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On the Influence of Emission of Gravitational Waves on the Evolution of Low-Mass Close Binary Stars

by

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ABSTRACT

The evolutionary changes of parameters under the influence of emission of gravitational waves are computed for low-mass short-period binaries, that contain a main-sequence or degenerate hydrogen-helium, helium or carbon secondaries, filling their Roche lobes. It is shown that some cataclysmic variables could evolve under the influence of gravitational radiation. However, the observed changes of periods of cataclysmic variables are not due to the evolution of the system and we suggest that they are possibly connected with the instabilities of mass transfer and/or accumulation of matter and momentum in the disc. Under certain conditions (initial $M_2/M_1 \gtrsim 0.83$) binaries with degenerate secondaries may transform into single objects with massive ($\sim 1 M_\odot$) discs around them. For close binaries containing neutron star primary and low-mass secondary that fills its Roche lobe, evolution under the influence of the gravitational radiation could provide $\dot{M}$ in the range, necessary for explanation of X-ray bursters: $(10^{-10}$-$10^{-8}) M_\odot$/year.

1. Introduction

The role of the gravitational radiation (GR) in the evolution of binary stars was discussed by many authors (*e.g.* Paczyński 1967, Tutukov 1969, Brancewicz 1970, Faulkner 1971, 1976, Vila 1971). The main result of these investigations was that the loss of energy and angular momentum *via* the gravitational waves emission is significant for evolution of systems with periods shorter than about 10^h. This means that the GR can play a role for W UMa type stars, cataclysmic variables, and a number of unique

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objects like the binary pulsar, 18^m binary HZ 29, system of two subdwarfs LB 3459. The GR may be significant for the evolution of a binary system containing a main-sequence star in pair with a neutron star. Such systems are usually suggested as models of the bulge X-ray sources (*e.g.* Sco X-1) and X-ray bursters. If the rate of mass exchange in such a system does not exceed that corresponding to the critical Eddington luminosity ($\sim 10^{-8} M_\odot$/year) its influence would not differ from the evolution of a system, containing a massive carbon-oxygen white dwarf primary (if we do not take into account the mass loss from the binary system, caused by the photoevaporation of the secondary matter; Massevitch *et. al.* 1979).

It is worthwhile to investigate systematically the evolution of close binaries under the influence of the GR since (*i*) the existence of the GR seems now to be convincingly proven by the observations of orbital period changes in the binary pulsar (Taylor 1978), and (*ii*) it was shown (Tutukov and Yungelson 1978a) that the common envelope stage of the evolution of a low or moderate mass binary system has to be followed by formation of a system containing a degenerate white dwarf primary and a low-mass main-sequence of degenerate secondary.

In this Paper we present the computations of the evolution of close binaries consisting of a compact primary and a low-mass main-sequence or degenerate secondary. We assume that the system evolves due to the GR losses. The results of the computations are applied to the analysis of the observational data on cataclysmic binaries, W UMa stars and some unique short-period stellar objects. This work is a part of the investigation of an evolutionary scenario for moderate and low-mass binaries and concerns the latest stages of their evolution.

2. Formulation of the Problem

Landau and Lifschitz (1962) give the formula for the rate of energy loss *via* gravitational waves for the system of two bodies with masses M_1 and M_2 on a circular orbit with semiaxis a. From this classical formula we get that due to the GR the semiaxis a of a circular orbit decreases with the rate

$$\dot{a} = -\frac{64G^3}{5c^5a^3} M_1 M_2 (M_1+M_2), \tag{1}$$

where c is the light velocity and G is the gravitational constant (all quantities in C. G. S. units). In further considerations we shall make the usual assumption that the axial rotation of the mass-losing component is synchronous with the orbital evolution. Due to the GR the separation between

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the components decreases. The secondary fills its Roche lobe and the mass exchange begins. Conservation of angular momentum requires that since this instant Eq. (1) should be replaced with

$$\dot{a} = \frac{2a}{M_1M_2}(M_2-M_1)\dot{M}_2 - \frac{64G^3}{5c^5a^3}M_1M_2(M_2+M_1), \tag{2}$$

where $\dot{M}_2$ is the rate of mass outflow from the star that fills the Roche lobe. (In derivation of the Eq. (2) we neglected the angular momenta of components). We may write $\dot{a} = (\mathrm{d}a/\mathrm{d}M_2)\dot{M}_2$ and we get

$$\dot{M}_2 = 10^{-8.95}\frac{M_1M_2(M_1+M_2)}{a^3\left[\dfrac{2a(M_2-M_1)}{M_1\cdot M_2} - \dfrac{\mathrm{d}a}{\mathrm{d}M_2}\right]}. \tag{3}$$

Here masses and separation are in solar units. Similar equation though in a somewhat different form was used by Faulkner (1971) and Vila (1971). The separation a is related to the radius of the secondary by the formulae (Paczyński 1971)

$$a = \begin{cases} R_2/\left(0.38+0.2\lg(M_2/M_1)\right) & \text{for } 20 \gtrsim M_2/M_1 \gtrsim 0.135, \\ R_2/\left(0.46224\left(M_2/(M_1+M_2)\right)^{1/3}\right) & \text{for } M_2/M_1 \lesssim 0.135. \end{cases} \tag{4}$$

