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SUPERNOVAE OF TYPE I AS END PRODUCTS OF THE EVOLUTION OF BINARIES WITH COMPONENTS OF MODERATE INITIAL MASS ($M \lesssim 9\ M_\odot$)[1]

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ABSTRACT

Analysis of the theory of evolution of low- and intermediate-mass binaries allows us to select promising scenarios that lead to presupernova systems consisting of an accreting electron-degenerate dwarf (made primarily either of oxygen, neon, and magnesium, of carbon and oxygen, or of helium) and a low-mass ($M \approx M_\odot$) star supplying hydrogen-rich matter at rates in the range $\sim 10^{-9}$–$10^{-6}\ M_\odot\ \mathrm{yr}^{-1}$ or a heavy disk composed of helium or of C and O supplying matter at an as yet undetermined rate. Some of these scenarios have an estimated frequency of realization comparable with the observed frequency ($\sim 10^{-2}\ \mathrm{yr}^{-1}$) of Type I supernovae (SNeI), but it is as yet impossible to identify conclusively a single one as the most likely explanation of the SNI phenomenon and to reject all other possibilities. It is therefore not excluded that SNeI are a mixture of products of different scenarios. Estimates of formation frequency are very preliminary, since some of them are strongly dependent on the distribution of unevolved binaries over the initial mass ratio q_0, especially for $q_0 \approx 0.1$–0.3, and most of them require knowledge of the processes that occur during a common envelope stage, the understanding of which is still very rudimentary.

We find that those systems having the highest formal probability ($\sim 0.008\ \mathrm{yr}^{-1}$) of evolving into the presupernova state are binaries with component masses in the range 5–9 $M_\odot$, semimajor axes between 70 and 1500 $R_\odot$, and orbital periods in the range 1 month to 6 yr. After experiencing two common envelope stages, these systems become two electron-degenerate dwarfs composed of carbon and oxygen; the dwarfs have individual masses in the range 0.7–1 $M_\odot$, are separated by a distance of 0.2 to 3.5 $R_\odot$, and orbit each other with periods between 12 minutes and 14 hr. The angular momentum carried away by gravitational waves forces the components closer together over a time ranging from 10^5 to 10^{10} yr, and when the semimajor axis is reduced to $\sim 0.01\ R_\odot$, the less massive of the two dwarfs is transformed into a heavy disk or rapidly rotating envelope about the more massive dwarf. When accretion from the disk raises the mass of the single stellar remnant to $\sim 1.4\ M_\odot$, a supernova explosion results. An interesting aspect of this scenario is that it predicts SN characteristics in accord with the observations: typical expansion velocities of the ejectum of $\sim 11{,}000\ \mathrm{km\ s}^{-1}$, no detectable hydrogen or helium in the spectrum at maximum light, the presence in the SN spectrum of incompletely burned heavy elements including Ca and Si, and a light curve whose peak half-width matches that of most observed SNeI light curves. Further, the longest period representatives of these systems can occur in elliptical galaxies 10^{10} yr after star formation has ceased.

The next (formally) most likely scenario (realization frequency $\sim 0.005\ \mathrm{yr}^{-1}$) begins with two stars each of mass about 3–4 $M_\odot$, separated by a distance between 70 and 460 $R_\odot$, and orbiting with periods between 20 days to 1 yr. Again after two common envelope stages, these systems evolve into two electron degenerate dwarfs, but this time the dwarfs are composed of helium and have masses between 0.4 and 0.5 $M_\odot$. The initial semimajor axes and orbital periods of the potential presupernova systems are, respectively, in the ranges 0.13–1 $R_\odot$ and 8 minutes to 3 hr. Within 2×10^5 to 5×10^8 yr the angular momentum loss by gravitational wave radiation brings the components close enough together to lead to the formation of a heavy disk which is then accreted by the initially more massive dwarf. Possible disadvantages of this scenario, when compared with the observations, are that it predicts typical expansion velocities on the order of $16{,}000\ \mathrm{km\ s}^{-1}$, the absence of Ca and Si in the SN spectrum, and a half-width for the peak of the light curve that is, in most instances, narrower than the observed one by about a factor of 2. If the (unknown) accretion rate from the heavy disk is large enough, the merging dwarf system will not produce an SN explosion but will instead produce a CO dwarf as a final product. In contrast, the double CO dwarf system will always produce an SN explosion, regardless of the rate of disk accretion. Further, in the framework of our approximations, most double helium dwarf systems are formed at such small initial separations that evolution to the potentially explosive state occurs in less than $\sim 5\times10^8$ yr, and thus such systems may not account for the occurrence of SNeI in elliptical galaxies. One last scenario which

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produces two degenerate dwarfs drawn together by gravitational wave radiation is a hybrid of the other two scenarios, and the product is also a hybrid: one dwarf composed of C and O, the other of helium. The realization frequency is $\sim 0.001\ \mathrm{yr}^{-1}$, and the most likely end product is an R CrB star rather than an SN.

Other scenarios lead to potential presupernova systems which consist of a carbon-oxygen dwarf and a main-sequence or red (super)giant companion which transfers hydrogen-rich material to the dwarf. The most restrictive aspect of these systems is that the occurrence of hydrogen and helium shell flashes prevents the dwarf from retaining matter accreted between flashes unless the accretion rate is within a relatively narrow interval. We find formal realization frequencies of $\sim 0.004\ \mathrm{yr}^{-1}$ for cataclysmic systems and $\sim 0.003\ \mathrm{yr}^{-1}$ for cataclysmic-like systems which may reach the explosive state, provided mass transfer within an appropriate interval can be maintained. The realization frequency of possibly "successful" symbiotic-like systems in which mass transfer from a giant is due to Roche lobe filling is $\lesssim 5\times10^{-4}\ \mathrm{yr}^{-1}$, whereas the frequency of producing symbiotic systems in which the dwarf captures mass from the wind emitted by an asymptotic giant branch star is $< 0.004\ \mathrm{yr}^{-1}$, but our upper limit requires the most favorable choice of an unknown parameter in an expression for mass capture.

Since the formation frequency of low-mass X-ray binaries is so small ($\sim 10^{-7}\ \mathrm{yr}^{-1}$) compared with the observed formation frequency of SNeI, we infer that such binaries are not the typical end product of evolution which produces SNeI. We strengthen this inference by demonstrating that low-mass X-ray binaries could be formed at the known frequency by a variety of capture and exchange capture processes in the cores of existing and disrupted globular clusters, as well as by similar processes occurring among field stars in the galactic bulge.

Subject headings: stars: binaries — stars: evolution — stars: interiors — stars: stellar statistics —
stars: supernovae

I. INTRODUCTION

a) *Precursors of Type I Supernovae as Electron-degenerate Dwarfs in Mass-exchanging Binary Systems*

The origin of Type I supernovae (SNeI; singular = SNI) continues to be a popular and still unsolved problem in modern astrophysics (e.g., Shklovski 1981; Wheeler 1982; Trimble 1982*a*). Several features strongly distinguish SNI events from SNII events. The absence of hydrogen from the spectra of SNeI is evidence that a highly evolved object has lost its original hydrogen-rich envelope by means of a stellar wind, by mass exchange in a close binary, or both. SNeI are also characterized by uniformity of light curves and spectra. The usual interpretation of the observational facts suggest that the immediate presupernova conditions are similar. However, since the limits of that similarity are not very well determined, similarity does not necessarily mean identity. This leaves open the possibility that there is more than one evolutionary scenario that culminates in the production of an SNI.

The fact that SNI events do not occur preferentially in spiral arms of spiral galaxies provides a means for estimating quantitatively an upper limit on progenitor mass. Since stars require more than $\sim 1\text{–}5\times10^7$ yr to diffuse out of the spiral arms in which they are born, stars with initial main-sequence mass greater than 10 $M_\odot$ (and hence with lifetimes less than 10^7 yr) are not good candidates. A most revealing property of SNeI is that they are the only type which occurs in elliptical galaxies. Although there is some evidence that star formation may be actively occurring in some elliptical galaxies, it is impossible to explain the observed SNI frequency in all such galaxies as being due to the formation of young massive stars (see, e.g., Maza and van den Bergh 1976; also van den Bergh 1980*a*, *b* who states: "[the fact] that the surface distribution of SNeI is similar to that of the total light of ellipticals suggests that most of the progenitors of SNeI themselves belong to an old stellar population"). The main constituents of such galaxies are old nuclear-burning stars of near solar mass and degenerate dwarfs.

Tammann (1978) has estimated that an SNI explosion occurs approximately every 500 yr in an elliptical galaxy with a luminosity of $\sim 10^{10}\ L_\odot$ in the *B* astrophotometric band. Most of the energy radiated by elliptical galaxies comes from stars with initial mass $\sim 0.8\ M_\odot$ which become, after evolution ends, carbon-oxygen (CO) white dwarfs with mass $\lesssim 0.5\ M_\odot$. Using the fine grid of evolutionary models constructed by Mengel *et al.* (1979), we estimate that every such star radiates during its life $\sim 3\times10^{51}$ ergs in the *B* band. So, the energy of one $\sim 0.8\ M_\odot$ star is enough to supply the energy flux from an elliptical galaxy with a luminosity of $10^{10}\ L_\odot$ for ~ 2.5 yr. This means that one SNI explosion occurs for every 200 stars of initial mass 0.8 $M_\odot$ which reach the end of their nuclear-burning evolution. In our Galaxy, the contemporary star formation (death) rate is $dN \approx (M_\odot/M)^{2.5}\, dM/M_\odot\ \mathrm{yr}^{-1}$ (Popova, Tutukov, and Yungelson 1982), which is equivalent to one star with mass exceeding $\sim 0.8\ M_\odot\ \mathrm{yr}^{-1}$. So, the estimated "observed" frequency of SNeI in our Galaxy is $\sim 0.5\times10^{-2}\ \mathrm{yr}^{-1}$. The real frequency of SNeI in the Galaxy must be somewhat higher than this. Since the frequency of SNeI per unit luminosity increases by a factor of 2 on passing through the Hubble sequence from E to Sb (Tammann 1982), we will take the "observed" frequency in our Galaxy (assuming it is an Sb) to be $\nu_{\mathrm{SNI}} \approx 10^{-2}\ \mathrm{yr}^{-1}$.

Our estimate that approximately one out of every 100 solar mass stars is somehow implicated in the formation of SNeI does not distinguish whether the "successful" star is a single star or whether it is a component of a binary system. However, the requirement of similarity makes it very natural and attractive to suppose that SNI events are produced by electron-degenerate dwarfs behaving, at the moment of explosion, as effectively single objects, with the common feature producing similarity being the near-uniqueness of the structure of a white dwarf whose mass is close to the Chandrasekhar limit. Historically, the first attempt at understanding SNI events in this fashion made use of the fact that, on cooling and contracting, single white dwarfs of mass near the Chandrasekhar limit experience electron capture, thus effectively lowering the Chandrasekhar limit and thereby, in some instances, causing collapse (Finzi and Wolf 1967). In retrospect, this channel appears too narrow to account for the observed frequency of SNeI (Shklovski 1978). Whelan and Iben (1973) hypothesized that a carbon-oxygen (CO) dwarf with initial mass $\sim M_\odot$ may have a giant companion filling its Roche lobe and that the mass of the CO dwarf grows by accretion until reaching the Chandrasekhar limit, thereupon exploding. In this paper, we propose several variants of this second scenario wherein the component filling its Roche lobe may include, in addition to a red giant, a degenerate C–O dwarf, a degenerate helium star, or a "heavy disk" which results from the disruption of the lower mass component in a system consisting of two degenerate dwarfs. The mass accreting star which ultimately explodes as a supernova may be, in addition to a degenerate CO dwarf, a degenerate helium dwarf, or possibly a degenerate O Ne Mg dwarf. We propose, additionally, a variant in which the degenerate primary accretes mass from a wind emitted by a secondary which does not fill its Roche lobe.

An examination of the energy budget of a CO supernova model provides further motivation for pursuing these scenarios. Let us suppose that, just prior to explosion, the degenerate dwarf of mass $\sim 1.4\ M_\odot$ is made up of 50% carbon and 50% oxygen. To estimate the thermonuclear energy produced in the explosion, we assume that the final product of the burning is nearly pure nickel (which later decays to iron). The total energy produced during the supernova explosion will then be $\sim 22 \times 10^{50}$ ergs (using nuclear data given by Wapstra and Gove 1971). The binding energy of the 1.4 $M_\odot$ CO dwarf is $\sim 5.1 \times 10^{50}$ ergs (Iben 1982*a*). The radiation losses, as obtained by integrating under the SNI light curve (Branch *et al.* 1983) are negligible ($\sim 10^{49}$ ergs). Therefore, $\sim 17 \times 10^{50}$ ergs are conceivably available for accelerating matter in the supernova envelope, which in the case of complete disruption has a mass of 1.4 $M_\odot$. Assuming complete disruption and assuming no other energy sinks such as extensive neutrino losses, the final average velocity of envelope material will be $\sim$11,000 km s^{-1}. This is very close to the observed velocities of matter in an SNI envelope (Pskovsky 1977*a*, *b*; Branch *et al.* 1983). The energy of 17×10^{50} ergs, which we have assumed to be completely converted into the initial kinetic energy of the envelope, is also close to the energy which, from the properties of extended supernova remnants, may be inferred to have been dissipated into the interstellar medium by the expanding envelope (Losinskaya 1980). A recent direct estimate by Seward, Gorenstein, and Tucker (1983) of the mass and energy of Tycho's SN, a classic SNI, gives additional support to the scenario of an exploding CO dwarf. These authors estimate a total energy of 14×10^{50} ergs for the outflowing remnant and impacted interstellar medium, a mass of $\sim 2\ M_\odot$ ($\sim 1.4\ M_\odot$, within the uncertainties) for the SN ejectum, and a mass of $\sim 2\ M_\odot$ for the interstellar matter swept up by this ejectum.

As another example, we can also estimate the energy budget for an exploding degenerate helium dwarf, assuming that the mass of such a dwarf is $\sim 0.8\ M_\odot$ (Nomoto and Sugimoto 1977). The energy liberated by the complete conversion of helium into Ni is $\sim 14 \times 10^{17}$ ergs g^{-1}, so the total nuclear energy available from this source is $\sim 22 \times 10^{50}$ ergs. The binding energy before explosion is $\sim 10^{50}$ ergs (Iben 1982*a*), so that $\sim 21 \times 10^{50}$ ergs might be converted into the kinetic energy of the exploding envelope. This is equivalent to velocities of $\sim$16,000 km s^{-1}, which agrees within the uncertainties with observational estimates for SNeI ejecta, although it may be somewhat on the high side of the observational estimates. Thus, both models can supply the necessary energy to accelerate the envelope to the observed energy.

