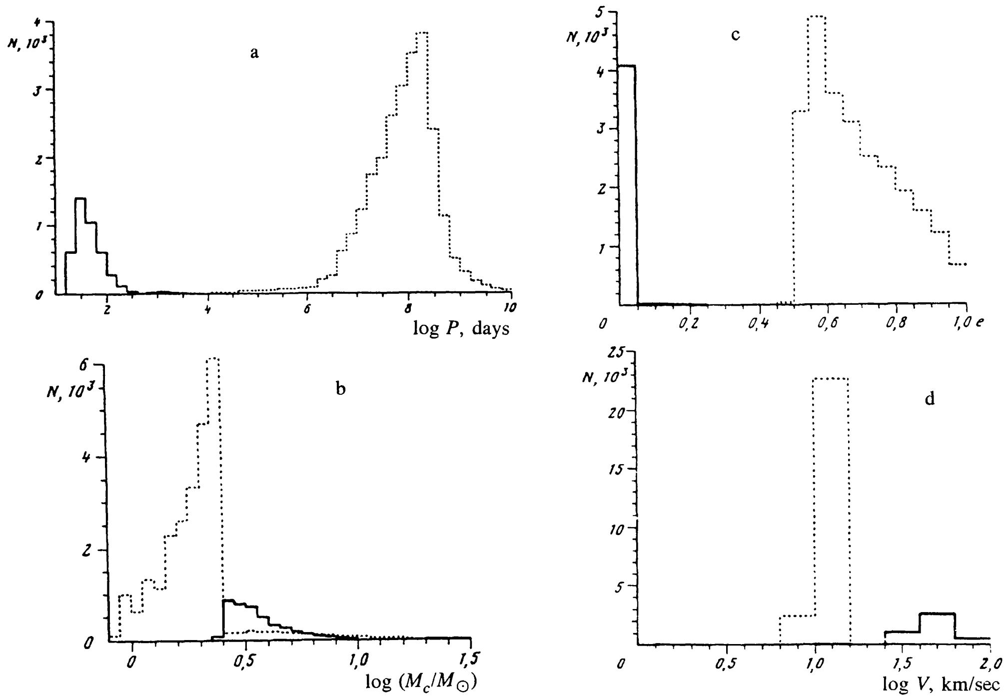

FIG. 9. Systems with giants and supergiants with nondegenerate helium cores as companions: solid line) close systems; dotted line) wide systems; a) distribution of NS + SG systems with respect to orbital period; b) distribution of NS + SG systems with repsect to the mass of the giant or supergiant; c) distribution of NS + SG systems with respect to orbital eccentricity; d) distribution of NS + SG systems with respect to rms spatial velocity.

tems is in the fairly narrow range 40-100 km/sec (Figs. 5e, f), which reflects the narrow ranges of initial semimajor axes of the orbits and of the masses of the components of the progenitors of those systems.

The masses of the helium dwarfs in NS + Hed systems are 0.25-0.3 $M_\odot$ (Figs. 5c, d). Systems with helium dwarfs of smaller mass, 0.15-0.25 $M_\odot$, merge rapidly (Fig. 5d).

It is interesting that the distribution of NS + Hed systems with respect to the mass of the neutron star is bimodal. Neutron stars with masses 1.4-1.6 $M_\odot$ pass through the phase of evolution in a common envelope on the thermal time scale of the donor ($\approx 10^7$ years). Massive neutron stars ($M \geq 1.92$ $M_\odot$), under our assumption, have passed through the semidetached phase of conservative evolution and have accreted all of the hydrogen envelope of their companion with an initial mass 0.8-1.08 $M_\odot$ and a degenerate helium core with a mass 0.15-0.25 $M_\odot$. Most of those systems, naturally, were converted into BH + Hed systems as a result of accretion.