The radii of the degenerate stars can be fairly accurately represented by the parabolic fit of the form $\lg(R_2/R_\odot) = A + B\lg(M/M_\odot) + C\lg^2(M/M_\odot)$ to the result of Zapolsky and Salpeter (1969). Here

$$(A, B, C) = \begin{cases} -1.42,\ -0.36,\ -0.068 & \text{for hydrogen-helium stars,} \\ -1.92,\ -0.41,\ -0.0685 & \text{for pure helium stars,} \\ -1.92,\ -0.33,\ -0.058 & \text{for pure carbon stars.} \end{cases} \tag{5}$$

Table 1

Initial parameters of computations

$M_1/M_\odot$	$M_2/M_\odot$	P_0(hours)	$a_0/R_\odot$	Chemical composition of the secondary
0.107	0.093	0.385	0.156	X=0.7, Z=0.03. Degenerate[+]
0.708	0.092	0.351	0.234	X=0.7, Z=0.03. Degenerate[+]
1.503	0.097	0.339	0.287	X=0.7, Z=0.03. Degenerate[+]
0.332	0.268	0.046	0.054	Helium, degenerate
0.444	0.356	0.034	0.049	Helium, degenerate
0.753	0.447	0.025	0.046	Helium, degenerate
0.109	0.091	0.126	0.074	Helium, degenerate
0.435	0.365	0.030	0.045	Carbon, degenerate
0.879	0.721	0.016	0.037	Carbon, degenerate
1.326	1.074	0.011	0.032	Carbon, degenerate
0.400	0.400	2.185	0.836	X=0.7, Z=0.03. Nondegenerate
0.800	0.800	4.401	1.67	X=0.7, Z=0.03. Nondegenerate

[+] The initial values of M_2 for these sequences were chosen in order to provide a continuous transition from the sequences with nondegenerate secondaries.

For the mass-radius relation of normal low-mass main-sequence stars ($M \lesssim 1M_\odot$) we adopt (Warner 1976)

$$\lg(R_2/R_\odot) = -0.1 + \lg(M/M_\odot). \tag{6}$$

Eqs. (2)-(6) describe the evolution of the quantities $M_2, \dot{M}_2, a, P, \dot{P}$ for any system containing a star which fills the Roche lobe, if one assumes that in the course of evolution this star remains in the state of thermal equilibrium. The initial parameters of computed systems are listed in Table 1.

3. Results and Discussion

Let us assume that due to the orbital angular momentum loss in the common envelope stage of evolution a system with a period of about or shorter than 10^h was formed and that the secondary just filled its Roche lobe.

For stars under consideration the time scale of nuclear evolution $T_{nuc} = 10^{9.9}/(M/M_\odot)^{3.3}$ years (Paczyński 1967) is longer than the time scale of the evolution under the influence of the GR. Therefore we neglect the effects of nuclear evolution.

If the secondary is a nondegenerate hydrogen-helium main-sequence star and $q = M_2/M_1 \gtrsim 4/3$ then the denominator of Eq. (3) is positive. This is due to the fact that for $q \gtrsim 4/3$ the value of the derivative of the radius of the (equivalent) Roche lobe by mass is greater than $\mathrm{d}R_2/\mathrm{d}M_2$. For such a mass ratio it would be impossible to keep the star within the Roche lobe. So we expect that the star could lose mass in the thermal time scale. It is possible that part of the matter will be lost in the time scale close to the dynamical one (Paczyński *et al.* 1969). For $q \gtrsim 4/3$ our analysis is invalid. If the secondary is degenerate, the denominator of the Eq. (3) is positive if $q \gtrsim 0.83$. The mass loss in this case will proceed in the time scale that will be probably close to the dynamical one. The accretion rate of the primary is limited by some critical value that produces the critical luminosity: 10^{-6}-10^{-5} $M_\odot$/year for degenerate dwarfs or $\sim 10^{-8}$ $M_\odot$/year for neutron stars. If the matter is lost by the secondary with the rate that exceeds the critical one, it can not be completely accreted and from a disc, which could exist 10^5-10^6 years (or $\sim 10^8$ years respectively). If $q \geqslant 0.83$ in a system with a degenerate secondary, the time scale of mass loss is so short (10-100 sec) that the whole secondary could transform into a massive disc around the primary. Such single star surrounded by a disc of comparable mass was suggested by Paczyński (1978, private communication) as a possible model of an X-ray burster.