We acknowledge that, as emphasized again by Wheeler (1978), conservative evolution of a single homogeneous star with mass exceeding the Chandrasekhar limit can also lead to a SN explosion after more than 10^9 yr of evolution. However, one must explain in this case why only one star in 100 evolves in such an uncommon way. The example of blue stragglers which are found in the H-R diagram near the expected location of stars with mass close to twice the mass of turnoff stars does not demonstrate the possibility: mass exchange in binaries can lead to the formation of stars whose evolution is similar to that of completely mixed single stars (Mermilliod 1982). Some of these stars may be a by-product of a low-mass close binary which merges under the influence of strong magnetic breaking (Kraicheva *et al.* 1978; Tutukov 1982). A partial increase in the mass of the mixing zone in the nuclei of product stars could increase the extension of the main sequence in the H-R diagram (Massevich *et al.* 1979; Maeder and Mermilliod 1981) to account, for example, for all blue stragglers in NGC 7789 (Breger and Wheeler 1980). The merging mechanism predicts a natural upper limit on blue straggler mass of $\sim 3.0\ M_\odot$, equal to twice the mass of the heaviest star having a convective envelope on the main sequence. It is possible that the evolution of the heaviest (1.5–3 $M_\odot$) merged stars with radiative envelopes and with therefore inevitably very fast rotation of the double core can be followed to a very unusual conclusion, but it is unclear that this conclusion will be an SN explosion.

b) *On Estimating the Realization Frequencies of Type I Supernova Scenarios*

Thus, a binary scenario for pre–SNI evolution remains the most promising avenue for exploration. The aim of this paper is to discuss evolutionary scenarios for low- and medium-mass double stars leading to the possibility of an SN explosion as the result of accretion of satellite matter by a degenerate dwarf and to estimate the realization frequency of each scenario. The star-formation rate, which for stars with masses exceeding $\sim 0.8\ M_\odot$ nearly coincides with the star death rate, we shall use in the form: $dN \approx (M_\odot/M)^{2.5}\, dM/M_\odot$ yr^{-1}. The distribution of binary stars over the primordial semimajor axes A will be taken as $dN \approx 0.2 d \log A$ for A in the range $10 \lesssim A/R_\odot \lesssim 10^4$ (Popova, Tutukov, and Yungelson 1982). The most uncertain part of the problem is the distribution of binaries over the primordial ratio of component masses (see, e.g., Abt and Levy 1976, 1978; Abt 1983; Jaschek 1978; Wolff 1978; Lucy and Ricco 1978). Unevolved close binaries usually have almost equal or quite comparable component masses (Popova *et al.*), but the method employed to obtain this result is not very effective for finding systems with a low mass ratio. We take this into account in first approximation by assuming that $N \propto q$, where $q = M_2/M_1$ and M_2 (the primordial secondary mass) $\leq M_1$ (the primordial primary mass). Putting all three contributions together, our final approximation to the realization frequency of any scenario is

$$\nu = 0.2q\int_{M_A}^{M_B}\frac{dM}{M^{2.5}}\Delta\log A\,, \qquad (1)$$

where $\Delta \log A$ is the appropriate range in the logarithm of the semimajor axis for the particular scenario, and M_A and M_B (both in solar units) are the limits to the range in appropriate primary mass. We recognize the crudeness of our approximation and therefore provide, for each scenario, estimates of the three individual elements which enter into this approximation so that the interested reader may reevaluate the realization frequency with another probability distribution of his choice. Examples of other approaches to estimating a realization frequency are given by Greggio and Renzini (1983) and Miller and Chevalier (1983).

To estimate the realization frequency of each scenario, it will be necessary to discuss briefly the details of the transformation of what we shall call the *primordial* binary system (consisting of two unevolved components) into the *initial* binary system (consisting of an electron-degenerate dwarf and mass-donating companion). This transformation will typically involve a common envelope stage (Sparks and Stecher 1974; Paczyński 1976; Taam *et al.* 1978) which serves two purposes: it divests the primordial primary of its unevolved envelope to reveal an electron-degenerate core (or a potential electron-degenerate core), and it brings the two components closer together so that mass transfer to (one of) the degenerate core(s) may eventually occur. If the *initial* system consists of two degenerate dwarfs, then two common envelope stages may be required to achieve the *initial* configuration.

To illustrate the concepts which will be utilized to describe the transformation of a primordial into an initial system, we examine the requirements for producing an electron-degenerate dwarf made primarily of oxygen, neon, and magnesium (an ONeMg core, for short). It is expected that such a dwarf, if it accretes enough mass at an appropriate rate, will experience electron capture which will cause it to collapse as a whole to form a neutron star (Nomoto *et al.* 1979), and, thus, the massive, rapidly expanding envelope characteristic of SNeI may not be produced. However, since we will have occasion to invoke the possible existence of "bare" ONeMg cores in a later section, a discussion of this case is appropriate at this juncture.

All single stars of initial main-sequence mass less than ~ 8–$9\ M_\odot$ ultimately develop an electron-degenerate CO core; the precise upper limit on initial main-sequence mass depends on the surface composition (e.g., Becker and Iben 1979, 1980). Single stars of initial mass less than the upper limit become CO white dwarfs after the ejection of a planetary nebula shell.

If its initial mass is between ~ 8 and $\sim 12\ M_\odot$ (precise limits again being a function of composition), a single star will ignite carbon relatively quiescently at its center and develop a core composed primarily of O, Ne, and Mg (Barkat, Reiss, and Rakavy 1974; Barkat 1975). This core becomes highly electron degenerate; when its mass reaches $\sim 1.38\ M_\odot$, electron captures on Ne and Mg reduce the pressure sufficiently that core collapse is initiated, and, although a deflagration wave due to oxygen burning forms, an ultimate collapse of the core to neutron-star dimensions cannot be averted (Nomoto *et al.* 1979; Miyaji *et al.* 1980; Nomoto 1981).

The situation can be altered significantly if the star which ultimately develops an electron-degenerate ONeMg core is in a binary system. Of interest here are those systems in which the primary grows to fill its Roche lobe immediately after the main-sequence phase. In such systems, characterized by a semimajor axis in the range $A \sim 20$–$2000\ R_\odot$, the entire hydrogen-rich envelope will be stripped off, exposing a helium core whose mass $M_{\rm He}$ is related to the main-sequence mass $M_{\rm MS}$ of the primary approximately by $M_{\rm He} \approx [0.23 + 0.0135(M_{\rm MS} - 10.)]M_{\rm MS}$, where both $M_{\rm He}$ and $M_{\rm MS}$ are in solar units. This expression is obtained by multiplying the maximum mass in the convective core in a main-sequence star by 0.75 (Iben 1967) and is valid for $M_{\rm MS} \gtrsim 3\ M_\odot$.

As the newly formed helium star evolves, carbon and oxygen are produced in a central region which attains a mass $M_{\rm CO} \approx 1.3 + 0.65(M_{\rm He} - 2.4)$ when helium finally vanishes at the center (Paczyński 1971*b*; Delgado and Thomas 1981). This relationship between $M_{\rm CO}$ and $M_{\rm He}$ is valid for $M_{\rm He} \gtrsim 1.5\ M_\odot$. If $M_{\rm He}$ is in the range $M_{\rm He} \approx 1\ M_\odot$ (Paczyński 1971*b*) to $M_{\rm He} \approx 2.5\ M_\odot$ (Nomoto 1982*a*, *b*), the helium envelope attempts to expand to red giant dimensions as the more highly processed CO core contracts. We estimate that, if $M_{\rm CO}$ *at this point* is larger than $\sim 1.2\ M_\odot$ (see, e.g., Figs. 2–4 in Nomoto 1983*a*), carbon is ignited at the center, and, following a series of convective shell flashes, an electron-degenerate ONeMg core is built up.

Since the primordial primary filled its Roche lobe to become a helium star, it will again fill its Roche lobe and lose its helium-rich envelope if $M_{\rm He} \lesssim 2.5\ M_\odot$ (and $M_{\rm CO} \lesssim 1.36\ M_\odot$, $M_{\rm MS} \lesssim 10.4\ M_\odot$). If $M_{\rm CO} \gtrsim 1.2\ M_\odot$ (and $M_{\rm He} > 2.25\ M_\odot$, $M_{\rm MS} \gtrsim 9.8\ M_\odot$), the result will be a "bare" electron-degenerate ONeMg core. If $M_{\rm CO} \lesssim 1.2\ M_\odot$, the result will be a "bare" electron-degenerate CO white dwarf. Thus, it appears that only for a very narrow range of primordial main-sequence masses, viz., $\Delta M_{\rm MS} \approx 0.6\ M_\odot$ centered on $M_{\rm MS} \sim 10\ M_\odot$, will an ONeMg white dwarf be formed. We emphasize that our estimates are based on an extrapolation of the results for single-star evolution to a situation in which mass loss by Roche lobe overflow plays a crucial role; these estimates are therefore highly uncertain and should be replaced by the results of calculations in which mass transfer is properly taken into account (Iben and Tutukov 1983*c*).

We are now in a position to use equation (1) to estimate the realization frequency of this scenario. With $\Delta \log A \approx \log(2 \times 10^3/20)$ and $\Delta M_{\rm MS}/M_{\rm MS}^{2.5} \approx 0.6/10^{2.5}$, we have $\nu_{\rm ONeMg} \approx 7.6 \times 10^{-4}\, q$. Whether or not the newly formed white dwarf will eventually accrete and retain enough matter from its companion to achieve a mass near the Chandrasekhar limit, it is clear that, since $q \ll 1$, the probability of SN formation via this scenario is at least one order of magnitude smaller than the observed frequency of SNeI. Since the primordial secondary must have a mass less than $\sim 0.8\ M_\odot$ if it is to live for $\sim 10^{10}$ yr, q must be less than ~ 0.1, and the frequency of formation of SNeI in elliptical galaxies via this scenario is at least two orders of magnitude less than the observed frequency. Finally, an additional reduction of the probability of SNI formation comes about because only a fraction of ONeMg dwarfs formed in binary systems can accrete enough mass from their companions to reach explosive conditions. In § VII we shall show that the observed X-ray flux from galactic sources (due to accretion on the neutron star remnant) reduces the SNI realization frequency of this scenario to possibly less than $\nu_{\rm SNI} \approx 5 \times 10^{-7}$, or about four orders of magnitude smaller than the observed SNI frequency. On a more positive note, we shall also show that the recently discovered incredibly fast pulsar PS 1937 214 (Baker *et al.* 1982) can be quite naturally understood in terms of this infrequently realized scenario.

c) *On the Occurrence and Consequences of a Common Envelope*

Unfortunately, although the invocation of common envelope stages, which are postulated to have the effect of simultaneously stripping the expanding component down to a bare degenerate dwarf and bringing this dwarf and its companion closer together, is (in our view) absolutely essential for understanding the formation of promising pre–SNI systems, there exists no simple or straightforward theory that may be used to demonstrate convincingly that common envelopes with the desired characteristics must occur in appropriate situations or that may be used to predict, with any pretence of success, what the final quantitative consequences of the postulated common envelope stages will be. However, a judicious comparison (e.g., Webbink 1979; Ritter 1983) between (relatively) highly secure theoretical predictions about the formation of evolved cores as a function of the mass of the primordial primary component and the characteristics of known highly evolved close binary systems, such as cataclysmic variables, double-core planetary nebula, and low-mass X-ray binaries, provides rather convincing evidence that common envelopes with desirable properties actually occur in nature and justify the use of a parameterized summary of the outcome that has nearly the credibility of the parameterized summary of stellar convective flow expressed by the mixing length recipe for fully developed convection.

A good example is the system U Gem, which is one of two known cataclysmic variables which is also a double-line spectroscopic binary and, therefore, permits a direct estimate of the individual masses of the components. From the orbital period of $P \approx 0\overset{\rm d}{.}1769$ and the spectrographic data, Stover (1981) derives masses $M_1 \approx 1.18\ M_\odot$ and $M_2 = 0.56\ M_\odot$ for the white dwarf and its main-sequence companion, respectively. These masses imply a semimajor axis of size $A \approx 1.6\ R_\odot$. Since $0.5\ M_\odot$ is the maximum mass for a helium core which can avoid conversion into a CO core in the primordial primary, the white dwarf is almost certainly composed of C and O. One may envision two ways in which a CO white dwarf of the observed mass might be formed: by mass loss through Roche lobe overflow when a fairly massive primary leaves the main sequence just after having exhausted hydrogen in its core, before the production of a CO core, and by Roche lobe overflow delayed until after the primary has already formed a CO core.

Using the expressions relating $M_{\rm MS}$, $M_{\rm He}$, and $M_{\rm CO}$ given in the previous subsection, we estimate that, if it loses its entire hydrogen-rich envelope after leaving the main sequence but before igniting helium at its center, a primary of mass $M_{\rm MS} \approx 9.7\ M_\odot$ will expose a bare helium star of mass $M_{\rm He} \approx 2.2\ M_\odot$, and this star will go on to form a CO core of mass $M_{\rm CO} \approx 1.18\ M_\odot$ before again expanding to fill its Roche lobe and losing the remainder of its helium-rich envelope. The radius of a zero-age main-sequence star of mass $9.7\ M_\odot$ is $\sim 3.5\ R_\odot$, and, during the main-sequence phase, this radius nearly doubles to $\sim 7\ R_\odot$; we infer that the primordial semimajor axis for this scenario must be larger than $A_0^{\rm min} \approx 7\ R_\odot$. Using the fact that angular momentum J is related to component masses and separation A by $J = (GA)^{1/2} M_1 M_2 M_t^{-1/2}$, where G is the gravitational constant and $M_t = M_1 + M_2$, we can determine the minimum angular momentum of the primordial system $J_0^{\rm min}$ and compare with the current angular momentum $J_{\rm now}$ to find $J_{\rm now}/J_0^{\rm min} \approx 0.141$. Thus (for this scenario) at least 80% of both the original mass and angular momentum has been lost from the system as a consequence of processes initiated by Roche lobe overflow.

The situation is even more extreme if we examine the alternate scenario for producing a CO white dwarf: Roche lobe overflow after, and not until, the primordial primary has already formed a CO core in its interior. The relationship between CO core mass $M_{\rm CO}$ and stellar radius for single stars is given by (see § II) $R \approx 1050\ R_\odot\ (M_{\rm CO}/M_\odot - 0.5)^{0.68}$. Therefore, if a core mass of $M_{\rm CO} \approx 1.18\ M_\odot$ is to be achieved before Roche lobe overflow occurs, the primordial separation of the system must be on the order of $A_0 \approx 2R(M_{\rm CO} = 1.18\ M_\odot) \sim 1600\ R_\odot$. By adopting $M_{\rm MS} = 1.2\ M_\odot$ as the minimum main-sequence mass of the primary (and thereby neglecting the fact that, as discussed in § III, mass loss from single asymptotic giant branch stars is extensive), we derive that (for this scenario) $J_{\rm now}/J_0^{\rm min} < 0.012$. This is obviously a considerable overestimate since, if the system loses angular momentum, it must also lose mass. In any case, both scenarios demonstrate that the angular momentum and semimajor axis of the precursor of U Gem must have been much larger than that which U Gem currently possesses, and one of the scenarios requires that the primary lose over 80% of its original mass.

Possibly even more compelling evidence for a mechanism which simultaneously brings stars closer together and produces mass outflow from the system is provided by those planetary nebulae which are associated with a central star that is known to be in a very short period binary. The most recent example is Abell 41 which surrounds a binary of period $P \approx 2$ hr 43 min (Grauer and Bond 1983). From the fact that the secondary does not appear to fill its Roche lobe, Grauer and Bond deduce upper limits of $M_2 < 0.32\ M_\odot$ and $R_2 < 0.31\ R_\odot$. Using the fact that the white dwarf primary cannot have a mass larger than $1.4\ M_\odot$, it follows that the current semimajor axis of the system must be less than $\sim 1.2\ R_\odot$. By whatever means the central white dwarf was formed, the primordial separation must have been considerably larger than the current separation, and the presence of an extended, detached nebula which cannot have been formed more than $\sim 10^4$ yr before the primary contracted to white dwarf dimensions, and which is now coasting outward at velocities characteristic of mass outflow from the surface of stars having red giant dimensions, demonstrates that mass loss from the system occurred at the same time that the secondary was drawing closer to the primary, through the outflowing matter. Thus, there is considerable observational support for the concept of a common envelope in which energy generated by the frictional drag on one or both members of the double core is converted into pressure waves which spread freely through the main body of the common envelope, thermalizing the atmosphere and forcing mass loss. At the same time, the frictional drag forces the components of the double core to spiral closer together.

If we assume, following Tutukov and Yungelson (1979*a*, *b*), that a fraction α of the energy released by double-core friction is used up in expelling the common envelope, then

$$\alpha \frac{GM_c M_2}{A_f} = \frac{GM_1^2}{A}, \tag{2}$$

where M_1 is the primordial mass of the primary, M_2 is the mass of the secondary, M_c is the mass of the degenerate core of the primary (or the mass of the helium core in the event of case B Roche lobe overflow), A and A_f are, respectively, the semimajor axis before and after the common envelope phase. This formulation allows us to explore, as a function of a single parameter, the consequences of the common envelope phases that appear in various scenarios in much the same fashion that stellar evolutionists have explored the effects of convection that appears in various stages of evolution as a function of a single parameter l/H in the mixing-length treatment of convection.

We can estimate the lifetime of a common envelope of mass M_e and radius R around a close binary of mass M_b and semimajor axis A. The rate of energy generation by the frictional drag force is $L_d \approx 0.1 \rho V^3 A^2$, where $\rho \sim M_e/R^3$ is the average density of the common envelope and V is the orbital velocity of the star filling its Roche lobe. The binding energy of the common envelope is $E_b \approx GM_e M_b/R$, and, if we suppose that all of the drag luminosity is spent on mass loss, then the time scale for loss of the common envelope is

$$\begin{aligned} \tau_l &\approx (10/\alpha)(R/V)(R/A) \\ &\approx (0.3/\alpha)(R/A)^2 A^{3/2} M_b^{-1/2} \text{ days}, \end{aligned} \tag{3}$$

where now M_b and A are in solar units, and the factor α has been introduced in the same spirit as in equation (2). This is effectively the time for a sound wave to propagate through the envelope and represents a lower limit to the time over which the envelope can be ejected. In reality, the drag force will cause a spin up of the common envelope and a concomitant reduction in the frictional action. In principle, this could continue until corotation of the whole system is achieved and the time scale for the loss of the common envelope approaches the thermal time scale of the envelope, $\tau_{\mathrm{KH}} \approx E_b/L$, where L is the luminosity flowing through the envelope.