**e) Neutron stars with carbon–oxygen dwarfs as companions.** We consider systems consisting of a neutron star and a degenerate carbon–oxygen dwarf (NS + COd, COd + NS). They are the most numerous among systems that have a neutron star as one component (Table I). In close systems, the neutron star can be the product of the evolution of either the primary or the secondary component. The distribution of the frequencies of formation and the population of systems containing a CO dwarf as a function of orbital period is given in Figs. 6a, b. The COd + NS systems are products of the evolution of close binaries that have passed through two to four stages with a common envelope. In the first case, momentum loss in these stages shortens the final orbital period to about 1 day, and in the second case to less than 8 h. The components of the vast majority of systems with periods less than 8 h merge due to the emission of gravitational waves, forming first Thorne–Zytkow objects and then single neutron stars (scenario such as 3.5). The orbital eccentricities of all of these systems must be close to zero, since the orbit is circularized when the progenitor of the white dwarf fills its Roche lobe, under our assumptions. Among the observed radio pulsars in binary systems, there are several objects with zero orbital eccentricities and short periods,[48] such as PSR 0655+65 with $P = 1$ day and $e = 10^{-4}$.

Among the companions of neutron stars in "young" NS + COd systems, dwarfs with masses of about 1 $M_\odot$ predominate (Fig. 6c). They do not appear in the closest systems, since their progenitors had relatively large masses, 6-9 $M_\odot$, on the one hand, and their formation was preceded by three or four stages with a common envelope, on the other. Because of their short periods and the large masses of their components, the vast majority of NS + COd systems containing a massive dwarf rapidly merge (Figs. 6c, d). The components of those NS + COd systems in which the progenitor of the white dwarf filled its Roche lobe after helium depletion in its core avoid merging.

Wide NS + COd systems are concentrated toward orbital periods of about $10^8$ days (Figs. 6a, b) and orbital eccentricities larger than 0.5. Their distribution with respect to period is determined by the $P$ distribution of their immediate progenitors, in which the deficit of systems with log $P$ (days) = 5-7 is due, in turn, to the disruption of systems in supernova explosions. The masses of the dwarfs in wide NS + COd systems are in the range 0.6-1.4 $M_\odot$, and the distribution with respect to mass reflects the initial mass spectrum of the secondary components and the conditions of the disruption of wide pairs in the first supernova explosion in the system.

Close binary systems with masses $M_1$ less than 11.4 $M_\odot$ but fairly close to that value and with an initial mass ratio close to unity can form COd + NS systems after a conservative first phase of mass transfer and a nonconservative second phase. Their distribution with respect to orbital period is given in Figs. 6a, b. It has two components, like the distribution of those systems with respect to orbital eccentricity (Figs. 6e, f).

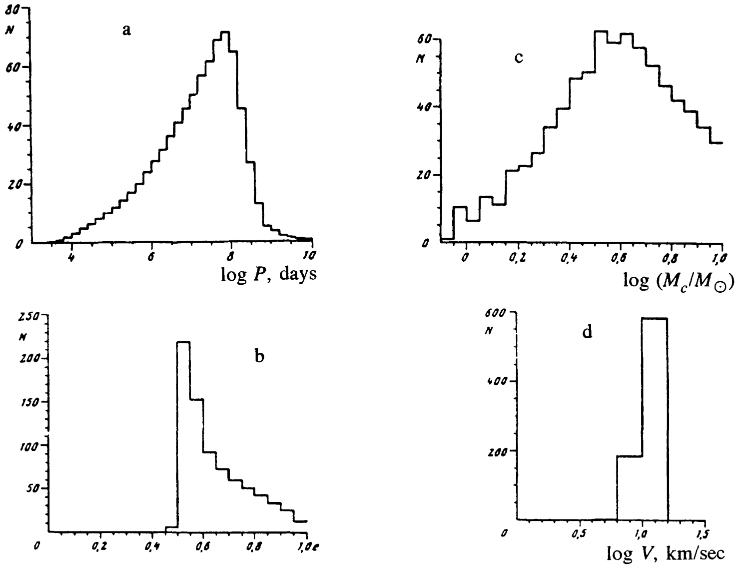

**FIG. 10.** Systems with asymptotic giant branch stars as companions: a) distribution of NS + AGB systems with respect to orbital period; b) distribution of NS + AGB systems with respect to the mass of the AGB star; c) distribution of NS + AGB systems with respect to orbital eccentricity; d) distribution of NS + AGB systems with respect to rms spatial velocity.