The initial parameters of computed systems were such that they satisfied the condition $q \lesssim 4/3$ (or $q \lesssim 0.83$ respectively). We had assumed that in the course of the evolution secondaries remain in thermal equilibrium. The changes of parameters of investigated systems that can be directly compared to the observational data – $P, P/\dot{P}, M_2, \dot{M}_2$ – are shown in Figs. 1-3. Since the parameters of the systems with helium and carbon secondaries depend very slightly on the total mass of the system we show only one representative curve for the total mass of the system,

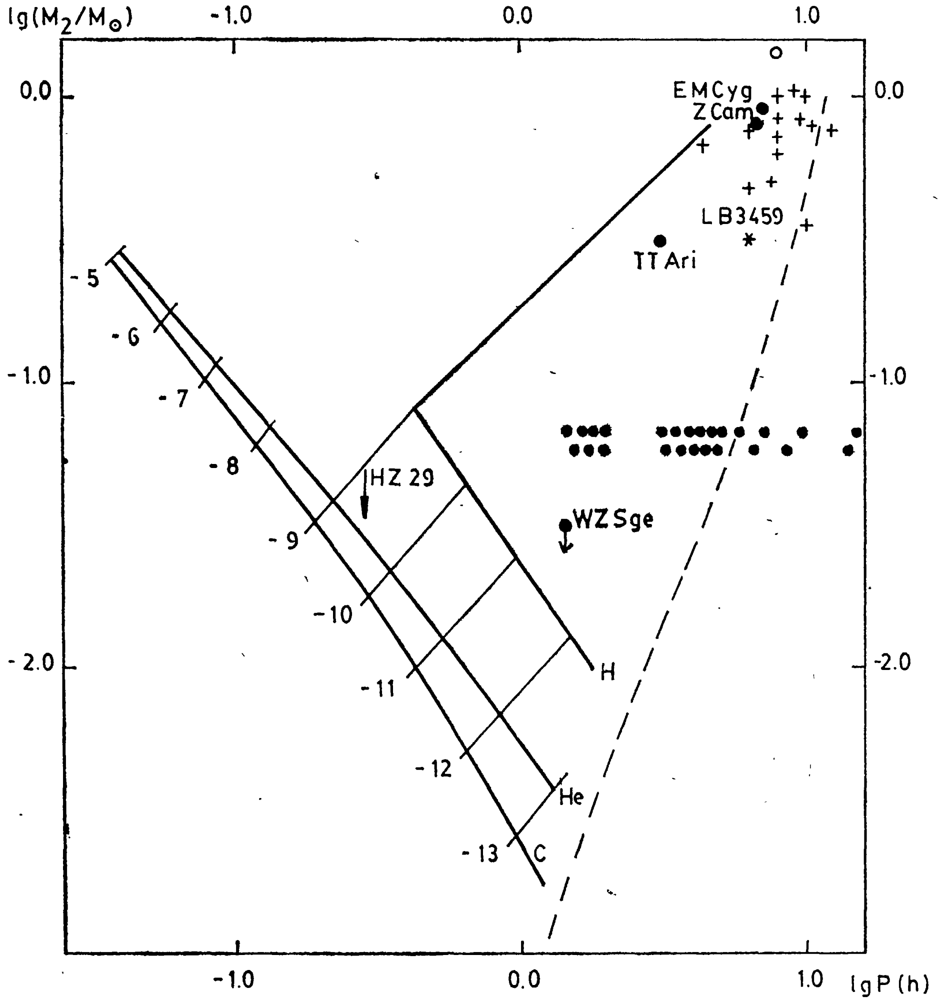

Fig. 1. The relation between the period of binary system and the mass of the secondary for the systems, that evolve under the influence of the gravitational radiation. Thin lines are the curves of constant mass-loss rates. Numbers indicate the values of $\lg \dot{M}_2$ in $M_\odot$/year. Dashed curve is the upper limit of periods of systems for which the gravitational radiation could play a significant role in 10^{10} years. Crosses are the W UMa stars. Open circle – the double pulsar. The known periods of cataclysmic variables (with unknown M_2) are indicated by dots. Asterisk – the system LB 3459.