In closing this discussion we remark that a necessary condition for the formation of a common envelope is that mass transfer onto the accreting star must be more rapid than this star can actually incorporate into a new quasi-static structure whose radius remains *inside* of its Roche lobe. It is the rejection of accreted matter on Roche lobe overflowing that sets the stage for the formation of the common envelope. We remark finally that, whenever the common envelope is initiated by a star with an electron-degenerate core, the inevitable conclusion is that this core becomes "bare" or "naked" on such a short time scale that neither the mass of the core nor the mass of its companion changes.

d) *Plan of Attack*

In § II we explore the limitations on mass accretion rates that will lead to the steady growth in mass of a degenerate dwarf. In § III we examine the necessary conditions for stable mass transfer by a Roche lobe filling red giant and estimate the realization frequency of systems consisting of a massive CO degenerate dwarf and a Roche lobe filling red giant. The prognosis for such systems becoming SNeI is not bright, but in § IV we show that systems in which the mass donor is an asymptotic giant branch (AGB) star *not* filling its Roche lobe have a better chance of evolving to explosive conditions; in these systems, the "driver" for mass transfer is an intrinsic wind from the red supergiant, rather than expansion driven by nuclear evolution in the interior as in the case of a Roche lobe filling donor.

In § V we discuss both cataclysmic systems, which consist of a degenerate dwarf and a low-mass Roche lobe filling main-sequence donor, and cataclysmic-like systems, in which the donor is more massive and can have evolved slightly off of the main sequence. For both systems, the components must be initially closer together than $\sim 10\ R_{\odot}$. For cataclysmic systems, the driving force for mass transfer is a magnetic stellar wind, or gravitational radiation, or both. For cataclysmic-like systems, the driving force is evolutionary change in the interior, and mass-transfer is on a thermal time scale. The combined frequency of realization of these systems is estimated to be close to the observed supernova rate, but the fact that, in both cases, the rate of hydrogen accretion is low enough that nova and dwarf nova events are to be expected may eliminate some fraction (perhaps most) of these systems from contention as SNeI precursors.

In § VI we explore the probability of forming *very* close binaries consisting of two electron-degenerate dwarfs that are driven ever closer together by the radiation of gravitational waves. Estimates of realization frequencies suggest that double degenerates consisting of either two CO dwarfs or two helium dwarfs are very likely SNI precursors.

In § VII we examine the effect of a supernova explosion on the binary systems for the various scenarios and attempt to derive therefrom some further clues as to the most likely SNI precursor systems. Then, in § VIII, we argue that low-mass X-ray binaries are not the end product of SNI explosions in binaries. Section IX contains a summary and conclusion.

In a related paper (Iben and Tutukov 1983*a*) emphasis is focused on the scenarios that lead to double CO degenerate dwarfs drawn together by gravitational wave radiation. In yet another paper (Iben and Tutukov 1983*b*) transients that occur in relevant mass-transferring binaries are discussed.

We close this introduction with one final technical detail: mass, radius, semimajor axis, and luminosity will sometimes be given explicitly in solar units ($M_\odot = 1.99 \times 10^{37}$ g, $R_\odot = 6.96 \times 10^{10}$ cm, $L_\odot = 3.86 \times 10^{33}$ ergs s^{-1}) and sometimes not. The choice of units should be clear from the context.

II. LIMITATIONS ON THE ACCRETION RATE FOR THE SUPERNOVA EXPLOSION OF AN ACCRETING ELECTRON-DEGENERATE DWARF

The behavior of an accreting electron-degenerate dwarf as it progresses toward an explosive state and the nature of the final explosion depend on the accretion rate. For example, accretion of hydrogen-rich matter at a rate much less than $\sim 10^{-9}\ M_\odot\ \text{yr}^{-1}$ on a CO white dwarf with mass exceeding 1.1–1.2 $M_\odot$ may lead to strong hydrodynamic flashes initiated in a degenerate hydrogen shell followed by the loss of almost all matter accreted between flashes, as in classical novae (e.g., MacDonald 1983). For higher accretion rates, shell flashes are more likely to be quasi-static and not of themselves lead to mass ejection. A massive ($M \gtrsim M_\odot$) CO dwarf accreting hydrogen-rich matter at a rate larger than $\sim 10^{-8}\ M_\odot\ \text{yr}^{-1}$, but less than $\dot{M}_{\text{steady}} \approx 1.3 \times 10^{-7}\ (M/M_\odot)^{3.57}\ M_\odot\ \text{yr}^{-1}$, remains rather compact even during hydrogen shell flashes, and very little mass is expected to be lost from the binary system during a very brief phase when the accreting star swells to giant dimensions (Iben 1982*b*).

However, CO dwarfs accreting at rates of 10^{-9}–$10^{-8}\ M_\odot\ \text{yr}^{-1}$ expand during hydrogen shell flashes beyond the limits of orbits of cataclysmic-like binaries with semimajor axis $\sim R_\odot$ (Paczyński and Zytkow 1979; Iben 1982*b*). White dwarfs accreting at rates larger than $\dot{M}_{\text{steady}}$ do not experience hydrogen shell flashes, but expand to a steady state radius $R \approx 1050\ (M/M_\odot - 0.5)^{0.68}\ R_\odot$ (Iben 1983), which, for a massive white dwarf, is also much larger than the orbits of typical cataclysmic variables. The formation of a common envelope in these latter two cases can lead to effective mass loss from the binary system, as is evident on comparing our estimate for the time scale of common envelope mass loss (eq. [3]) with typical lifetimes τ_n for the hydrogen-burning phases of accreting white dwarfs which experience shell flashes. Values of τ_n range from a few months (for an accretion rate of $10^{-7}\ M_\odot\ \text{yr}^{-1}$ on a dwarf of mass 1.4 $M_\odot$) to several thousand years (for $\dot{M} = 10^{-10}\ M_\odot\ \text{yr}^{-1}$ and $M_{\text{WD}} \approx 0.8\ M_\odot$) (Iben 1982*b*).

Thus, for those close binaries which develop a common envelope during shell flashes, the effective accretion rate will be reduced by mass loss from the entire system. Wider systems may avoid common envelope formation, but for very close systems, heavy mass loss during the red supergiant portion of the hydrogen-burning phase could prevent the mass of the CO dwarf from reaching the Chandrasekhar limit. In those close binaries in which accretion rates are greater than $\dot{M}_{\text{steady}}$, the common envelope stage may persist until the "feeding" star is completely "eaten"; the remnant star will continue as a single AGB star, lose its acquired envelope via a strong wind, and ultimately eject a planetary nebula before the underlying white dwarf reaches the Chandrasekhar limit. For accretion rates less than $\sim 10^{-8}\ M_\odot\ \text{yr}^{-1}$, the fraction of the time τ_n which an accreting dwarf can spend at large radius is a function of the mass remaining in the hydrogen-rich envelope. Once this mass drops below a critical value (whether as a consequence of nuclear burning or as a consequence of common envelope mass loss), the accreting star will shrink and consume the remainder of its nuclear fuel as a compact star, well within its Roche lobe. For $A \gtrsim R_\odot$ and $10^{-7} > \dot{M}_{\text{acc}} > 10^{-9}\ M_\odot\ \text{yr}^{-1}$, one may estimate that effective accretion rates exceed one-half the interflash accretion rate for all $M_d \gtrsim M_\odot$ (Iben 1982*b*).

In summary, accretion at a rate greater than $10^{-9}\ M_\odot\ \text{yr}^{-1}$ and less than $\dot{M}_{\text{steady}}$ may lead to the growth of a helium shell above the CO core. What happens when this helium shell ignites depends on the accretion rate. If the accretion rate is in the range $4 \times 10^{-8} \lesssim \dot{M}(M_\odot\ \text{yr}^{-1}) \lesssim \dot{M}_{\text{steady}}$, then stationary burning or a sequence of weak helium shell flashes leads to a continuous increase in the CO core mass up to a carbon deflagration supernova explosion (Fujimoto 1980). Fujimoto predicts that, for $10^{-9} \lesssim \dot{M}(M_\odot\ \text{yr}^{-1}) \lesssim 4 \times 10^{-8}$, the helium flash will develop into a detonation of carbon in the underlying core and to an SN explosion which may disrupt the core entirely. If the shock generated by the helium shell flash is not too strong, a CO dwarf may remain after the SN explosion. However, the most probable result of carbon ignition in an accreting CO dwarf of mass close to the Chandrasekhar limit is the complete disruption of the dwarf, although the formation of a remnant CO dwarf in a few instances is also not excluded. Since the details of the computation of an SN explosion are complicated and not completely understood, the formation of neutron stars, at least in some cases, is also still a possibility.

In the foregoing, we have assumed that the accreted material is composed primarily of hydrogen. If the accreted material contains no hydrogen, then the limits on accretion rate that can lead to growth of the CO core toward the Chandrasekhar limit are modified. For example, if the accreted material is primarily helium, at least five distinct regimes have been identified (see, e.g., Fujimoto 1980; Fujimoto and Taam 1982; Fujimoto and Sugimoto 1982; Nomoto 1982*b*; Woosley, Axelrod, and Weaver 1983). (1) For dwarf masses larger than $\sim 1.15\ M_\odot$ and for accretion rates $\lesssim 10^{-9}\ M_\odot\ \text{yr}^{-1}$, the matter in the growing helium shell atop the CO white dwarf is not compressed or heated sufficiently to experience a single helium shell flash; the end result will be an SN explosion triggered by carbon ignition at the center when the total mass of the layered white dwarf reaches $\sim 1.4\ M_\odot$. (2) For CO dwarf masses on the order of 0.5 $M_\odot$ and for accretion rates in the range $10^{-9} < \dot{M}(M_\odot\ \text{yr}^{-1}) < 4 \times 10^{-8}$ a single, outward propagating detonation wave converts the freshly accreted helium into ^{56}Ni, and so on, and ejects it, leaving a white dwarf (Weaver and Woosley 1980). (3) For CO dwarf masses on the order of 1.2 $M_\odot$ and for accretion rates in the range $10^{-9} \lesssim \dot{M}(M_\odot\ \text{yr}^{-1}) \lesssim 4 \times 10^{-8}$, the final result will be a "double" detonation which begins in the helium shell and propagates into the C-O core; the SN explosion completely disrupts the star. (4) If $\dot{M} > 4\times10^{-8}$ $M_\odot$ yr^{-1}, finite-amplitude helium flashes will occur, the C-O core will grow until, again when $M \approx 1.4$ $M_\odot$, carbon ignition at the center will lead to an SN event. (5) Finally, the upper limit on accretion rate is $\sim 10^{-6}$ $M_\odot$ yr^{-1}, rather than $\dot{M}_{\text{steady}}$; for values of $\dot{M}$ much larger than 10^{-6} $M_\odot$ yr^{-1}, the accretion dwarf is expected to expand until the accretion luminosity equals the Eddington luminosity. Many more calculations should be done to delineate the boundaries of the parameter space occupied by representatives of each type of explosion. Furthermore, all limits on $\dot{M}$ depend, of course, on the temperature distribution in the dwarf, and very little of the required exploration has yet been done.

Suppose now that the accreting white dwarf is composed primarily of helium, which means that its initial mass must be less than ~ 0.5 $M_\odot$. If the accreted matter is hydrogen rich and the accretion rate exceeds $\dot{M}_{\text{RG}}$, the rate of growth of the degenerate helium core in a single star on the first red giant branch, the hydrogen-rich envelope will expand to giant dimensions in response to hydrogen burning at its base, and the formation of a common envelope will prevent the increase in mass of the helium dwarf. The critical accretion rate, as given by equation (10) in § III*c*, varies from $\sim 10^{-8}$ $M_\odot$ yr^{-1} when $M_{\text{He}} \approx 0.4$ $M_\odot$ to $\sim 10^{-7}$ $M_\odot$ yr^{-1} when $M_{\text{He}} \approx 0.5$ $M_\odot$. Assuming an analogy with the case of the accreting CO dwarf, we may guess that, for accretion of hydrogen-rich matter at rates between 0.1 and 0.01 of the critical one, expansion and common envelope formation will be avoided. Indeed, Nomoto and Sugimoto (1977) find that, accretion at the rate $\dot{M} \lesssim 2\times 10^{-8}$ $M_\odot$ yr on a helium dwarf of initial mass $M_{\text{He}} = 0.4$ $M_\odot$ will proceed uneventfully until the mass of accreted matter exceeds 0.25–0.4 $M_\odot$, at which point a detonation which completely disrupts the star is set off. In the absence of additional numerical results for different initial masses and accretion rates, we shall assume that accretion of hydrogen-rich matter at rates 0.01–1 $\dot{M}_{\text{RG}}$ will, in general, lead to an SN explosion if accretion can be sustained until the helium dwarf attains a mass ~ 0.65–0.8 $M_\odot$.

We expect that the accretion of helium-rich matter on a helium dwarf will lead to the ignition of a core helium flash if $\dot{M} > \dot{M}_{\text{RG}}$ when M_{He} reaches ~ 0.5 $M_\odot$. The dwarf will then be converted into a CO dwarf, and evolution should proceed as already outlined for an initial CO dwarf. Accretion at rates less than $\dot{M}_{\text{RG}}$ should lead to a detonation SN when M_{He} reaches ~ 0.65–0.8 $M_\odot$, just as in the case of the accretion of hydrogen-rich matter. The accretion of matter by an electron degenerate ONeMg dwarf has not been studied numerically at all. But, taking into account that the mass-radius relationship for an ONeMg dwarf cannot differ significantly from the relationship for a CO dwarf (until electron captures become important), we may suppose that the limitations on the accretion rate of hydrogen- and helium-rich matter by CO dwarfs are valid in this situation. The final outcome, of course, is quite different.

Two final, and yet numerically unexplored cases are the accretion of carbon- and oxygen-rich matter on a CO dwarf and the accretion of similar matter on an ONeMg dwarf. It is not difficult to guess that, for almost all accretion rates in the range 10^{-11}–10^{-6} $M_\odot$ yr^{-1}, the accreting dwarf will remain compact until its mass reaches ~ 1.38–1.39 $M_\odot$, whereupon it will be completely disrupted if its core is made of carbon and oxygen, or it will collapse if its core is made of oxygen, neon, and magnesium. These statements derive from experiments (Ergma and Tutukov 1976; Taam 1980*a*, *b*; Iben 1982*a*) which follow explicitly the growth of a CO dwarf in response to assumed steady burning of accreting hydrogen-rich matter (all thermal instabilities suppressed). There is an absolute maximum to the effective accretion rate that is set by the requirement that the rate at which gravitational energy is released be less than the Eddington limit. For a dwarf of solar mass, this means that $\dot{M} \lesssim 5\times10^{-6}$ $M_\odot$ yr^{-1}. For rates larger than this an extended envelope will be formed. However, the formation of an extended envelope does not automatically imply evolution into a common envelope stage and consequent mass loss from the system. Since the gravitational energy released in the process of accretion is inversely proportional to the radius of the accreting star, expansion may be halted at a new equilibrium radius that is well within the Roche lobe of the accreting star. This possibility requires testing by numerical experiment. The interval ~ 1–5×10^{-6} $M_\odot$ yr^{-1} for CO dwarfs must also be explored numerically since it is expected that, at such large accretion rates, carbon ignition at the center will occur for smaller central densities and therefore for smaller core masses than in the case of ignition at the center of a single star (Ergma and Tutukov 1976). The character of the resulting explosion has also yet to be explored numerically, but one may anticipate a weaker deflagration than in the case of smaller accretion rates.