The closest systems ($P \lesssim 3$ days) had a phase of a common helium envelope, which additionally reduced the semimajor axis of the system's orbit and the mass of the supernova progenitor. As a result, the orbital eccentricities of those systems were less than 0.6. The remaining systems were relatively wide and have periods 30-1000 days and orbital eccentricities greater than 0.8. We cannot yet state with certainty that there are COd + NS systems among the systems containing radio pulsars.

The dispersion of the spatial velocities of wide systems remains small, as usual. Supernova explosions in NS + COd and COd + NS systems accelerate them to 25-100 km/sec. The potentially "fastest" systems with $v > 100$ km/sec are among the closest systems and their components merge (Figs. 6g, h).

**g) Neutron stars with oxygen−neon dwarfs as companions**. The components of close binary systems having initial masses 9-11.4 $M_\odot$ can, according to Refs. 59 and 60, produce oxygen−neon dwarfs (ONed). Some of them can have neutron stars as companions. The distribution of such systems with respect to orbital period is given in Figs. 7a, b. The NS + ONed systems pass through one stage with a common envelope, as a rule, before the neutron star is formed and two stages with a common envelope before the dwarf is formed. The components of essentially all of them merge, forming Thorne−Zytkow objects first and then single neutron stars. Only initially relatively wide systems, in which at least one of the components filled its Roche lobe after helium burnup in its core, avoid merging. The ONed + NS systems are wider than NS + ONed systems, since they originate from main sequence systems with an iniyial mass ratio of components close to unity, in which the first mass transfer is conservative. In a considerable share of ONed + NS systems, however, the components also merge (Figs. 7a, b). Periods reaching 1000 days are a consequence of supernova explosions in systems with low-mass companions, leading to orbital eccentricities close to unity.

Massive dwarfs [$\log(M/M_\odot) > 0.1$] predominate in "young" NS + ONed systems, since the wider and more massive the initial system, the more favorable are the conditions for the formation of a dwarf. More energy is expended on the ejection of the common envelope in this case, however, and the final system is very close. As a result, after the closest and most massive systems "leave" due to merging, the distributions with respect to the masses of the dwarfs in NS + ONed and ONed + NS systems are similar (Figs. 7c, d).

The circular orbits of NS + ONed systems are a consequence of their passage through phases with a common envelope. The large eccentricities of some ONed + NS systems are a consequence of supernova explosions (Figs. 6e, f).

The NS + ONed and ONed + NS systems differ considerably with respect to spatial velocity (Figs. 7e, f). Essentially all NS + ONed systems have rms velocities 25-60 km/sec. The velocities of ONed + NS systems exceed 60 km/sec and reach 240 km/sec, which is the result of supernova explosions in systems that are low-mass by the time of the explosion.

No examples of neutron stars paired with oxygen−neon dwarfs have yet been detected. It cannot yet be ruled out, however, that ONe dwarfs may be the companions of radio pulsars that have masses comparable to the latter, such as PSR 1534+12, the sum of the masses of whose components is[47] 2.68 $M_\odot$.

**h) Neutron stars with nondegenerate helium stars as companions.** The companion of a neutron star in the course of the evolution of a close binary system may turn out to be a nondegenerate helium star (Hes). The distribution of NS + Hes systems with respect to orbital period is given in Figs. 8a, b. If the mass of the helium star exceeds 5-7 $M_\odot$, its intense mass loss produces the phenomenon of a Wolf−Rayet star. The first such system, with an orbital period 4.8 h, was recently identified[61] with the x-ray source Cyg X-2. The lifetime of massive helium stars is short, however, and systems with low-mass helium stars, down to a minimum $\sim 0.3\ M_\odot$, for which steady helium burning in the core is still possible,[62] should predominate in the population of NS + Hes systems (Figs. 8c, d). Since most of the progenitors of NS + Hes systems pass through two stages with a common envelope, the components converge to periods less than 1 day, as a rule (Fig. 8a). The low-mass helium remnants of main sequence stars are able, over their lifetime[60] $\tau \approx 1.4 \cdot 10^7 (M_{\rm He}/M_\odot)^{-3.7}$, to fill their critical surface due to the emission of gravitational waves, forming low-mass x-ray sources. This may explain the origin of sources with orbital periods down to 10 min, such as 1E 2259+586 and MXB 1820−30 (Refs. 33 and 35). The evolution of such sources probably ends with catastrophic destruction of the donor and the formation of a single millisecond radio pulsar.[32] Some systems, for which $M_{\rm He}/M_{\rm ns} > 1.2$ at the time of Roche lobe filling by the helium star, form Thorne−Zytkow objects, whose evolution also ends with the formation of a single neutron star.