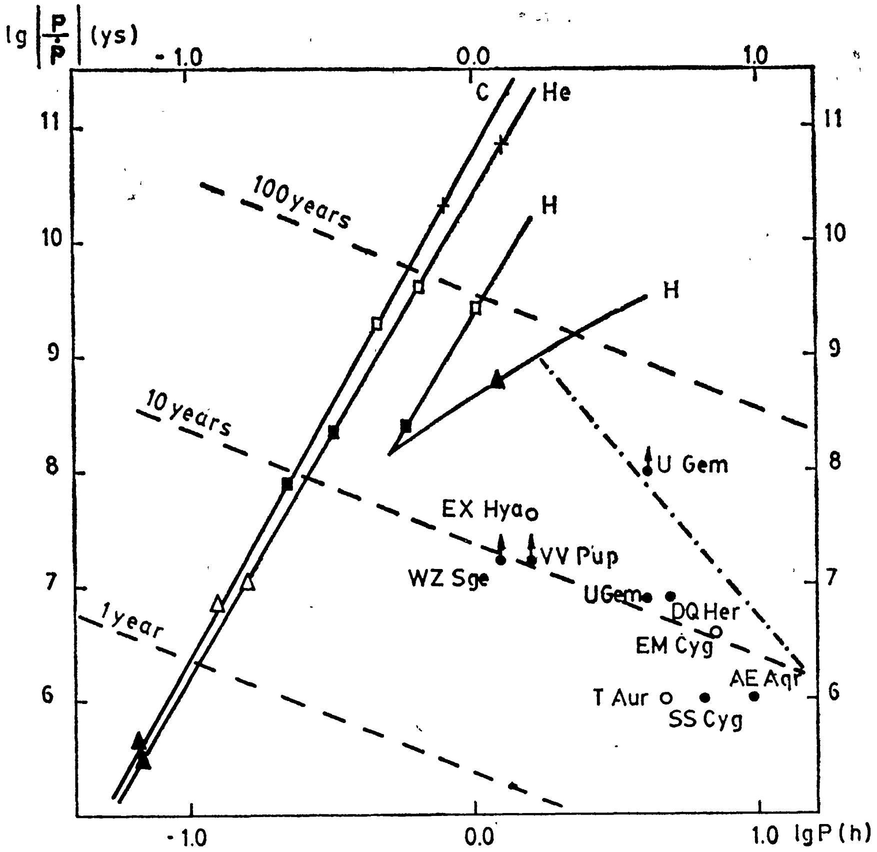

Fig. 2. The relation between the period of binary system and the time scale of period change. The curves are drawn for the same systems as in Fig. 1. The dashed lines indicate the time intervals necessary for discovery of period changes. Dash-dotted line — period changes in the thermal time scale of main-sequence stars. Filled circles — stars with positive $P/\dot{P}$. Open circles — stars with negative $P/\dot{P}$. Crosses correspond to the semiamplitude of radial velocity $K = 3$ km/sec, open squares to $K = 10$ km/sec, filled squares to $K = 30$ km/sec, open triangles to $K = 100$ km/sec, filled triangles to $K = 300$ km/sec ($\sin i = 1$).

$0.8\ M_\odot$ in each figure. For the system with the main sequence secondary the total mass is $1.6\ M_\odot$.

If the secondary is a nondegenerate main-sequence star the separation a and the period P decrease until mass of the star is lower than $0.1\ M_\odot$. From this moment on we assume that the star becomes degenerate and evolves along the line for degenerate hydrogen-helium stars. For degenerate stars a and P increase from the very beginning of the mass exchange. Actually the thermal time scale $T_{th} = 10^{7.5} M_2^2/RL$ years (M, R, L — in solar units) rapidly grows with decreasing mass of the star and for $M_2 \lesssim 0.4 M_\odot$ becomes longer than the time scale of the evolution under the influence of the GR. Since then the mass losing star can not be considered as a star in the thermal equilibrium. Therefore the real evolutionary