III. A CARBON-OXYGEN DWARF ACCRETING MATTER FROM THE HYDROGEN-RICH ENVELOPE OF A ROCHE LOBE FILLING RED (SUPER)GIANT

a) *General Considerations*

Stars with degenerate cores have the well-known property that, to first approximation, the luminosity and the rate at which core mass grows depend only on the mass of the degenerate core. Once total stellar mass, surface composition, and l/H are specified, the stellar radius is also a function only of the core mass. This provides us with an opportunity to estimate analytically the mass exchange rate in a system wherein the secondary or Roche lobe filling component has a degenerate core. The accreting primary can be a main-sequence star, degenerate dwarf, or neutron star. Let us assume that the radius R of a star with a degenerate helium or CO core can be written as

$$R/R_\odot = c(M_c/M_\odot)^\gamma, \tag{4}$$

where M_c is the core mass, and that the rate at which the core mass increases can be written as

$$\dot{M}_c = b(M_c/M_\odot)^\beta\ M_\odot\ \text{yr}^{-1}. \tag{5}$$

The usual dependence of the Roche lobe radius R_{R} on the mass M of the Roche lobe filling star R can be represented as

$$R_{\mathrm{R}} = 0.52A(M/M_t)^{0.44}, \tag{6}$$

where A is the semimajor axis of the orbit (which is assumed to be circular) and M_t is the total mass of the system. This last equation is an approximation to the Paczyński (1971*a*) logarithmic formula, and it is valid for $0.1 \lesssim M_2/M_1 \lesssim 1.5$.

If we also assume (for the present) that orbital angular momentum and total stellar mass are conserved during evolution (this is called the "conservative" case), the condition for filling the Roche lobe at any epoch becomes

$$\frac{R}{R_0} = \frac{M_0^{1.56}(M_t - M_0)^2}{M^{1.56}(M_t - M)^2} = \left(\frac{M_c}{M_{c_0}}\right)^2, \tag{7}$$

where index zero indicates initial values. Equation (7) can be solved numerically because $M_c(t)$ is known from evolutionary computations. To find the mass exchange rate $\dot{M}$, we find derivatives of both sides of equation (7), taking into account the dependence of $\dot{M}$ and R on the degenerate core mass, to find

$$\dot{M}(M_\odot\ \mathrm{yr}^{-1}) = -\frac{\gamma b M}{(1.56 - 2q)M_\odot}\left(\frac{M}{M_t}\right)^{0.44(\beta-1)/\gamma} \times\left(\frac{0.52A}{cR_\odot}\right)^{(\beta-1)/\gamma}, \tag{8}$$

where $q = M/(M_t - M)$. Although $\dot{M}$ varies during mass exchange, this variation is relatively small (Webbink *et al.* 1982; Taam 1983), and equation (8) with q set equal to the initial mass ratio q_0 provides a satisfactory estimate of the mass exchange rate.

Since $\dot{M}$ must be negative, $1.56 - 2q$ must be positive, or $q < 0.78$. It is easy to show that this condition is fulfilled for all systems in which the Roche lobe filling component has a deep convective envelope. From equation (6) it follows in the conservative case that $d\ln R_{\mathrm{R}}/d\ln M = 2q - 1.56$. To the extent that the secondary is completely convective and radiation pressure can be neglected ($\Gamma = [d\ln P/d\ln\rho]_{\mathrm{ad}} = 5/3$), it may be represented by a polytrope of index $n = 3/2$. The binding energy of an index n polytrope is $E_b = -(3/2)(GM^2/R)(5-n)$ and, by setting $\dot{E}_b = -GM\dot{M}/R$, we obtain $d\ln R/d\ln M = (2n-4)/3$. Thus, setting $n = 3/2$, we have that $d\ln R/d\ln M = -1/3$, and the condition that $d\ln R_{\mathrm{R}}/d\ln M = d\ln R/d\ln M$ translates into $q_0 \lesssim 0.61$.

Since a real red giant usually has a radiative-conductive core whose mass is not negligible, and since radiation pressure is in fact appreciable in supergiants and the ionization rate is variable along the radial coordinate, the $n = 3/2$ approximation is not strictly applicable, and the absolute value of the derivative $d\ln R/d\ln M$ can be larger than $-1/3$. Therefore, even systems for which q_0 is somewhat larger than 0.61 can avoid the immediate formation of a common envelope when the star with a convective envelope fills its Roche lobe. Only concrete numerical experiments can determine, in any given case, the precise limitation on q_0 for avoiding dynamical mass exchange (e.g., Iben and Tutukov 1983*c*). In the following we will, however, adopt the approximate condition, $q_0 < 0.61$.

b) Limits on the Mass of the Donor

The maximum mass which an accreting CO core can achieve without exploding is $\sim 1.39\ M_\odot$ and, from the condition that $q_0 \lesssim 0.61$, the maximum mass of a red (super)giant component when mass transfer begins must therefore be less than $\sim 0.85\ M_\odot$. To estimate the lower limit on the initial mass of the CO dwarf component, we need to know the maximum mass of the oldest red giants. Toward this end, we have constructed an analytical approximation to the numerical evolutionary computations of Mengel *et al.* (1979) for the time it takes a star of initial mass M_G to reach the tip of the red giant branch:

$$T_G = 7.6\times 10^9 \frac{(1+30Z)}{(M_G/M_\odot)^{3.70}}\left(\frac{0.23}{Y}\right)^{1.6}\ \mathrm{yr}. \tag{9}$$

This formula is valid for $0.7 \lesssim M/M_\odot \lesssim 0.9$, $0.2 \lesssim Y$ (helium abundance by mass) $\lesssim 0.3$, and $0 \lesssim Z$ (heavy element abundance by mass) $\lesssim 0.04$. We rewrite this in terms of the initial main-sequence mass M_G of the giant as

$$\frac{M_G}{M_\odot} \approx 0.83 \frac{(1+30Z)^{0.27}}{T_{15}^{0.27}}\left(\frac{0.23}{Y}\right)^{0.43}, \tag{9a}$$

where T_{15} is the cosmological time in units of 15×10^9 yr. Assuming that $T_{15} \approx 1$ and that the Big Bang values of Y and Z are $Y_0 = 0.23$ (Gautier and Owen 1983) and $Z_0 \approx 0$, we have that $M_G \approx 0.83\ M_\odot$. Decreasing Y_0 to 0.215 increases the initial mass of the giant to $0.85\ M_\odot$, making it impossible for systems consisting of an accreting white dwarf and a red (super)giant filling its Roche lobe to exist unless mass loss from the red giant precedes the Roche lobe filling stage. As another example, if we suppose that $Y_0 \approx 0.23$ and that $\Delta Y \approx 3.0\Delta Z$ (e.g., Serrano and Peimbert 1981), we have that, on changing the heavy element abundance Z from zero to 0.03, M_G changes from $0.83\ M_\odot$ to $0.85\ M_\odot$, again making it impossible for systems consisting of an accreting white dwarf and a red (super)giant filling its Roche lobe to exist unless mass loss from the red giant precedes the Roche lobe filling stage.

Thus, we see that the stability criterion limits the masses of components of systems which can produce supernovae to very narrow intervals: the initial mass of the degenerate CO dwarf must be in the range 1.31–1.39 $M_\odot$, and the initial mass of its companion must lie in the range ~ 0.8–$0.85\ M_\odot$. Such systems clearly belong to a very old population with an age no shorter than $\sim 12\times 10^9$ yr. Whether or not real single stars can produce CO white dwarfs of initial mass as large as those required in this scenario is itself a problem comparable with that of producing supernovae (see § III*e*).

Suppose, now, that the degenerate dwarf companion of a red (super)giant is composed of helium. Since the maximum mass which a helium core may achieve without of itself burning helium is $\sim 0.5\ M_\odot$, it is clear from the condition for stability against dynamic mass exchange ($q < 0.61$) that a helium white dwarf cannot be fed by a red (super)giant with $M \gtrsim 0.8\ M_\odot$ filling its Roche lobe. Instead, a common envelope will form, and all of the matter initially in the hydrogen-rich envelope of the red (super)giant will be lost from the system.

As an aside, we are in a position to place a lower limit on the mass of the neutron star in the oldest low-mass X-ray binaries consisting of a neutron star and a red giant filling its Roche lobe (Webbink *et al.* 1983). Since $q < 0.61$ and since the initial mass of the red giant donor must exceed $\sim 0.8\ M_\odot$, the mass of the neutron star in such systems must be larger than $\sim 1.31\ M_\odot$.

c) *The Case of a Donor with a Degenerate Helium Core*

After the common envelope stage which transforms the primordial binary consisting of two main-sequence stars into one consisting of a CO degenerate dwarf and a less evolved companion, the first potential opportunity for mass transfer occurs when the companion leaves the main sequence, develops an electron-degenerate helium core, and becomes a red giant. The radius and the hydrogen-burning rate of the red giant (of Population I composition) can be approximated from the numerical results of Mengel *et al.* (1978) for $Y = 0.2$, $Z = 0.01$, $0.8 \le M/M_\odot \le 2.2$ by

$$\frac{R}{R_0}=10^{3.5}\left(\frac{M_c}{M_\odot}\right)^{4} \quad \text{and} \quad \dot{M}_c(M_\odot\ \mathrm{yr}^{-1})=10^{-5.36}\left(\frac{M_c}{M_0}\right)^{6.6}. \tag{10}$$

Thus, in equation (8), we may insert $c = 10^{3.5}$, $\gamma = 4$, $b = 10^{-5.36}$, and $\beta = 6.6$, and the mass *exchange* rate becomes

$$\dot{M}(M_\odot\ \mathrm{yr}^{-1})=\frac{10^{-10.05}}{1.56-2q}\left(\frac{M}{M_\odot}\right)^{1.62}\left(\frac{M_\odot}{M_t}\right)^{0.62}\left(\frac{A}{R_\odot}\right)^{1.4}. \tag{11}$$

This estimate agrees within a factor of 2 with the results of numerical integration of equation (7) by Webbink *et al.* (1983) and by Taam (1983). Taking into account the initial masses of the components ($M_0 \sim 0.8\ M_\odot$, $q_0 \leqslant 0.61$), we have

$$\dot{M}(M_\odot\ \mathrm{yr}^{-1}) \approx 1.4\times10^{-10}(A/R_\odot)^{1.4}. \tag{11a}$$

From this last equation we see that, for $\dot{M}$ in the permissible range from $10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$ to $\sim 10^{-6}\ M_\odot\ \mathrm{yr}^{-1}$, A may range from $\sim 4\ R_\odot$ ($M_{\mathrm{He}} \approx 0.16\ M_\odot$) to $460\ R_\odot$ ($M_{\mathrm{He}} \approx 0.5\ M_\odot$). Comparison with equation (10) shows that typically $\dot{M}_c < \dot{M}$ as M_c grows from $\sim 0.1\ M_\odot$ to a maximum which is less than $\sim 0.5\ M_\odot$, and that the total mass that will be transferred is roughly $0.8–0.11(A/R_\odot)^{0.25}\ M_\odot$, or between $0.6\ M_\odot$ ($A \approx 4\ R_\odot$) and $0.3\ M_\odot$ ($A = 460\ R_\odot$). Therefore, if accretion begins with a CO dwarf of mass $\gtrsim 1.3\ M_\odot$, an accreting CO dwarf can reach the Chandrasekhar limit. We have neglected mass loss due to a stellar wind from the red giant since it is known that, for such stars, wind mass loss does not exceeds $\sim 20\%$ of the initial mass (e.g., Renzini 1981).

If the accreting star is a neutron star, then the X-ray luminosity of the accretion disk will be $\sim 10^2\ (A/R_\odot)^{1.4}\ L_\odot$, and all systems with $A \lesssim 40\ R_\odot$ ($P \lesssim 20^{\mathrm{d}}$) will be bright X-ray sources with a luminosity lower than the Eddington limit. This possibility, as an explanation for low-mass "bulge" X-ray sources, is discussed in detail by Webbink *et al.* (1983). It is worth pointing out that, because the maximum semimajor axis for systems where the secondary has a degenerate helium core is $\sim 460\ R_\odot$ ($P \approx 2$ yr), a significant fraction of systems with neutron stars may have super-Eddington accretion.

d) *The Case of a Donor with a Degenerate* CO *Core*

When a low- or intermediate-mass star exhausts helium at its center, it begins the ascent of the AGB, burning helium in a shell surrounding a growing electron-degenerate CO core. Hydrogen does not again burn in a shell until the helium-burning shell nearly touches the hydrogen-helium discontinuity. In a low-mass star ($M \lesssim 2.3\ M_\odot$) which experiences a core helium flash as a red giant (RG) when the mass in the electron-degenerate helium core reaches $M_{\mathrm{He}} \approx 0.5\ M_\odot$, hydrogen burning recommences on the AGB when the mass in the CO core reaches $M_{\mathrm{CO}} \approx 0.53\ M_\odot$. When hydrogen reignites, the star undergoes a series of thermal instabilities which causes hydrogen and helium to burn alternatively in shells, and we say that the star is in the thermally pulsing (TP-AGB) phase. During the preceding "E-AGB" phase, which lasts for approximately $t_{\text{E-AGB}} \sim 10^7$ yr, the luminosity varies as (Iben and Renzini 1983) $L \approx 170\ L_\odot \exp(1.91t/t_{\text{E-AGB}})$, and the stellar radius is related to the luminosity L by (Iben 1983) $R \approx 0.59(L/L_\odot)^{0.68}$. To obtain this last relationship we have assumed the mixing length to scale height ratio to be $l/H = 1.0$, the metal abundance to be $Z = 0.02$, and have neglected a weak dependence on stellar mass.

It follows from $d\ln R_{\mathrm{R}}/d\ln M = 2q - 1.56$ that, if the E-AGB star can fill its Roche lobe, mass will be transferred at the rate

$$\dot{M} \approx \frac{M}{(1.56-2q)M_\odot}1.3\times10^{-7}\ M_\odot\ \mathrm{yr}^{-1}, \tag{12}$$

and, after a time Δt(yr),

$$\left(\frac{M}{M_0}\right)^{1.56}\left(\frac{M_t-M_0}{M_t-M}\right)^{2}=\exp\left(-1.3\times10^{-7}\Delta t\right). \tag{12a}$$

Although the accretion rates given by equation (12) when $q \lesssim 0.61$ would permit a steady growth of a primary CO white dwarf of initial mass greater than $\sim 1\ M_\odot$, mass transfer in an isolated binary cannot occur since the maximum radius of a low-mass star during the E-AGB phase ($R_{\max} \approx 73\ R_\odot$) is smaller than the radius of a precursor hydrogen-burning red giant when it experiences the helium core flash ($R_{\mathrm{flash}} \approx 170\ R_\odot$). Thus, even though mass transfer during the earlier RG phase might increase the mass of the primary up to $\sim 1.3\ M_\odot$, further potential mass transfer will be postponed until the secondary enters the TP-AGB phase and the radius of the potential donor exceeds R_{flash}.

Even though, from the point of view of isolated binary star evolution, the exercise we have just completed would appear to be of entirely academic interest, three-body interactions in the dense cores of globular clusters could produce a few systems in which mass transfer during the E-AGB phase is important. Examples of appropriate interactions are (1) the displacement by a white dwarf of the main-sequence companion of a horizontal branch star, and (2) a series of "hardening" collisions which reduce the semimajor axis of a system consisting initially of a widely spaced horizontal branch star and a white dwarf. Of course, due to the short lifetime of the horizontal branch phase ($\sim 10^8$ yr) relative to the age of a typical cluster, the probability that appropriate stellar encounters occur will not be very large (see § VIII).

During the thermally pulsing phase, the average rate at which core mass grows due to nuclear burning is

$$\dot{M}_c(M_\odot\ \mathrm{yr}^{-1}) = 6.0\times10^{-7}\, X_{\mathrm{H}}^{-1}\left(\frac{M_c}{M_\odot}-0.5\right), \qquad (13)$$

where X_{H} is the surface hydrogen abundance by mass. The *mean* stellar radius may still be approximated by $R/R_\odot = 0.59\ (L/L_\odot)^{0.68}$. Ignoring, for the moment, the fact that the radius departs from its mean value during and immediately after thermal pulses, and using the fact that, for such stars, $L \propto (M_c/M_\odot - 0.5) \propto \dot{M}_c$, we can find $\dot{M}$ from equation (8) by setting $\gamma = 0.68$, $\beta = 1$, and $b = 8.0\times10^{-7}$ (choosing $X_{\mathrm{H}} = 0.75$):

$$\dot{M}(M_\odot\ \mathrm{yr}^{-1}) = -\frac{\gamma b M}{(1.56-2q)M_\odot} \approx -\frac{5.4\times10^{-7}}{1.56-2q}\frac{M}{M_\odot}. \qquad (14)$$

This estimate is valid for $0.55 \lesssim M/M_\odot \lesssim 8$ if the denominator of the last equation is positive. Remarkably, the mass-exchange rate in this case does not depend on the initial orbital period of the binary.