The rms spatial velocities of NS + Hes systems are 40-100 km/sec (Fig. 8e), and their orbital eccentricities are zero after the last phase with a common envelope.

No NS + Hes objects are formed in wide binary systems under our assumptions.

**i) Neutron stars with giants and supergiants as companions.** A star with an initial mass exceeding 2.5 $M_\odot$ spends part of its life in the phase of helium burning in a nondegenerate core. Here we arbitrarily call such stars supergiants (SG). Their companions can also be neutron stars. The distribution of the relatively few NS + SG systems in our Galaxy (Table I) with respect to orbital period is given in Fig. 9a. The products of the evolution of close binaries have typical orbital periods $\leq 100$ days. They have all passed through a nonconservative phase with a common envelope, which considerably decreased the distance between the components.

For systems with $M_2 < 2.5\ M_\odot$, the distance between the components turns out to be so small that the secondary component essentially always fills its Roche lobe before He ignition in the core. This is reflected in the distribution of NS + SG systems with respect to the mass of the companion (Fig. 9b). The range of orbital semimajor axis in which Roche lobe filling is possible after He ignition in the core increases abruptly at $M = 2.5\ M_\odot$, so systems with $M \approx 2.5\ M_\odot$ dominate among NS + SG objects (Fig. 9b). The distributions of wide systems with respect to period and to the mass of the companion reflect the disruption of a considerable fraction of systems with small orbital eccentricities in supernova explosions.

The spatial velocities of wide systems are low, while close systems are accelerated to $v \approx 200$ km/sec by mass loss in supernova explosions (Fig. 9d). The products of the evolution of close systems have zero eccentricity (Fig. 9c), naturally, while the eccentricities of wide systems exceed about 0.5, as is usual for them. They increase over the initial values as a result of supernova explosions in those systems. No examples of NS + SG systems are yet known.

Stars with degenerate carbon–oxygen cores surrounded by layered helium and hydrogen sources of energy liberation and extended hydrogen envelopes are usually called asymptotic giant branch (AGB) stars. Such objects can also have neutron stars as companions. All of these are wide systems, naturally, since after the components converge in the course of mass transfer, preceding the formation of a neutron star, it is impossible for an asymptotic giant branch star to be formed in the close system. The distributions of NS + AGB systems with respect to orbital period, the mass of the visible component, orbital eccentricity, and spatial velocity (Figs. 10a-d) are similar to the distributions of their progenitors – wide NS + MS systems. Some of the differences are due to the strong dependence of the lifetime of AGB stars on mass.

Some NS + AGB systems may be detected as symbiotic stars, i.e., cool stars whose spectra contain indications of the presence of a source of high-energy photons. The wind from the giant in an NS + AGB system can be ionized by x rays produced in accretion of the wind matter onto the neutron star. According to Ref. 63, the companion of the source of hard x rays GX 1+4 is an M6 III giant, whose spectrum is similar to those of symbiotic stars. That system may be an example of NS + AGB objects.

## CONCLUSION

Let us enumerate our main results.

**1.** We have constructed a model of the family of neutron stars in the Galaxy that are components of detached systems or are single stars.

**2.** The observed degree of duplicity of radio pulsars and their distribution with respect to spatial velocity are explained in their main features in the proposed model without assuming that the young neutron star receives a kick at the time of the star's supernova explosion.

**3.** The lack of radio pulsars in binary systems with orbital periods 300-20,000 years, which are detectable, may indicate the impossibility of their formation in wide systems, because the rotation of the neutron stars formed in those systems is too slow, for example. The progenitors of neutron stars in close systems can acquire considerable angular momentum as the result of mass transfer.

We have described above the main characteristics of the model of the population of single neutron stars and neutron stars in detached binary systems. The observed analogs of such objects are radio pulsars in binary systems. To compare the theoretical model of the population of neutron stars with observations, we must supplement the evolutionary scenario modeling by an analysis of the evolution of the radio emission of neutron stars, so as to identify the radio pulsars among them.

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