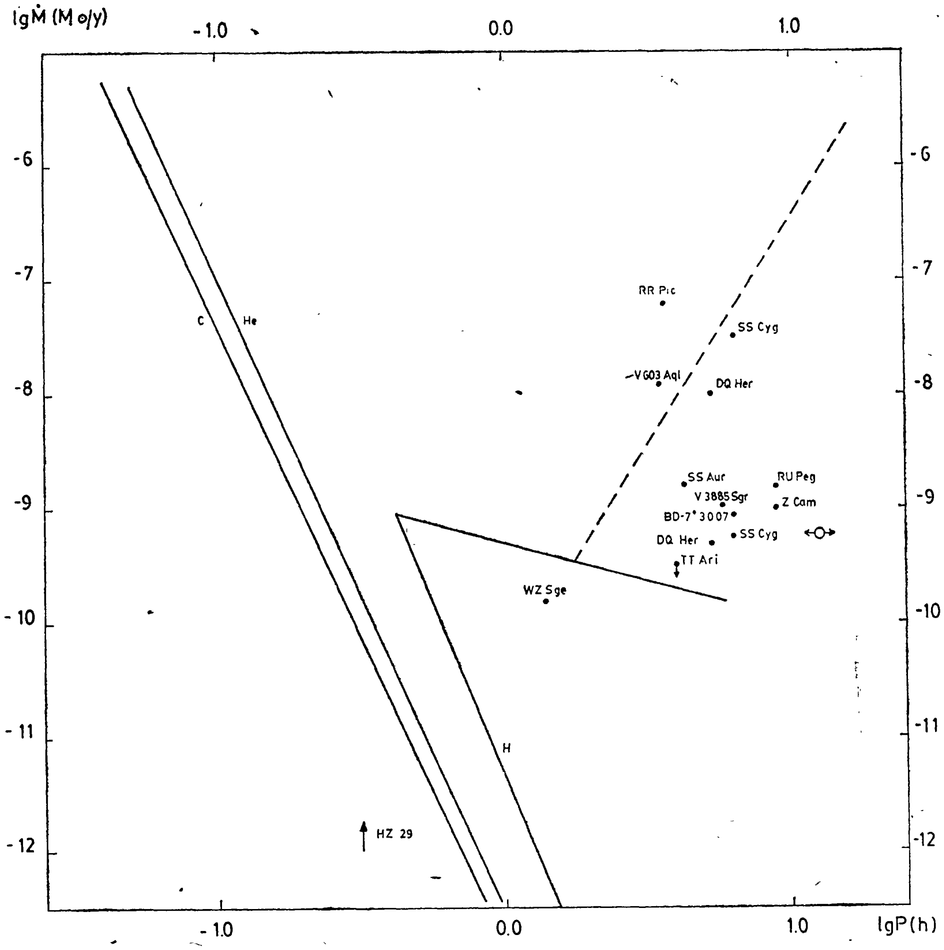

Fig. 3. The relation between the periods and mass loss rates. Curves are drawn for the same systems as in Figs. 1, and 2. Dashed line shows $\dot{M}$ for the mass loss in the thermal time scale of the main-sequence stars. Open circle with arrows shows the value of $\dot{M}$ estimated from the "mean" magnitude of the dwarf novae.

track will not abruptly change its direction when degeneracy sets in, but transition between the two branches will proceed smoothly. Actually systems with the secondaries not in the thermal equilibrium have to evolve through the region limited by "degenerate" and "nondegenerate" tracks in Figs. 1-3. Mass loss rate $\dot{M} \sim 10^{-5}$ $M_\odot$/year (the critical one) gives the upper limit to our computed curves. The lower limit to the mass loss rates is given by the requirement that the GR emission has to play a significant role in the evolution of a binary system in the lifetime of our Galaxy ($\sim 10^{10}$ years).

In the accreting star is a carbon-oxygen or helium star the accretion can create conditions necessary for explosive nuclear burning in the center (Tutukov and Yungelson 1976, Ergma 1976). Explosion will manifest itself as a supernova. If the amount of energy released during the explosion exceeds the binding energy of the star, the star will be completely disrupted. The accurate value of the central density for which the disruption occures can be determined only in hydrodynamical computation. After the explosion and complete disruption of the primary the secondary gets spatial velocity up to $\sim 10^3$ km/sec. If the primary survives the explosion the stationary burning in $\sim 10^7$ years will turn helium star into a carbon-oxygen star with helium envelope. Explosive carbon ignition completely disrupts the star if $\varrho_c \lesssim 10^9 \mathrm{g/cm^3}$ (Ivanova *et al.* 1977). If $\varrho_c \gtrsim 10^9 \mathrm{g/cm^3}$ a neutron star forms. The system could be disrupted also in this latter case due to the supernova envelope ejection. The energy estimate shows that the encounter of supernova envelope with the secondary could destroy the secondary if its mass does not exceed $\sim 0.1\, M_\odot$.

4. Comparison between the Observed and Computed Parameters of Low-mass Binaries

The most promising class of stars to be compared with our models are the cataclysmic variables (CV's). From observations we may determine their periods P and rates of the changes of orbital periods. If in addition the inclination of the orbit can be determined, we may find the amplitude of radial velocity K and masses of components. Only two CV's are both double-line spectroscopic and eclipsing binaries (EM Cyg and Z Cam) with accurately determined masses of components. Usually the relative rate of mass exchange (or loss) $\dot{M}_2/M_2$ is determined through the rate of period change $\dot{P}/P$, assuming that all mass lost by the secondary is either transferred onto the primary or lost by the system. But $\dot{M}_2/M_2$ does not probably determine $\dot{P}/P$ uniquely. The observed period and its changes depend also on changes in the mass, angular momentum and size of the disc around the primary (Smak 1972). The rate of mass accretion by the disc can be estimated from the luminosity of the hot spot, if the distance to the CV is known.

In Fig. 1 we plotted the positions of EM Cyg and Z Cam (Robinson 1976), WZ Sge (Krzemiński and Smak 1971), TT Ari (Cowley *et al.* 1975), and some systems for which the GR emission may influence the evolution, like the double pulsar (Taylor 1978), system of two subdwarfs LB 3459 (Dearborn and Paczyński 1978) and fourteen W UMa stars (Kopal 1959). For the secondary of WZ Sge, only the upper limit of mass $\sim 0.03 M_\odot$ is known. Such low value of mass excludes the possibility that it is a main-sequence star. We also have not arguments in favour of the assumption that it is a degenerate hydrogen star with mass about $0.015 M_\odot$ (this is suggested by its position respectively to the $P-M_2$ line for degenerate hydrogen stars). In Fig. 1 are also shown the known periods of CV's (Warner 1976, 1978, Angel *et al.* 1977). It is interesting that there are no systems with periods from 2^h to $3^h\,12^m$. The upper value of periods in the short-period group is very close to the upper value of periods for degenerate hydrogen stars, but we do not have enough information to identify them with any type of models. The mean apparent magnitude of three most luminous stars in the group with $P \leqslant 2^h$ is about 3^m fainter than that of the three most luminous stars in the group with $P \geqslant 3^h\,12^m$. We have estimated that the mean galactic lattitude of stars in both groups is the same: $27° \pm 6°$ and it is close to the mean value of lattitude of stars uniformly distributed over the sky: 32°5. Thus, regretfully, we cannot estimate the difference in the average distances to these stars. Additional information is necessary if we want to decide which group is on the average more luminous and to conclude about mass exchange rates and evolutionary status.