Comparing equations (14) and (13) we see that, for parameters in the permissible range for SNI formation, the rate at which the hydrogen-rich envelope of the donor decreases due to mass exchange is comparable to the rate at which it decreases due to nuclear burning. Since the initial CO core mass of a thermally pulsing AGB star exceeds $\sim 0.53\ M_\odot$, and since the initial mass of the secondary must be in the range 0.8–0.85 $M_\odot$, at most only $\sim 0.1\ M_\odot$ of hydrogen-rich matter is available for accretion. Thus, the initial mass of an accreting CO dwarf which might achieve an explosive state is confined to the range $\sim$1.3–1.4 $M_\odot$. As an example, if we assume $M_1 = 1.35\ M_\odot$, $M_2 = 0.8\ M_\odot$ ($q = 0.59$), then $\dot{M}$ given by equation (14), viz. $\dot{M} = 1.1\times10^{-6}\ M_\odot\ \mathrm{yr}^{-1}$, exceeds by almost a factor of 2 the maximum growth rate possible for a CO core of mass 1.35 $M_\odot$ (eq. [13]), and one therefore expects the formation of a giant common envelope around the mass-exchanging components. With $M_1 = 1.35\ M_\odot$ and $M_2 = 0.65\ M_\odot$ ($q = 0.48$), $\dot{M} \approx 6.3\times10^{-7}\ M_\odot\ \mathrm{yr}^{-1}$, and the accreting white dwarf will burn hydrogen as rapidly as it is accreted without being forced to expand. Thus, extremely delicate "tuning" is required if the accreting CO core is to reach the explosive stage. It is necessary to add that the total mass of exchangeable material depends strongly on the initial separation; the appropriate initial semimajor axis A is limited to the rather narrow interval 600–1500 $R_\odot$.

It is worth commenting that an observational search for possible representatives of this scenario would not be very rewarding. A white dwarf of mass $\sim 1.35\ M_\odot$ accreting at the rate $\dot{M} \sim 6\times10^{-7}\ M_\odot\ \mathrm{yr}^{-1}$ liberates gravitational potential energy and nuclear energy at a constant rate of $\sim 4.5\times10^4\ L_\odot$, corresponding to $M_{\mathrm{BOL}} \approx -7$. However, because hydrogen burns quiescently, no circumstellar shell of the planetary nebula variety is present about the white dwarf, and most of the light from the dwarf, being emitted primarily in the far-UV, is prevented by interstellar extinction from reaching the Earth. Thus, even if systems of the sort under discussion were responsible for all SNeI, so that there would be 0.1 $M_\odot\ \nu_{\mathrm{SNI}}/\dot{M} \approx 2\times10^3$ of them in the Galaxy, they would not be easily detectable.

A further potential obstacle to the transfer of sufficient matter to the CO dwarf is the fact that a bright AGB star is expected to lose matter via a stellar wind. The mass-loss rate in this wind can be estimated by the Reimers's (1975) relation as $\dot{M}_w(M_\odot\ \mathrm{yr}^{-1}) \approx -4\times10^{-13}\ LR/M(M_\odot/L_\odot R_\odot)$. For a red supergiant with $M_{\mathrm{CO}} \approx 0.60\ M_\odot$, $\dot{M}_w \approx 10^{-6}\ M_\odot\ \mathrm{yr}^{-1}$. If the matter in the stellar wind escapes from the binary system, the probability of obtaining the critical mass for an SN explosion is reduced even further, both because less mass is available for transfer and because the permitted range of appropriate initial semimajor axes is reduced. However, since it is probable that, for a given mass and radius, the mass-loss rate decreases with decreasing metallicity, this argument is less compelling for Population II stars. Furthermore, as we shall discuss in § IV, the possibility of accreting mass from the stellar wind bypasses the need for Roche lobe overflow, and the added freedom in permissible semimajor axes may, in fact, make wind transfer more effective than Roche lobe overflow transfer.

A final, and probably fatal, difficulty for accretion by Roche lobe transfer from a low-mass AGB star arises when one takes into account the oscillatory character of stellar evolution in the AGB stage. Thermal pulses, also known as helium shell flashes, lead to sharp dips and rises in luminosity and radius, which periodically interrupt the otherwise monotonic growth of radius which is corrolated with increasing core mass (see, e.g., Iben 1982*a*, *b*). For a short period following each flash, the stellar radius becomes larger than the preflash radius. When the radius of the star increases beyond the Roche lobe radius, one may estimate the mass-exchange rate as $\dot{M} \approx \rho R^2 c_T$, where ρ is the density of the red supergiant envelope, R is its radius, and c_T is the thermal velocity in this envelope. We find that, over the 10^1–10^3 yr duration of the large-radius phase of a thermal pulse, the mass-exchange rate will be of the order of 10^{-4}–$10^{-2}\ M_\odot\ \mathrm{yr}^{-1}$. This means that the matter lost from the AGB star will flow into a common envelope. From equation (3) with $R \approx 170\ R_\odot$ we see that the common envelope will be lost from the binary system in ~ 100 days (choosing $\alpha \approx 1$) and that the mass of the primary CO dwarf cannot increase at all during the common envelope stage. However, the two stars will be brought closer together during this stage, and, for a time, the system will appear as a planetary nebula excited by photons from two hot and bright CO white dwarfs, the less massive one burning helium and the more massive one burning newly acquired hydrogen near its surface. Whether or not the two white dwarfs will engage in subsequent effective mass exchange will be addressed in § VI.

In summary, we conclude that the likelihood that a system which consists of an AGB secondary and a degenerate CO dwarf primary will become an SNI is significantly impaired by severe restrictions: the need for a high initial CO core mass ($>1.3\ M_\odot$); the narrow range of favorable initial parameters ($q_0 < 0.61$); and the occurrence of an excessively large mass-exchange rate during helium shell flashes and the consequent probable formation of an extended common envelope. The most probable result of mass exchange in such systems will be a double-core planetary nebula (see § III*h*).

e) *Do* CO *Degenerates of Sufficiently Large Initial Mass Form?*

All of the previously discussed scenarios for SNI formation that rely solely on mass transfer from a secondary red (super)giant by way of Roche lobe overflow require that the initial mass of the CO white dwarf primary be on the order of or larger than 1.3 $M_\odot$. Are such large initial core masses achieved in nature? Comparison of the observed number-luminosity distributions of red supergiants with theoretical number-luminosity distributions of AGB stars in the Magellanic Clouds reveals the absence of red carbon supergiants (C stars) with CO core mass larger than $\sim 0.85\ M_\odot$ (see, e.g., Iben 1981*a*, *b* for a discussion). Frogel and Blanco (1983) find that, even in the youngest ($\lesssim 10^8$ yr) Magellanic Cloud globular clusters, there are only a handful of AGB stars brighter than $M_{\rm BOL} \approx -6$ and none brighter than $M_{\rm BOL} \approx -6.5$, whereas $M_{\rm BOL} \lesssim -7$ is expected for CO core masses $\gtrsim 1.3\ M_\odot$. However, Wood, Bessel, and Fox (1982) have argued that long-period variable M stars in the Clouds are AGB stars with core masses that extend all the way to the Chandrasekhar limit. Even so, the frequency of such stars appears to be at least 10 times less than expected on the assumption of a Salpeter-like mass function and a 10^6 yr AGB lifetime (Iben and Renzini 1983). Presumably mass loss via a potent wind, or planetary nebula ejection, or both, is responsible for these results.

The observed distribution of white dwarfs with respect to mass in our Galaxy (Weidemann 1979; Schonberner 1981) supports the existence of an upper limit to white dwarf mass considerably less than 1.4 $M_\odot$, but, because of the pressure of many selection effects, the proper transformation of the observed distribution of dwarfs into the real absolute distribution is a difficult task. Recent studies of white dwarfs in young galactic clusters reveal no white dwarfs of mass larger than $\sim 1.2\ M_\odot$ (Koester and Reimers 1981; Weidemann and Koester 1983), but once again, there are many uncertainties that enter into these mass estimates, and the statistics of small numbers plays a role. We conclude that, although the frequency of formation of CO dwarfs of initial mass as large as 1.3 $M_\odot$ appears to be at least an order of magnitude less than that expected on the basis of a Salpeter-like mass function and a 10^6 yr AGB lifetime, some small fraction of real AGB stars may produce a CO dwarf of initial mass $\gtrsim 1.3\ M_\odot$.

f) *Estimates of the Type I Supernova Formation Frequency for the Red Giant Plus* CO *Degenerate Dwarf Scenario*

Since, as we have seen, the *a priori* likelihood of producing an SNI by the transfer of matter through the Roche lobe of an AGB star onto a CO degenerate dwarf is exceedingly small, we return to the case of a red giant of mass $\sim 0.8\ M_\odot$ and a CO degenerate dwarf and derive a crude estimate of the frequency with which SNeI might be formed by such systems.

Given the uncertainty as to the actual relationship between primordial stellar mass M_0 and final CO core mass $M_{\rm CO}^f$ for single stars, we adopt a compromise relationship:

$$M_{\rm CO}^f = 0.446 + 0.106\ M_0. \tag{15}$$

This relationship, which is close to one derived by assuming a Reimers's mass-loss rate with efficiency $\eta = 2$ (Iben and Renzini 1983) and which is also close to the semiempirical one found by Weidemann and Koester (1983), states (whether or not this is true in reality) that no single stars of mass less than 9 $M_\odot$ (the upper mass limit for the formation of an electron-degenerate CO core) become SNeI and that very few AGB stars develop CO cores as massive as 1.3 $M_\odot$.

The integral over mass in equation (1) is now taken to extend from $M_A = 9.4 M_{\rm CO} - 4.2$, as given by equation (15), to $M_B = 9$. Here $M_{\rm CO}$ is to be interpreted as the minimum initial mass of the CO dwarf which will ultimately become a SNI. The value of q to be inserted in equation (1) is $\sim 0.8/8 = 0.1$, and the appropriate range in semimajor axes is approximated by $\Delta \log A \approx 0.68 \log[(1.4-0.5)/(M_{\rm CO}-0.5)]$, this latter following from the fact that the radius of an AGB star is proportional to $(M_{\rm CO}-0.5)^{0.68}$ and from the requirement that a common envelope must be formed if the CO degenerate dwarf and the 0.8 $M_\odot$ star are to become close enough together for Roche lobe filling to eventually occur. Putting all factors together, we have that

$$\nu_{\rm SNI}({\rm yr}^{-1}) \approx 0.009 \log\left(\frac{0.9}{M_{\rm CO}-0.5}\right) \times\left[(9.4M_{\rm CO}-4.2)^{-1.5} - 0.037\right] \tag{16}$$

is the frequency with which systems can produce CO white dwarfs of mass $\geqslant M_{\rm CO}$ and accrete enough matter through the Roche lobe of a red giant companion to become SNeI.

For $0.7 \leqslant M_{\rm CO} \leqslant 1.3$, expression (16) is roughly equivalent to

$$\log \nu_{\rm SNI}({\rm yr}^{-1}) \approx -0.8 - 3.2 M_{\rm CO}. \tag{17}$$

As we have shown in § III*b*, if the red giant of mass 0.8 $M_\odot$

possesses a deep convective envelope and if $d \log R/d \log M \approx -1/3$, mass transfer through the Roche lobe is dynamically unstable if the mass of the CO component is less than ~ 1.3 $M_\odot$. Therefore, the most probable mass exchanging systems of this type will begin with $M_{\rm CO} \gtrsim 1.3$, and, for them, $\nu_{\rm SNI} \approx 10^{-5}$ yr^{-1}, almost three orders of magnitude smaller than the observed frequency of $\sim 10^{-2}$ yr^{-1}. If, however, the convective envelope is not deep enough, then the criterion for stability is altered to $q \lesssim 0.78$, and it is possible to begin with $M_{\rm CO}$ as small as $\sim M_\odot$. The frequency of such systems becomes $\sim 10^{-4}$ yr^{-1}, still far below the observed SNI frequency (even in ellipticals, where, as we may estimate from Tammann 1982, $\nu_{\rm SNI} \approx 0.005$ yr^{-1} per 10^{10} $L_\odot$ in the B band).

One effect, not taken into account in deriving the R-L relationship used to estimate the value of $\Delta \log A$ used in equation (1), is the fact that, for appropriate conditions (on core mass, total mass, and surface composition), thermally pulsing AGB stars can dredge up to their surfaces matter which has been freshly processed in the convective shell during a thermal pulse (see, e.g., Iben and Renzini 1983). The newly added elements provide a further source of opacity which can act to increase the rate at which mean stellar radius increases with increasing luminosity and core mass. However, even if this effect produced an extra factor of 2 increase in radius over the AGB lifetime and hence an increase in $\Delta \log A$ to at least ~ 0.3, our estimates of $\nu_{\rm SNI}$ would be adjusted upward by no more than a factor of 2.

One might hope that the frequency of SNeI formation via the scenario of CO white dwarf plus red giant with a helium core could be increased by choosing an average metallicity for primordial systems more appropriate to Population II. That is, lowering Z might be expected to reduce the mass-loss rate from the progenitor of the CO dwarf to such an extent that a star with initial mass as small as 2–3 $M_\odot$ could produce a CO dwarf with mass 1.3–1.4 $M_\odot$. However, the relative number of low metal abundance stars in our Galaxy is too small for this possibility to go very far toward accounting for the observed SNI frequency.

g) On the Frequency of Planetary Nebula Byproducts

Although the traditional scenario appears at this writing to fall woefully short of accounting for SNeI, the shortfall is due to many restrictions on initial conditions, and there should be many more primordial systems which pass through a common envelope phase to produce a system consisting of a CO dwarf and a companion which evolves to the red giant or supergiant phase, even though the characteristics of the system are inappropriate to lead to a SNI explosion. It is expected that the newly formed CO dwarf will retain a thin layer of hydrogen which it burns at high luminosity and high surface temperature, and the emitted photons may excite nebular emission from the ejected common envelope. Since the nebula has been formed as a consequence of the formation of a common envelope, the components of the central star of the resulting planetary nebula should be at a separation smaller than that of the primordial precursor. The frequency of formation of such double-core planetary nebulae is worth estimating.

In order to achieve a system in which the degenerate CO core of the primary is "revealed" by mass loss during a common envelope stage, all we require is that the primordial semimajor axis of the system be somewhere in the range ~ 460–2000 $R_\odot$. Since there are no effective restrictions on primary mass (except that $M_B \lesssim 9$ $M_\odot$) or on q, the fraction of all binaries with A in the appropriate range is simply $\sim 0.2 \log(2000/460) \approx 0.1$, and thus the frequency at which planetary nebulae are formed via this scenario is $\nu_{\rm PN} \approx 0.1$ yr^{-1}. Since, by assumption, the AGB evolution of the primary is aborted earlier than in the case of a single star of the same mass (and also in the case of a star in a binary with $A > 2000$ $R_\odot$), the mass of the ejected planetary nebula will be larger than that formed by a single star.

The semimajor axis of the resulting central "star" (CO subdwarf plus main-sequence companion) depends on the details of the common envelope event. If we choose $\alpha \approx 1$ in equation (2), we might guess that, for primordial systems of low mass (say $M_1 \approx M_2 \approx 1$ $M_\odot$, $M_c \approx 0.5$ $M_\odot$), the semimajor axis will shrink by only a factor of 2, whereas, for more massive systems (say $M_1 \approx M_2 \approx 4$ $M_\odot$, $M_c \approx 0.8$ $M_\odot$), the semimajor axis will shrink by a factor on the order of 5. If the primordial mass ratio q is very small, one can achieve even greater shrinkage and produce very short period systems such as the central star of the planetary nebula Abell 41, which has a period of $2^{\rm h}$ $43^{\rm m}$ (Grauer and Bond 1983). For example, with primordial masses of $M_1 \approx 9$ $M_\odot$, $M_2 \approx 0.3$ $M_\odot$, primordial semimajor axis $A \approx 300$ $R_\odot$ (Kopal 1959), and $M_c \approx 1$ $M_\odot$, we have from equation (2) with $\alpha \approx 1$ that $A_f \approx R_\odot$ and $P_{\rm orbital} \approx 3^{\rm h}$. If α is smaller than 1, then the primordial masses need not be as diverse. For example, with $\alpha \approx 1/2$ and $M_2 \sim 0.3$ $M_\odot$, a value of $M_1 \sim 5$ $M_\odot$ also gives $P_{\rm orbital} \sim 3^{\rm h}$.