In Fig. 1 we indicated HZ 29 – the shortest period CV: $P = 18^m$, (Smak 1967) – by an arrow near the curve for degenerate He stars. The reason for the assumption that the secondary in HZ 29 could be a helium or a carbon-oxygen star lies in the complete absence of any hydrogen traces in its spectrum. If the secondary of HZ 29 is a He star, the mass exchange rate in this system has to be $\sim 10^{-10} M_\odot$/year.

The probability of discovery of a star is proportional to the product of the volume of space in which the stars with the given absolute stellar magnitude are seen and of the life time T of the star. The volume of the space V in which the CV's are observed is determined by the luminosity L of the secondary, the disc and the hot spot. If we assume that the limiting stellar magnitude for the discovery of the CV's is 10^m, then the stars with $M_v \lesssim 5^m$ are seen within the disc which is ~ 200 pc thick. Radius of this disc is proportional to $L^{0.5}$. If L is determined by the hot spot, thickness of the disc ~ 200 pc corresponds to $M_2 \sim 10^{-8} M_\odot$/year for $M_1 = 1 M_\odot$, $R_{disc} = 0.1 R_\odot$. For $M_2 \sim 10^{-8} M_\odot$/year the luminosity of the spot exceeds that of the secondary if $\dot{M}_2 \lesssim 0.6 M_\odot$. The life time of the secondary $T \sim M_2/\dot{M}_2$. Then $VT \sim \dot{M}_2^{0.5} M_2$ for $\dot{M}_2 \lesssim 10^{-8} M_\odot$/year and $VT \sim M_2$ for higher $\dot{M}_2$. Fig. 1 allows to estimate that for degenerate helium and carbon-oxygen stars the value of VT changes from $10^{-0.6}$ for $M_2 = 0.3 M_\odot$ to 10^{-5} for $M_2 = 0.003 M_\odot$, for degenerate hydrogen secondaries – from $10^{-1.5}$ for $M_2 = 0.1 M_\odot$ to 10^{-4} for $M_2 = 0.01 M_\odot$, and for normal main-sequence stars – from 1 for $M_2 \approx 1 M_\odot$ to $10^{-1.5}$ for $M_2 \approx 0.1 M_\odot$. Thus the conditions of visibility favour the discovery of CV's with the main-sequence secondaries, in agreement with observations.

Another factor that determines the relative probability of discovery of systems of different types is the ratio of the numbers of pairs with two degenerate stars to the number of pairs with one main-sequence component. However the present knowledge of close binaries formation and evolution does not allow us to determine this factor.

In Fig. 2 we plotted the "tracks" of stars in the plane $\lg P - \lg|P/\dot{P}|$. There are also drawn the lines that indicate how long it is necessary to observe the system in order to discover the variations of the period:

$$\tau = \sqrt{2\Delta\varphi P\cdot(P/\dot{P})}\,, \tag{7}$$

where $\Delta\varphi$ is the minimal phase shift necessary for discovery of period changes. We assumed after Krzemiński (private communication) that for CV's $\Delta\varphi \approx 0.02$. If the time scale of period changes is longer than $\sim 10^8$ years, the discovery of period changes may be difficult due to the incompatibility of the Universal Time and the atomic time scales that limits the relative accuracy of the UT by a factor of $\sim 10^{-7.5}$ (Vogt *et al.* 1979). If we want to discover the period changes of the CV's that evolve due to the GR losses, we have to observe them for several decades.

On all curves in Fig. 2 are also shown the points where for the system with the given total mass the semiamplitude of radial velocity is equal to 3, 10, 30, 100, and 300 km/sec (for $\sin i = 1$). Positions of these points indicate, that the average values of radial velocities for systems with degenerate and main-sequence secondaries differ about 30 times. This can help to distinguish the two kinds of systems.

The dash-dotted line in Fig. 2 shows the period changes for a system containing a white dwarf primary and a main-sequence secondary that loses mass on the thermal time scale. For the low-mass stars under consideration luminosity $L \sim M^4$. Mass radius relation for main-sequence stars is given by Eq. (6). If we assume that $M_1 = M_2$, then from the Kepler low and the Eqs. (4) and (6) it follows that $P \propto a^{1.5}M_2^{-0.5} \propto R_2^{1.5}M_2^{-0.5} \propto M_2$. Thus we obtain

$$P/\dot{P} \approx 10^{9.7}/P^3 \quad \text{years}, \tag{8}$$

where P is in hours. Eq. (8) gives for the rate of mass loss

$$\dot{M}_2 \approx 10^{-10.4}P^4 \quad M_\odot/\text{year}. \tag{9}$$

Eqs. (8) and (9) are probably accurate to a factor about 3.