The fate of the central "star" of a double-core planetary nebula is also a function of the primordial mass of the system. That is, in low-mass systems with $A_f \approx 200$–1000 $R_\odot$, the primordial secondary can in many instances ($A_f \gtrsim 460$ $R_\odot$) eventually also become an AGB star; as it swells into its Roche lobe, another common envelope will be formed, and the system will again go through a planetary nebula stage. On this occasion, both central stars will contribute to the excitation of the nebula. The more massive CO dwarf will be resurrected or rejuvenated as a hydrogen-burning star for a brief moment, thanks to the acquisition during the common envelope stage of a thin layer of hydrogen-rich matter, and the less massive, newly formed CO dwarf will burn the thin layer of hydrogen-rich matter left on its surface, and do so on a longer time scale than its more massive companion.

If the primordial system is more massive, the semimajor axis formed during the first common envelope stage may be too small to allow the primordial secondary to become an AGB star. Instead, mass transfer of the "case B" variety and the formation of a second common envelope will produce a very close binary (say, $A_{ff} \approx 3$–100 $R_\odot$) consisting of a CO dwarf with a newly acquired thin skin of hydrogen-rich matter burning at high luminosity and a helium dwarf of low luminosity. As a consequence of hydrogen burning, radiation from the CO dwarf excites matter in the ejected common envelope. The mass of the nebula should in this instance be quite large (> $M_{\odot}$). The period of the double-core central star should be in the range 15 minutes to 100 days.

Thus, planetary nebula formation via a first common envelope phase is followed by planetary nebula formation via a second common envelope phase, and the rate at which binaries produce planetary nebulae via common envelope ejection (beginning with a system in which the primary becomes an AGB star before the first common envelope stage) is therefore on the order of $\nu'_{\rm PN} = 2\nu_{\rm PN} \approx 0.2$ yr^{-1}.

Another scenario for producing a short-period, double-core planetary nebula begins with an intermediate-mass close binary in which the primary overflows its Roche lobe after having established a large (> 0.5 $M_{\odot}$) helium core, but before helium is ignited in this core ("case B" mass transfer in the traditional nomenclature). In this case (see § VI for more detail) we require a semimajor axis in the range $6 \lesssim A/R_{\odot} \lesssim 460$, a primary mass in the range $3.8 \lesssim M_1/M_{\odot} \lesssim 8$, and no restriction on primordial mass ratio (i.e., $q = 1$ in eq. [1]), giving a formation frequency of $\nu \approx 3 \times 10^{-2}$ yr^{-1}. Since the helium core burns ($^4{\rm He} \rightarrow {}^{12}{\rm C} + {}^{16}{\rm O}$) on a long time scale and does not become hot enough to emit significant amounts of ionizing radiation, the ejected common envelope does not, in general, appear as a planetary nebula.

However, the helium star eventually transforms into a CO dwarf; when the second common envelope stage occurs as a result of the expansion of the primordial secondary, the CO dwarf burns the hydrogen in a newly acquired thin skin captured from the common envelope, and the system finally appears as a planetary nebula. Thus, the frequency of planetary nebula formation beginning with "case B" mass transfer is ~ 0.03 yr^{-1}, and the combined rate of planetary nebula formation due to common envelope ejection is $\nu'' \approx 0.23$ yr^{-1}.

For completeness, we remark on one last scenario which produces a planetary nebula composed of helium. In this case, we need to first form a "bare" helium star of mass in the range $1 \lesssim M_{\rm He}/M_{\odot} \lesssim 2$. This is accomplished by choosing $6.8 \lesssim M_1/M_{\odot} \lesssim 12$, $20 \lesssim A/R_{\odot} \lesssim 400$, and $q = 1$, giving a formation frequency of $\nu \approx 6 \times 10^{-3}$ yr^{-1}. Again, no planetary nebula is formed in this stage. However, in the process of developing an electron-degenerate CO core, the helium star expands, and, once it fills its Roche lobe, a common envelope composed by helium is formed and ejected as a planetary nebula excited by radiation from the hot CO core.

The frequencies which we have estimated should not be immediately translated into observational probabilities before a discussion of the lifetimes of both the envelopes and the central stars. In the case of the first common envelope ejection, we do not expect significant differences from usual planetary nebulae either with regard to the mass of the exciting star or with regard to nebular velocity; the only major difference is that the average mass of the ejected common envelope will be larger than the mass of the usual planetary nebula. In the case of the second common envelope ejection, expansion velocities may be larger than those of usual nebulae, and this may significantly shorten their lifetimes as observable nebulae. Finally, in the case of helium common envelopes, the mass of the central star (~ $M_{\odot}$) will be larger than that of central stars in usual nebulae (~ 0.6 $M_{\odot}$), and, since the fading time t_f of the central stars goes as $t_f \propto M_{\rm core}^{-10}$ (e.g., Iben 1983), these nebulae will be of shorter duration than the usual nebula.

In summary, planetary nebula formation via a first common envelope phase is followed by planetary nebula formation via a second common envelope phase, and the fraction of all binaries that produce planetary nebulae with a double-core central "star" of short period ($P < 1$ yr) is ~ 0.23. Roughly half of the nebulae produced should be indistinguishable from usual nebulae, apart from the duplicity of the central star. The shortest period central stars formed via these scenarios will have periods on the order of a few hours ($A \approx 2$–3 $R_{\odot}$) and will consist of a hot subdwarf with a CO core and a faint star with an electron-degenerate helium core.

IV. ACCRETION FROM A STELLAR WIND

An alternative to the traditional scenario opens up if we relax the assumption that the Roche lobe must be filled in order for mass transfer to occur. It is well known that bright supergiants have strong stellar winds which abstract mass at rates that can be estimated by the Reimers expression. Some stellar wind material may be captured by the dwarf companion, as in the case of nuclear-powered symbiotic stars (Tutukov and Yungelson 1976) or in the case of massive X-ray binaries (Davidson and Ostriker 1973; Tutukov and Yungelson 1973). Following these latter authors, the efficiency of accretion can be estimated by

$$\Sigma \approx \frac{G^2 M_d^2}{4\left(v_{\rm orb}^2 + v_w^2\right)^2 A^2}, \tag{18}$$

where M_d is the mass of the degenerate dwarf, A is the semimajor axis of the system, v_w is the wind velocity at the position of the dwarf, and $v_{\rm orb}$ is the orbital velocity of the dwarf. The wind velocity is the most important parameter in equation (18). Since the semimajor axis is of the order of the red supergiant radius, the dwarf is in the acceleration zone for the stellar wind matter, and this makes the estimation of v_w very difficult. The usual assumption, viz., that v_w is close to the escape velocity from the red supergiant surface, may not be applicable here.

Setting the wind velocity equal to a fraction α_w of the escape velocity, $v_w = \alpha_w (2GM/R)^{1/2}$, and assuming that the supergiant loses most of its initial mass, we may estimate the total mass accreted by a degenerate dwarf of mass $M_{\odot}$ from the wind emitted by a red supergiant of mass M and radius R to be

$$\Delta M \approx 0.25 M_d^2 M_t^2 M^{-3}\left(1 + 2\alpha_w^2 M_t M^{-1} A R^{-1}\right)^{-2}, \tag{19}$$

where $M_t = M_d + M$.

We see that a low-mass supergiant which almost fills its Roche lobe as it loses mass via a self-generated wind may be very effective in feeding a degenerate dwarf companion, provided α_w is small. Assuming $A \approx 2R$ and $M_d = M_{\odot} = M/2$, we find from equation (19) that $\Delta M \approx 0.3 M_{\odot}(1 + 6\alpha_w^2)^{-2}$. If $\alpha_w = 1/2$, a dwarf companion can accrete only ~ 0.05 $M_{\odot}$, which demands, once more, a large ($\gtrsim 1.35\ M_\odot$) initial mass for the degenerate dwarf, if a supernova explosion is to result. However, decreasing α_w still further will significantly increase the efficiency of accretion from a stellar wind.

If $\Sigma \dot{M}$ exceeds $10^{-6}\ M_\odot\ \mathrm{yr}^{-1}$, as might occur during helium shell flashes in low-mass supergiants or via a superwind which puts an end to the AGB life of the feeding star, the efficiency of accretion will be decreased as a result of the formation of a common envelope. In these cases the final result of mass transfer is a double-core planetary nebula.

To estimate the maximum frequency of supernova descendants of CO dwarfs which are fed by accretion from a wind, we suppose that the semimajor axis must be within a factor of 2 of the radius of the supergiant star at the time of the largest rate of mass transfer (so, $\Delta \log A \approx 0.3$) and that the total mass accreted is $\sim 0.3\ M_\odot$. Using equation (15) and a Salpeter-like birth rate function, we estimate in the usual way that a fraction ~ 0.06 of all binary systems forming every year will produce a degenerate dwarf with mass exceeding $\sim 1.1\ M_\odot$. Further, a fraction ~ 0.06 of these systems will have an appropriate semimajor axis. Therefore, the maximum frequency of successful systems will be $\nu_{\mathrm{SNI}} \approx 4 \times 10^{-3}\ \mathrm{yr}^{-1}$. Considering the very preliminary character of this estimate, we can firmly conclude only that this scenario may provide some fraction of observed SNeI, but it would be incautious to place much confidence in the quantitative estimate.

On the other hand, there are direct observational consequences of this variant of the traditional scenario that could be explored to test our crude estimate. Following each hydrogen shell flash, an accreting CO dwarf burns hydrogen quiescently at a luminosity given by $L \approx 6 \times 10^4\ L_\odot\ (M_{\mathrm{CO}}/M_\odot - 0.5)$. If we were to assume that every SNI explosion is preceded by the accretion and burning of $\sim 0.3\ M_\odot$ of hydrogen-rich matter, then $\sim 4 \times 10^{51} \times 10^{-2}\ \mathrm{ergs} = 4 \times 10^{49}$ ergs of thermonuclear energy is liberated per year by all presupernova systems. If the accreting stars remain compact during much of each quiescent hydrogen-burning phase (as they must if most of the matter accreted between flashes is not to be lost via a common envelope), this liberated energy is in the form of "hard" or potentially ionizing quanta appearing at the rate of $\sim 6 \times 10^{52}\ \mathrm{s}^{-1}$, or at $\sim 20\%$ of the rate at which all ionizing radiation is produced in our Galaxy (Mezger 1978). This rate of production of ionizing photons also exceeds by a factor of ~ 10 the production of ionizing photons by planetary nebula nuclei.

This last comparison can be understood in another way. At any one time, in only a fraction of all accreting systems will the accreting stars be in the luminous "on" phase which follows a hydrogen shell flash. Most of the time the accreting star will be in the low luminosity "off" phase, with the accreting disk supplying most of the emitted energy. By comparing the typical duration of the "on" phase with the duration of the "off" phase, as provided by theoretical accretion models (Iben 1982*b*), one finds that if hydrogen-accreting CO dwarfs in the mass range 1–1.4 $M_\odot$ are responsible for all SNeI, the total number of systems of our Galaxy in the "on" phase is on the order of a few thousand. But this is precisely the estimated number of planetary nebulae in the Galaxy. The fact that the hypothesized accretion systems produce ~ 10 times as much ionizing radiation is due primarily to the fact that the mass of the typical central star of a planetary nebula is only $\sim 0.6\ M_\odot$, compared to a mass of $\sim 1.2\ M_\odot$ for the typical hypothesized system; the luminosity of the typical planetary is thus $L_{\mathrm{PN}} \approx 6 \times 10^4\ L_\odot\ (0.6\text{–}0.5) \approx 6 \times 10^3\ L_\odot$, compared with the luminosity during the "on" phase of the accreting component:

$$L_{\mathrm{on}} \approx 6 \times 10^4\ L_\odot (1.2\text{–}0.5) \approx 4 \times 10^4\ L_\odot .$$

Are there any observational counterparts of accretion systems during the "on" phase? It would be very difficult to find the hot component in the absence of dense matter with a high emission measure nearby. The amount of matter emitted during the brief common envelope phase following each flash, if it occurs, is probably too small (typically of mass $M_e \approx 10^{-4}\text{–}10^{-6}\ M_\odot$) to be surely detectable, but the matter built up by the intrinsically driven wind of the red giant (supergiant) component might have built up about the system a diffuse cloud of sufficient density to be excited to detectability by the hot component. In this connection it is interesting to note that the total number of symbiotic stars (characterized by both a luminous red and a luminous blue component and by the presence of nebular material) in the Galaxy has been estimated to be on the order of 10^3 (Boyarchuk 1975), similar to the required number of SNeI precursors of the hypothesized type in the "on" phase. Furthermore, mass outflow rates in symbiotics are characteristically on the order of $10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$, so their formation rate is $\sim 10^{-2}\ \mathrm{yr}^{-1}$ (Tutukov and Yungelson 1976). This is close to our tentative estimate of ν_{SNI} for wind mass transfer systems. Kenyon and Webbink (1983) argue that the accreting component in perhaps half of all symbiotics is a main-sequence star, accepting matter at the exceedingly large rates of $\sim 10^{-4}\text{–}10^{-3}\ M_\odot\ \mathrm{yr}^{-1}$, but there are several systems (e.g., RR Tel) which can be understood simply only in terms of an accreting white dwarf which undergoes thermal flashes. Thus, it is not excluded that the wind scenario under discussion could be responsible for some fraction of SNeI events. It is ironic that whereas in the traditional scenario AGB stars contribute so inefficiently to SNI formation, these same stars possibly save the day in the wind mass transfer scenario.

Another possible objection to accepting (roughly half of all) symbiotic stars as SNeI precursors is that some of the light from the hot component will be intercepted by the cool companion and be reradiated in the visible; because of orbital motion and possible eclipses, that portion of the reradiated light which is directed toward us should vary in intensity with the orbital frequency, but no periodic variation in light from symbiotics has yet been detected. The objection is answered by noting that the (super) giant component of symbiotics is, in many instances, so bright that the light which is intercepted and reradiated may easily go undetected (Taranova and Judin 1981).

V. CATACLYSMIC SYSTEMS AND CATACLYSMIC-LIKE SYSTEMS IN WHICH MASS EXCHANGE IS ON A THERMAL TIME SCALE

Having explored the formation frequency and fate of mass-exchanging binaries composed of an electron-degenerate dwarf and a giant star, we turn our attention to systems consisting of a degenerate dwarf and a main-sequence star. These must, of course, be close binaries if the main-sequence star is to fill its Roche lobe. Historically, systems of this general type in which the mass-donating main-sequence star is less massive than $\sim 1.2\ M_{\odot}$ and is separated from its dwarf companion by less than $\sim 10\ R_{\odot}$ have been called cataclysmic variables. In them, mass transfer at rates deduced from the observations requires some mechanism of angular momentum loss to enable the main-sequence star, whose radius is essentially proportional to its mass, to continuously fill its Roche lobe. For sufficiently small separations, $A \lesssim 3\ R_{\odot}$, the radiation of gravitational waves can produce angular momentum loss at required rates (see § VI) but, for larger values of A, some other mechanism must be invoked, and this mechanism has acquired the designation of magnetic stellar wind (MSW).

If the mass of the main-sequence star is greater than $\sim 1.5\ M_{\odot}$, continuous filling of the Roche lobe can be achieved as a consequence of the evolutionary expansion of the stellar radius in response to nuclear transformations in the interior. We shall call systems of this type cataclysmic-like.

a) *Cataclysmic Systems Containing an Electron-degenerate* CO *or* ONeMg *Dwarf and a Main-Sequence Star*

A substantial fraction of cataclysmic variables with orbital period longer than ~ 3 hr can sustain accretion onto the degenerate component at a rate in the range 10^{-9}–$10^{-7}\ M_{\odot}\ \mathrm{yr}^{-1}$ (e.g., Patterson 1983) through the driving force of a MSW (Patterson 1982; Tutukov 1983). If the MSW is to be effective, the initial separation of the main-sequence star from the dwarf must be less than $\sim 10\ R_{\odot}$ (Tutukov 1983). The component filling its Roche lobe can be a typical main-sequence star. It can also be a moderately evolved star with almost no hydrogen in its core, but it must still be near the main sequence if the mass exchange rate is to remain in the observed range (see, e.g., Taam, Flannery, and Faulkner 1980). The fact that mass transfer rates are in the appropriate range, coupled with the fact that large amounts of potentially transferable matter are available, makes these systems promising as contributors to the observed SNI rate.