In Fig. 2 we indicate the positions of CV's with known values of P and $P/\dot{P}$: U Gem (two determinations cited by Warner (1978)), EX Hya (Vogt *et al.* 1979), DQ Her (Smak 1972), WZ Sge (Krzemiński and Smak 1971), EM Cyg, T Aur, and VV Pup (Pringle 1975), SS Cyg and AE Aqr

(Warner 1976). All stars change their periods on the time scales that are shorter than not only the time scale of the GR but even the thermal time scale. It is possible, as already mentioned above, that the observed variations of periods are not due to the evolution of the binary system as a whole but rather to some instabilities in the disc or in the secondary. Therefore we also do not discuss the evolutionary meaning of the sign of observed $P/\dot{P}$.

If the secondary of HZ 29 is a helium or a carbon degenerate star, then this system has to have the time scale of evolutionary period changes $P/\dot{P} \sim 10^8$ years, as indicates its position respective to the curves for these stars in Fig. 2. Such changes could be discovered in about 20 years. The assumption that this system has the secondary that fills its Roche lobe is based on the observations of broad spectral lines and instability of light curve, that is ascribed to the exchange of matter. If the secondary does not fill its Roche lobe then from Eq. (2) it follows that the evolutionary time scale is $P/\dot{P} \sim 10^5$ years. if $M_1 = M_2 = 1M_{\odot}$. Still we cannot exclude the possibility that HZ 29 is a single pulsating star. The discovery of evolutionary changes of the period in this system could help to choose one of the listed possibilities.

In Fig. 3 are shown the "tracks" of investigated systems in the plane $\lg P - \lg \dot{M}$. The value of $\dot{M}$ can also give some evidence about the evolutionary status of the system, if we exclude the influence of variations created by the instabilities in the envelope of mass losing star. We would like to point out that our estimates of $\dot{M}$ for nondegenerate stars are in good agreement with the results of evolutionary computations of Chau and Lauterborn (1977). In Fig. 3 is also shown the $\dot{M}$-line for stars losing mass on thermal time scale.

It is possible to determine $\dot{M}$ for a CV if we know the stellar magnitude of the hot spot and the distance to the system. We estimated $\dot{M}$ for some CV's taking values of magnitudes and distances from Warner (1976) and assuming that all energy of the impact is radiated away in the spot. The relation between $\dot{M}$ and the absolute stellar magnitude of the spot M_v is $\lg \dot{M} = -6.3 - 0.4 M_v$, where $\dot{M}$ is in $M_{\odot}$/year. For dwarf novae M_v of the spot is roughly equal to the M_v of the system. For nova-like stars M_v of the spot is $1^{\mathrm{m}}.5$ fainter than that of the whole system (Warner 1976). For old novae RR Pic and V603 Aql $\dot{M}$ were estimated by Tylenda (1977), who had compared observed and computed distributions of continuous radiation from accretion discs. For DQ Her $\dot{M}$ is given by Warner (1978) on the basis of the observed changes in the 71^{s} oscillation period (higher value of $\dot{M}$) and by Gorbatzky (1974) on the basis of observed deceleration of the envelope expansion (lower value of $\dot{M}$). For SS Cyg $\dot{M}$ was estimated

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by Ricketts *et al.* (1979) on the grounds of observed X-ray emission (in the minimum and the maximum of brightness). For this star we showed in Fig. 3 the value of $\dot{M}$ for quiescent phase and the average $\dot{M}$ per cycle, assuming that the maximum brightness lasts ~ 10 % of the whole cycle, as indicated by the X-ray light curve. The value of $\dot{M}$ for the minimum brightness is close to our estimate of $\dot{M}$ on the grounds of optical data. For WZ Sge we show the lower limit of the mass transfer rate, found by Krzemiński and Smak (1971) from the estimated luminosity of the hot spot. We also show the "mean" value of $\dot{M}$ obtained from "mean" magnitude of dwarf novae in the minimum of brightness $M_v = 7^m\!.5$ (Kraft and Luyten 1965). In our estimates we assumed for simplicity that the temperature of the spot is 10^4K. This allowed us not to take into account the bolometric correction. As seen from Fig. 3 estimates of $\dot{M}$ show that some of the CV's may lose mass in the time scale of the GR. Let us note that for SS Cyg there are estimates of $\dot{M}$ both in the minimum and in the maximum. For this star $\dot{M}$ based on X-ray data averaged over the cycle is about 50 times higher than $\dot{M}$ in the minimum of brightness. This difference could not be explained within the accumulation model of dwarf novae (Osaki 1974). Classical novae RR Pic, V603 Aql, DQ Her, as can be seen in Fig. 3 have average accretion rates ~ 100 times higher than those corresponding to the evolution under the influence of the GR. However, their values of $\dot{M}$ agree with the values of $\dot{M}$ predicted for main-sequence stars, that lose mass on the thermal time scale. Vogt *et al.* (1979) arrived to the same conclusion for the dwarf nova EX Hya. The definite conclusions about the role of the GR emission in the evolution of the CV's would be possible only after we have many more accurate estimates of $\dot{M}$ for individual systems.