We shall estimate the frequency of producing cataclysmic systems in which the degenerate dwarf is composed of carbon and oxygen. There are two ways to produce systems of the required characteristics: $A \lesssim 10\ R_{\odot}$, $M_2 \sim 1\ M_{\odot}$. The first is to delay the first instance of mass transfer until the primary develops a CO core on the AGB. This means that the primordial semimajor axis must be in excess of $\sim 460\ R_{\odot}$. If the initial cataclysmic system is to have a separation less than $10\ R_{\odot}$, we see from equation (2), with $\alpha \approx 1$, that M_2^2/M_1 must be quite large. Choosing $M_2 \approx 5$–$9\ M_{\odot}$, which implies $M_c \approx 0.7$–$1.0\ M_{\odot}$, we have that $A_f < 10\ R_{\odot}$ requires that $A < 600\ R_{\odot}$. Finally, with $\Delta \log A \approx \log(600/460) \approx 0.12$, $q \approx 1/7$, $M_A \approx 5$, and $M_B \approx 9$, equation (1) gives a frequency of realization: $\nu_{\mathrm{COMS}}^{\mathrm{wide}} \approx 1.2 \times 10^{-4}\ \mathrm{yr}^{-1}$.

The second way to achieve a suitably close system with a degenerate CO core is to begin with a primordial system in which the primary develops a helium core of mass in the range $M_{\mathrm{He}} \approx 0.5$–$1.0\ M_{\odot}$ and then evolves into contact with its Roche lobe somewhere in the Hertzsprung gap (case B mass transfer). The radius of the remnant helium core shrinks within its Roche lobe, and nuclear burning converts it rapidly into an electron degenerate CO core. The mass of the helium remnant can be estimated from the relationship $M_{\mathrm{He}} \approx 0.1\ M_1^{1.2}$, which is an approximation to the numerical results of van den Linden (1980) when M_1 is in the interval $3 < M_1/M_{\odot} < 9$. For $M_{\mathrm{He}} = 0.5$–$1.0\ M_{\odot}$ we have $M_1 = 3.8$–$6.8\ M_{\odot}$. There is at present no reason to restrict M_1 to values less than $6.8\ M_{\odot}$, so we set $M_B = 9$ in equation (1). The requirement of a fairly massive primordial primary ensures that the semimajor axis of the system which emerges from the common envelope stage will be suitably small. In fact, the requirement that $2.5 < A_f/R_{\odot} < 10$ translates into $80 < A/R_{\odot} < 320$. Setting $\Delta \log A \approx 0.6$, $q \approx 0.2$, $M_A = 3.8$, and $M_B = 9$, equation (1) gives a realization frequency: $\nu_{\mathrm{COMS}}^{\mathrm{close}} \approx 1.6 \times 10^{-3}\ \mathrm{yr}^{-1}$.

Thus, the total realization frequency for systems of CO core ($M_{\mathrm{CO}} \approx 0.6$–$1\ M_{\odot}$) and main-sequence star ($M_2 \sim 1\ M_{\odot}$) at a separation $A \approx 2.5$–$10\ R_{\odot}$ is $\nu_{\mathrm{COMS}}^{\mathrm{all}} \approx 2 \times 10^{-3}\ \mathrm{yr}^{-1}$. This estimate is consistent with the estimated birth rate of cataclysmic variables (Trimble 1982*b*). Decreasing α could increase the estimated number of such systems, but any estimate will be unreliable because of its strong dependence on our assumption as to the relative number of systems with low initial mass ratio.

The white dwarf in a small fraction of cataclysmic variables may be composed of oxygen, neon, and magnesium. In § I we estimated the formation frequency of bare ONeMg dwarfs to be $\nu_{\mathrm{ONeMg}} \approx 8 \times 10^{-4} q\ \mathrm{yr}^{-1}$ and with $q \approx 0.1$ we see that perhaps only one out of 25 cataclysmic variables might contain such a dwarf.

For testing the viability of this scenario, the exploration of the X-ray pulsar 1E2259+58.6 is especially important. Fahlman *et al.* (1982) have found that this pulsar, with period $P \approx 3.5$ s, may be a member of a low-mass close binary with orbital period ~ 2 hr and that the mass of the secondary may be ~ 0.3–$0.7\ M_{\odot}$. This binary appears to be at the very center of a SN remnant of age $\sim 10^4$ yr. Confirmation of duplicity will be evidence not only that the cataclysmic binary scenario for SN production is realized in nature but also that a neutron star may be formed as a consequence of the collapse-explosion of a degenerate ONeMg dwarf as a supernova (see § VII).

Cataclysmic binaries in which $\dot{M} \lesssim 10^{-9}\ M_{\odot}\ \mathrm{yr}^{-1}$ should experience nova outbursts (e.g., MacDonald 1983). During these outbursts they may lose not only the matter accumulated between outbursts, but, judging from the heavy overabundance of CNO elements in nova ejecta (Antipova 1974), also some of the original dwarf as well (Starrfield, Sparks, and Truran 1974; MacDonald 1983). Observations indicate that the mass-transfer rate in cataclysmic variables is correlated with the orbital period P_{orb} (the smaller $\dot{M}$, the smaller P_{orb}) and that the average value of $\dot{M}$ is $\sim 10^{-9}\ M_{\odot}\ \mathrm{yr}^{-1}$ for $P_{\mathrm{orb}} \sim 3$ hr (Patterson 1983). Assuming that cataclysmic systems evolve in the direction of decreasing P_{orb} (due to angular momentum loss by a magnetic stellar wind when $P_{\mathrm{orb}} > 3$ hr and due to the radiation of gravitational waves when $P_{\mathrm{orb}} < 3$ hr), we infer that those systems which cross the $P_{\mathrm{orb}} = 3$ hr threshold cannot become SNeI. Instead, both components will continue to lose mass, and a very few may ultimately become single-degenerate dwarfs (Ruderman and Shaham 1983).

Novae for which orbital characteristics have been determined have $P_{\rm orb}$ in the range 3–7 hr, and we may legitimately ask whether or not the accreting dwarf in these cataclysmic systems can grow in mass in spite of the occurrence of hydrogen shell flashes. Masses of white dwarfs in cataclysmic binaries for which reliable masses can be determined appear to be larger, on average, than masses of typical white dwarf descendants of single stars (Ritter 1983), and this may be taken as evidence that mass transfer can occur in these binaries at rates in excess of 10^{-8}–10^{-9} $M_\odot$ yr^{-1} for a long enough time to increase the initial mass of the degenerate dwarf companion by several tenths of a solar mass. Whether some systems can transfer enough mass to achieve explosive conditions before $P_{\rm orb}$ decreases below 3 hr or whether they are all destined to become short-period systems which avoid explosions cannot as yet be answered.

b) Systems Consisting of an Electron-degenerate Helium Dwarf and a Main-Sequence Star

It is appropriate to emphasize again that degenerate helium dwarfs can be formed only in close binaries as a consequence of mass exchange. Stars with $M \gtrsim 0.8$ $M_\odot$ which evolve in isolation or which, as components of wide binaries do not fill their Roche lobes, complete their nuclear burning evolution as CO white dwarfs. Since about one-half of all stars are members of close binaries with $2.5 < A < 500$ $R_\odot$, we conclude that about one-half of all products of stellar evolution will be helium dwarfs with mass distributed according to the law $dN \approx 4 d \log M_{\rm He}$ (see eq. [10]). Masses of most helium dwarfs will be confined to the approximate range 0.2–0.5 $M_\odot$. The lower limit arises because of a magnetic stellar wind, which prevents all binaries with $A \lesssim 10$ $R_\odot$ from evolving in the usual way (Kraicheva *et al.* 1978). The upper limit is due to the fact that, once the mass of the helium core in a single star exceeds 0.5 $M_\odot$, helium burning will have commenced in this star and forced the stellar radius to shrink. For the same reason, masses of CO dwarfs exceed ~ 0.5 $M_\odot$. So, if the mass of a degenerate dwarf is found to be smaller than ~ 0.5 $M_\odot$, we can predict that this dwarf is composed of helium and that it is a member of a close binary.

A helium dwarf of mass ~ 0.4–0.5 $M_\odot$ can accrete at a fairly high rate in a close binary if its Roche lobe filling companion is of mass ~ $M_\odot$ and is close to the main sequence (Taam, Flannery, and Faulkner 1980). It is also to be noted that, in cataclysmic-like systems consisting of a helium dwarf of mass $M_{\rm He} \approx 0.4$–0.5 $M_\odot$ and of a main-sequence star of mass $M_2 \approx 0.6$–1.0 $M_\odot$, mass exchange may be powered by a MSW. A preliminary estimate of the range of accretion rates in such systems is $\dot{M} \approx 10^{-9}$–10^{-7} $M_\odot$ yr^{-1}. Increasing the mass of the main-sequence donor increases the amount of potentially exchangeable matter, but it also tends to make the mass-exchange process unstable to the formation of a common envelope (Tutukov, Federova, and Yungelson 1982). If the mass of the main-sequence donor is too small, the compressional heating of the helium dwarf is not sufficient to initiate an explosion (Mazurek 1973). It is evident that an adequate determination of the initial conditions which will lead to SNI explosions in this case requires further concrete numerical exploration.

Let us first consider a primordial binary in which the secondary is of mass $M_2 \approx 0.5$ $M_\odot$ and in which the primary fills its Roche lobe when it reaches the Hayashi border with a growing electron-degenerate helium core after leaving the main sequence. Using the approximation given by equation (10), we see that, in order to form a helium core mass of ~ 0.4–0.5 $M_\odot$, the semimajor axis of the unevolved primordial system must be in the interval corresponding to $\Delta \log A \approx 0.4$, with $A \approx 160$ $R_\odot$ appropriate for $M_{\rm He} = 0.4$ $M_\odot$. In order for the new system (now consisting of a helium-degenerate dwarf and a main-sequence star, each of mass ~ 0.5 $M_\odot$) to evolve into contact under the influence of a MSW, the initial semimajor axis must be less than $A \lesssim (40/\alpha)(M_1/M_\odot)^2$ $R_\odot$ (see eq. [2]). Choosing $\alpha = 1$, we have that M_1 must be larger than 2 $M_\odot$ if a core mass as large as $M_{\rm He} = 0.4$ $M_\odot$ is to be "bared," and at the same time a final semimajor axis as small as $A_f = 10$ $R_\odot$ is to be achieved. Since single stars of main-sequence mass larger than ~ 2.3 $M_\odot$ do not produce electron-degenerate helium cores, we set $M_A = 2.0$ and $M_B = 2.3$ in equation (1). Finally, choosing $q \approx 0.25$ and $\Delta \log A \approx 0.4$, we estimate the realization frequency of this scenario to be $\nu^{\rm wide}_{\rm HeMS} \approx 10^{-3}$ yr^{-1}.

The second scenario involves primordial systems in which Roche lobe filling occurs during the evolution of the primary through the Hertzsprung gap, before the helium core becomes degenerate. Once again using $M_{\rm He} \approx 0.1$ $M_1^{1.2}$, we have that, to achieve $0.4 \leqslant M_{\rm He}/M_\odot < 0.5$, one must begin with a main-sequence star of mass in the range $3.2 \lesssim M_1/M_\odot \lesssim 3.8$. The initial semimajor axis in this case is limited to the range $\Delta \log A_0 \approx \log(10/2.5) \approx 0.6$. Taking into account all limitations on the initial parameters ($q \approx 0.3$, $M_A \approx 3.2$, $M_B \approx 3.8$, $\Delta \log A \approx \log(360/5)$, we estimate from equation (1) that the frequency of formation of appropriate systems is $\nu^{\rm close}_{\rm HeMS} \approx 1 \times 10^{-3}$ yr^{-1}.

Adding in binaries with primaries of mass ~1 $M_\odot$ filling their Roche lobes on the Hayashi border ($\alpha = 1$), the total frequency of formation of cataclysmic systems containing a helium dwarf becomes $\nu^{\rm all}_{\rm HeMS} \approx 2 \times 10^{-3}$ yr^{-1}. We emphasize once again that, since our estimates depend on rather uncertain guesses as to the initial masses of the helium dwarf and of the main-sequence star, they must be viewed as extremely tentative. However, $\nu^{\rm all}_{\rm HeMS}$ is a sufficiently large fraction of the observed frequency of SNeI that systems composed of a helium dwarf and a low-mass main-sequence star with mass exchange provided by a magnetic stellar wind deserve to be studied in greater detail. It is particularly important to obtain a better understanding of how the critical mass of the degenerate dwarf (when it explodes) depends on the mass-exchange rate so that a better estimate can be made of those sets of initial masses of components that lead to an SN explosion.

c) Cataclysmic-like Systems

Members of another class of binaries which could sustain mass exchange at rates appropriate for SNI formation consist of a CO degenerate dwarf and a primary which is a Roche lobe filling star with a radiative envelope transferring matter on a thermal time scale τ_{KH}. A good example of such evolution has been studied by Taam, Flannery, and Faulkner (1980). For stars near the main sequence, we may estimate $\tau_{\mathrm{KH}} \approx 3\times10^7$ yr $(M^2/RL) \approx 3\times10^7\ M^{-3}$ hr, where mass, radius, and luminosity are all in solar units. Then, a crude estimate of the mass-transfer rate is $\dot{M} \approx M/\tau_{\mathrm{KH}} \approx 3\times10^{-8}$ $M^4\ M_\odot$ yr^{-1}, which, for a primordial secondary of mass $M_2 = 0.8\ M_\odot$, gives $\sim 10^{-9}\ M_\odot$ yr^{-1} and, for $M_2 = 2.4\ M_\odot$, gives $\sim 10^{-6}\ M_\odot$ yr^{-1}. Numerical experiments show that, when mass loss from a star of mass larger than $\sim 0.8\ M_\odot$ is on a thermal time scale, stellar radius responds according to $1 \lesssim d\log R/d\log M \lesssim 2$ (Tutukov, Federova, and Yungelson 1982). Thus, since $d\log R/d\log M = 2M_2/M_d - 1.56$, where M_d is the mass of the degenerate dwarf, an additional restriction on the initial system is that the initial mass ratio must be in the range $1.2 \lesssim M_2/M_d \lesssim 1.8$ (Tutukov *et al.*). Finally, the primary must also be out of the Hayashi border if mass is to be exchanged on a thermal time scale. Filling of the Roche lobe in such systems may be the result of the evolutionary expansion of the primordial secondary or, if $M_2 \lesssim 1.5\ M_\odot$, it could be due to a magnetic stellar wind. The total amount of potentially exchangeable matter is about one solar mass. Hence, even a CO dwarf with an initial mass as small as ~ 0.6 $M_\odot$ has the potential of reaching the Chandrasekhar limit if no mass is lost from the system. The only major difference between the ingredients used to estimate the realization frequency of binaries of this sort and those used to estimate the realization frequency of cataclysmic binary formation is that the primordial secondary mass and primordial mass ratio is very slightly larger. Hence, we estimate ν(cataclysmic-like) $\approx 3\times10^{-3}$ yr$^{-1} \approx \nu$(cataclysmic).

VI. EVOLUTION TO THE SUPERNOVA STAGE OF BINARY SYSTEMS CONSISTING OF TWO DEGENERATE DWARFS

a) On the Formation of Double-degenerate Systems

The formation and behavior of double-degenerate systems have been touched upon by Tutukov and Yungelson (1979*a*) and by Webbink (1979). The single known mechanism for decreasing the semimajor axis of such binaries is the radiation of gravitational waves. The existence of this mechanism is supported by the orbital period evolution of the double pulsar (Taylor 1981) and by the existence of a minimum orbital period for cataclysmic binaries (Paczyński 1981). Gravitational wave radiation can be effective in producing an explosive system in a time less than 10^{10} yr only if the initial orbital separation satisfies

$$\frac{A_0}{R_\odot} \lesssim 3.3\left(\frac{M_1}{M_\odot}\right)^{1/4}\left(\frac{M_2}{M_\odot}\right)^{1/4}\left(\frac{M_1+M_2}{M_\odot}\right)^{1/4}. \qquad (20)$$

This estimate follows from the formalism of Landau and Lifshitz (1962). To achieve such a small separation, the primordial precursor system must lose almost all of its initial orbital angular momentum and a significant fraction of its total initial mass. The most effective way to lose mass and orbital angular momentum is once again by way of a common envelope.