For the model that explains the X-ray bursts by the thermonuclear flashes near the surface of an accreting neutron star (Joss 1978, Tutukov and Ergma (1979), accretion rates $10^{-10}M_\odot/\text{year} \lesssim \dot{M} \lesssim 10^{-8}M_\odot/\text{year}$ are required. Fig. 3 shows that such accretion rates are possible in binary systems that evolve under the influence of the GR emission.

5. Flux of the Gravitational Waves from the Short-period Binaries

Let us estimate the flux of the gravitational waves emitted by the short period binaries, containing two degenerate stars or one degenerate and one nondegenerate star. The characteristic period of waves, emitted by the given system is equal to the half of the orbital one. If the secondary fills its Roche lobe, the periods range from several minutes for helium or carbon secondaries to several tens of minutes for nondegenerate hydrogen-helium secondaries. ($M_2 = 0.5M_\odot$). The initial stellar mass function for the binaries of in Galaxy is (Tutukov and Yungelson 1978b)

$$\mathrm{d}N \propto \frac{a}{6} \frac{\mathrm{d}(M/M_\odot)}{(M/M_\odot)^{2.5}}, \tag{10}$$

where a is the relative number of systems that become close enough ($P \lesssim 10^\mathrm{h}$) for the GR to play role in their evolution. Let us assume for simplicity that the typical mass of a star in such a system is $\sim 0.5 M_\odot$. Then $E_g \approx 10^{47.3} (r/R_\odot)^{-1}$ ergs per system are radiated as gravitational waves. Here r is the radius of mass losing star. In the lifetime of our Galaxy only stars with $M \gtrsim 1 M_\odot$ have already achieved the final stages of their evolution. Their birthrate is ~ 0.1 per year. Then the capacity of the GR of systems under consideration is

$$L_g \approx a \frac{10^{5.2}}{r/R_\odot} L_\odot. \tag{11}$$

The total flux of the GR near the Earth is determined by radiation of all short-period variables and is equal to

$$F_g \approx a \frac{10^{-6.2}}{r/R_\odot} \quad \mathrm{ergs/sec/cm^2}. \tag{12}$$

Mironovski (1965) had estimated that the W UMa stars are the most powerful source of permanent gravitational radiation in our Galaxy. Flux of their radiation is $\sim 10^{-9}$ ergs/sec/cm^2 near the Earth. From Eq. (12) it immidiately follows that if $a \gtrsim 10^{-5}$ then F_g is greater than that produced by the W UMa stars.

6. Conclusion

We had investigated numerically the evolution of low-mass short period binaries under the influence of gravitational waves emission and had applied our results to the analysis of the observational data. The main results may be summarized as follows.

(1) The analysis of relations $P-M_2$ and $P-\dot{M}_2$ (Fig. 1 and 3) indicates that it is possible that some cataclysmic variables evolve under the influence of gravitational radiation, but additional evidence is necessary. The observed rapid variations of orbital periods ($P/\dot{P} \leqslant 10^8$ years) of the CV's are probably caused not by the angular momentum loss and mass exchange, but by other reasons: changes of mass and size of the disc and the instabilities of the mass outflow from the secondary. Our analysis shows that the value of the average mass loss rate from the secondary can be a good criterion for finding the systems that evolve under the

influence of the GR, if we neglect the possible instabilities of the mass outflow.

(2) If in the course of evolution binaries containing helium or carbon secondaries are formed, then it is most probable that they may be discovered among the objects that are lying in the galactical plane and are variable on the time scales of several minutes. It is possible that HZ 29 is an example of such systems.

(3) If in a system of two degenerate stars at the beginning of mass transfer $M_2/M_1 \gtrsim 0.83$ it is possible that the secondary transforms into a disc surrounding the ex-primary.

(4) If the relative number of binaries that evolve into the short-period systems ($P \lesssim 10^{\rm h}$) containing two degenerate stars exceeds $\sim 10^{-5}$ then the flux of their gravitational radiation near the Earth will exceed that radiated by the W UMa stars.

All the estimates in this paper were obtained under the assumption of conservation of total mass of close binary systems. The analysis of the nonconservative case is much more difficult due to the lack of reliable observational data on all forms of the mass loss from the systems and lack of knowledge of angular momentum loss with the matter that escapes from the system.

To investigate in more detail the role of the GR in the evolution of binaries new observational data on period changes, hot spot brightnesses, rates of mass exchange and mass loss from the systems, as well as the distances to the cataclysmic variables are highly desirable. Also highly desirable are the computations of the evolution of the main-sequence stars with initial masses $M \lesssim 1.5 M_\odot$ in the binaries containing degenerate or neutron stars.

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