We shall examine the formation of three types of "explosive" binaries consisting of two electron-degenerate dwarfs at an initial separation $\lesssim 3\ R_\odot$, the necessary condition for merging on a time scale less than the Hubble time. The first type of system consists of two degenerate CO dwarfs, the second of a degenerate CO dwarf and a degenerate He dwarf, and the third of two helium degenerate dwarfs (see also Webbink 1984). The first system may evolve to the explosive stage if the total mass of the two dwarfs exceeds 1.4 $M_\odot$. The second system may evolve into a double detonation SN if total mass exceeds $\sim 1\ M_\odot$ (Fujimoto 1980) and accretion is fast enough; it can also evolve into a carbon deflagration SN if total mass exceeds 1.4 $M_\odot$ and accretion is slow enough.

Nomoto and Sugimoto (1977) have estimated that a helium-accreting helium dwarf of initial mass exceeding ~ 0.4 $M_\odot$ will explode as a supernova if the accretion rate is smaller than $2\times10^{-8}\ M_\odot$ yr^{-1}. The explosion occurs when the mass of the accreting dwarf reaches ~ 0.65–$0.8\ M_\odot$. However, the lower limit on the total initial mass of the double-dwarf system, which depends on the accretion rate and, possibly, on the thermal history of the dwarf, is not firmly established, and since, as we shall show, the frequency of formation of such systems depends very strongly on this lower limit, it is important that it be established more carefully by means of further numerical calculations.

To estimate the frequency of formation of double degenerates with an initial semimajor axis smaller than $\sim 3\ R_\odot$ and with appropriate total mass, we must carefully follow the evolution of the semimajor axis. The primordial system consists of two stars with masses M_1 and M_2 $(M_1 > M_2)$ and semimajor axis A. After the first common envelope stage, the remnant of the primary with mass M_{1R} and the second star with mass M_2 will form a binary with semimajor axis which, according to equation (2), is equal to

$$A_f = \frac{\alpha M_{1R} M_2}{M_1^2} A. \qquad (21)$$

After the second common envelope stage, the remnant of the primordial secondary is of mass M_{2R} and the semimajor axis of the system becomes

$$A_{ff} = \alpha^2 \frac{M_{1R}^2}{M_1^2}\frac{M_{2R}}{M_2} A. \qquad (22)$$

i) *Two Carbon-Oxygen Dwarfs*

Let us first consider the formation of a system of two degenerate CO dwarfs at an initial separation less than ~ 3 $R_\odot$. We assume that the primordial system consists of two main-sequence stars of comparable mass M_{MS}. If, when it first fills its Roche lobe, its radius is in the range 10–200 $R_\odot$ $(10 \lesssim A/R_\odot \lesssim 460)$, the primordial primary will lose mass until it exposes a helium core. Mass loss will be on a thermal or on a dynamic time scale, a common envelope will be formed, and most of the mass lost by the primary will escape from the binary system. If the mass of the remnant helium star is in the range 0.7–1.0 $M_\odot$ (corresponding to primordial mass in the appropriate range 5.1–6.8 $M_\odot$), the remnant helium star will convert all of its helium into carbon and oxygen without at the same time expanding appreciably and losing additional mass through Roche lobe overflow (e.g., Paczyński 1971*b*). The primordial secondary will evolve in much the same way as has the primordial primary, first exposing a helium core after a brief common envelope stage, and then evolving into an electron-degenerate CO dwarf. From equation (22), with $M_{1R} \approx M_{2R} \approx 0.85\ M_\odot$ and $M_1 \approx M_2 \approx 5.9\ M_\odot$, we have that $A_{ff} \approx \alpha^2 3\times10^{-3}\ A$. Thus, with $\alpha \approx 1$, a primordial semimajor axis less than $A \approx 460\ R_\odot$ will easily ensure that the initial separation of the degenerate dwarfs will be less than $\sim 3\ R_\odot$. In order that the primordial secondary can produce a helium core before Roche lobe filling, we require further that $A_f \approx \alpha A/7 \gtrsim 10\ R_\odot$ or that $A \gtrsim 70\ R_\odot$. Setting $\Delta \log A \approx \log(460/70) \approx 0.8$, $q=1$, $M_A = 5.1$, and $M_B = 6.8$, we estimate from equation (1) that, via this channel, the formation frequency of double CO dwarfs of total mass $\gtrsim 1.4\ M_\odot$ and separation $\lesssim 3\ R_\odot$ is $\nu_{\rm COCO}^{\rm close} \sim 3\times10^{-3}\ {\rm yr}^{-1}$.

There is, at this point, really no reason to restrict M_{1R} to less than $\sim 1\ M_\odot$. In principle, expansion of the bare core during helium burning will simply lead to another common envelope stage, with the final remnant mass M_{1R} being somewhat less than the mass of the remnant left after the first common envelope stage. Detailed calculations are necessary to establish the relationship between M_1 and the final remnant M_{1R} in this case, but a preliminary exploration (Iben and Tutukov 1983*c*) suggests that only a small fraction of the helium star will be lost during the second common envelope phase, and we may therefore extend the range in M_1 all the way to $\sim 9\ M_\odot$ with the result that the frequency of double CO dwarf presupernova production by the "close" channel increases to $\nu_{\rm COCO}^{\rm close} \approx 5\times10^{-3}\ {\rm yr}^{-1}$. The orbital and stellar characteristics of the systems following this scenario are illustrated schematically in Figure 1.

Another way to achieve an initial system of two CO degenerate dwarfs at a suitably small initial separation is to select primordial parameters in such a way that (1) the first common envelope stage does not occur until the primordial primary becomes a thermally pulsing AGB star (radius larger than $\sim 230\ R_\odot$) with an electron degenerate CO core of mass M_{1R} and that (2) the second common envelope stage converts the primordial secondary into a bare helium core of mass $M_{2R} \gtrsim (1.4\ M_\odot - M_{1R}) \geqslant 0.5\ M_\odot$, which evolves into a CO degenerate dwarf of the same mass. The mass of the primary can in this case be as large as 9 $M_\odot$, and the mass of the first CO degenerate remnant will be between 0.8 and 1 $M_\odot$ (the initial mass of the CO core for M_1 between 5 and 9 $M_\odot$) or larger. If the mass M_{2R} is to make up the difference between 1.4 $M_\odot$ and M_{1R} and at the same time not expand during the conversion of He into C and O, then M_2 must be between 4.5 and 6.8 $M_\odot$. From equation (20) we infer that, if $A_{ff} \lesssim 3.5\ R_\odot$, the initial double-degenerate system will evolve to the explosive stage in less than 10^{10} yr, and from equation (22) we have (with $\alpha \approx 1$) $A_{ff} \approx A[(0.9)^2\times0.8]/[7^2\times5.7] \approx 2.3\times10^{-3}\ A$, giving finally that $A < 1500\ R_\odot$. Note from equation (21) that $A_f < 160\ R_\odot$, and this ensures that the second common envelope stage will produce a bare helium star which will evolve into a degenerate CO dwarf. We are now in a position to use equation (1). Setting $\Delta \log A \approx \log(1500/460) \approx 0.5$, $q \approx (4.5+6.8)/(5+9) \approx 0.84$, $M_A \approx 5$, and $M_B \approx 9$, we obtain $\nu_{\rm COCO}^{\rm wide} \approx 3\times10^{-3}\ {\rm yr}^{-1}$. The various stages through which representatives of this scenario pass are illustrated schematically in Figure 2.

Adding together the realization frequencies for the two channels, we have that the grand total frequency of formation of presupernova systems consisting of two CO degenerate dwarfs is $\nu_{\rm COCO}^{\rm all} \approx 8\times10^{-3}\ {\rm yr}^{-1}$. Note, however, that only products of "wide" systems can contribute to the formation of SNeI in ellipticals in which star formation ceased over 10^{10} yr ago. This result helps account for the decrease in the SNI frequency per unit luminosity that is encountered as one moves through the Hubble sequence from early- to late-type galaxies.

ii) *A Carbon-Oxygen Dwarf and a Helium Dwarf*

The channels for the formation of systems containing a CO degenerate dwarf and a helium degenerate dwarf at an initial separation less than $\sim 3\ R_\odot$ are qualitatively essentially identical to those leading to the formation of two CO degenerates. The major quantitative difference is that the mass of the primordial secondary must be small enough to eventuate in a helium dwarf of mass less than $\sim 0.5\ M_\odot$. The smaller primordial mass ratio results in narrower limits for an acceptable primordial semimajor axis, and the smaller mass of the second remnant results in the necessity for a larger mass for the first, CO, remnant. Let us first examine primordial "wide" systems with $A \gtrsim 460\ R_\odot$. Choosing $M_1 \approx 6$–$9\ M_\odot$, $M_{1R}^{\rm CO} \approx 0.9$–$1.2\ M_\odot$, $M_2 \approx 2$–$4\ M_\odot$, $M_{2R}^{\rm He} \approx 0.25$–$0.5\ M_\odot$, and $\alpha \approx 1$, we have that $A_f \approx A/18$ and $A_{ff} \approx A/414$. From equation (20), A_{ff} must be less than $\sim 2.8\ R_\odot$ if the preparation time scale is less than 10^{10} yr, and this implies that $A < 1200$, or that $\Delta \log A \approx \log(1200/460) \approx 0.4$. With $q \approx 0.4$, $M_A \approx 6$, and $M_B \approx 9$, equation (1) then gives $\nu_{\rm COHe}^{\rm wide} \sim 0.6\times10^{-3}\ {\rm yr}^{-1}$.

In a similar way, we find that, beginning with a "close" primordial system with $A \lesssim 460\ R_\odot$, the frequency of realization of an appropriate initial configuration is $\nu_{\rm COHe}^{\rm close} \approx 0.7\times10^{-3}\ {\rm yr}^{-1}$ and that the total realization frequency of potentially explosive systems is on the order of $\nu_{\rm COHe}^{\rm all} \approx 10^{-3}\ {\rm yr}^{-1}$. Figure 3 illustrates the system characteristics for various stages followed by representatives of the "wide" scenario.

iii) *Two Degenerate Helium Dwarfs*

To create a system consisting of two degenerate helium dwarfs with total mass exceeding $\sim 0.8\ M_\odot$ and with an initial semimajor axis small enough that the potentially explosive stage is reached in less than 10^{10} yr, we begin once again with a primordial binary consisting of two main-sequence stars of comparable mass M at a separation A. We achieve the first degenerate helium dwarf of mass 0.4–0.5 $M_\odot$ by choosing the mass of the primordial primary to be in the range 3.2–3.8 $M_\odot$ and by choosing the primordial semimajor axis to be smaller than $\sim 460\ R_\odot$. A lower limit on A_f of $\sim 8\ R_\odot$ is set by demanding that the primordial secondary does not fill its Roche lobe until it has developed a helium core. Setting $M_{1R} \approx 0.45\ M_\odot$, $M_1 \approx M_2 \approx 3.5\ M_\odot$, $\alpha \approx 1$, and $A_f > 8\ R_\odot$ in equation (21), we have then that $A \gtrsim 60\ R_\odot$. It follows that the

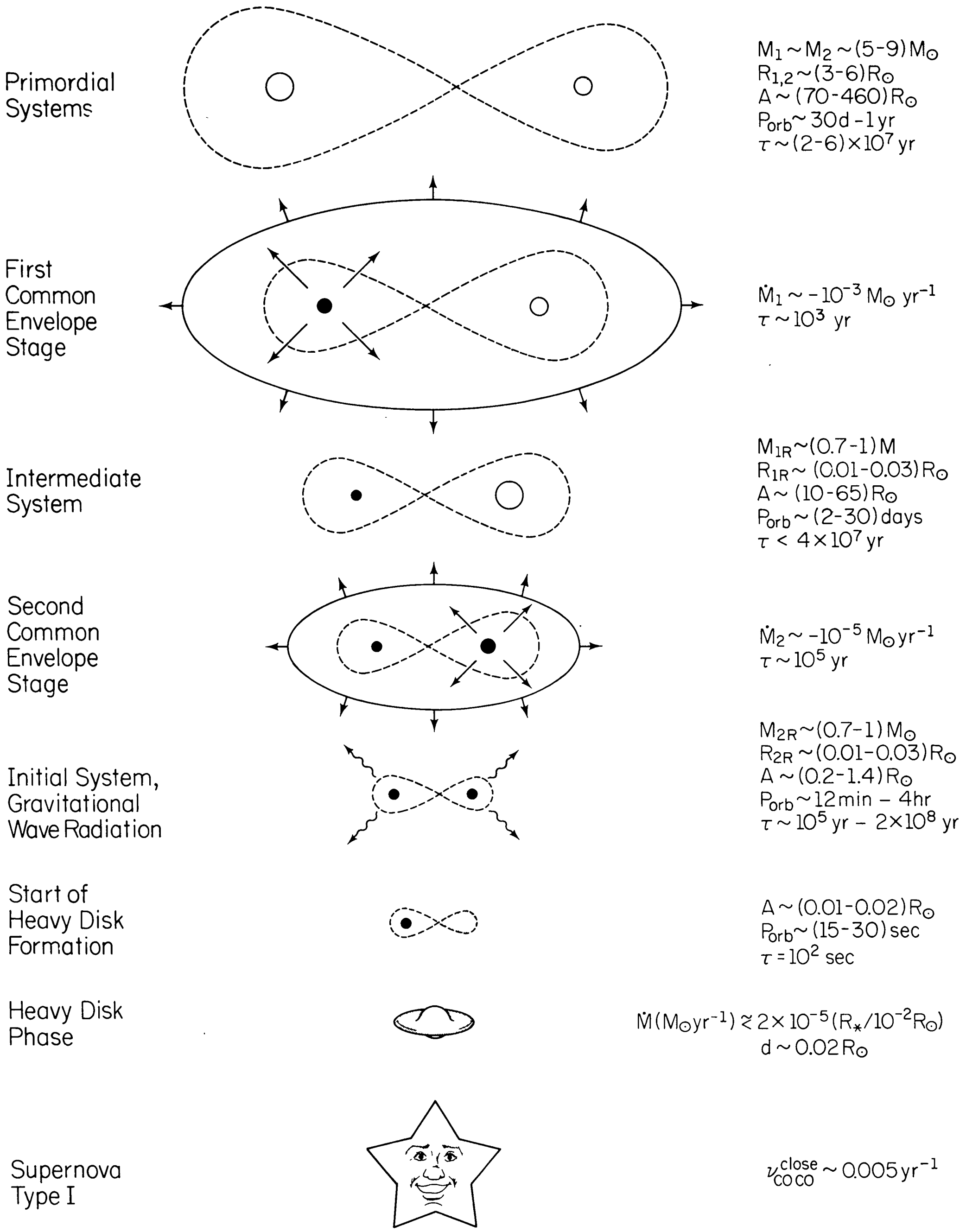

FIG. 1.—Schematic describing the scenario which produces an initial system containing two degenerate CO dwarfs from primordially "close" systems. Component masses and radii, system semimajor axes and orbital periods, and time scales for the various phases are shown.

range of permissible initial semimajor axes must satisfy $\Delta \log A \approx 0.9$. Inserting this, along with $q \approx 1$, $M_A = 3.2$, and $M_B = 3.8$ in equation (1), we estimate the formation frequency of explosive systems of double-degenerate helium dwarfs to be $\nu_{\text{HeHe}} \approx 5 \times 10^{-3}$ yr^{-1}. The evolution of system characteristics is traced schematically in Figure 4.

From equation (20) with $M_1 \approx M_2 \approx 0.85\ M_\odot$ it follows that the requirement for evolving to the final merged stage is less than 10^{10} yr is that the initial separation of the two degenerate dwarfs to be less than $\sim 2.1\ R_\odot$. But from equation (22), with the usual approximations, we have that $A_{ff} < A/430 \approx 460\ R_\odot/430 \approx R_\odot$, and merging will occur well within the Hubble time.

The time scale for shortening the semimajor axis as the result of the action of gravitational wave radiation is proportional to the fourth power of the semimajor axis. Thus, since an initial semimajor axis of $\sim 2.1\ R_\odot$ implies a time scale of 10^{10} yr for evolution to the merging state, the maximum initial separation of $\sim 1\ R_\odot$ for potentially explosive double helium-degenerate dwarfs means that evolution to the explosive state

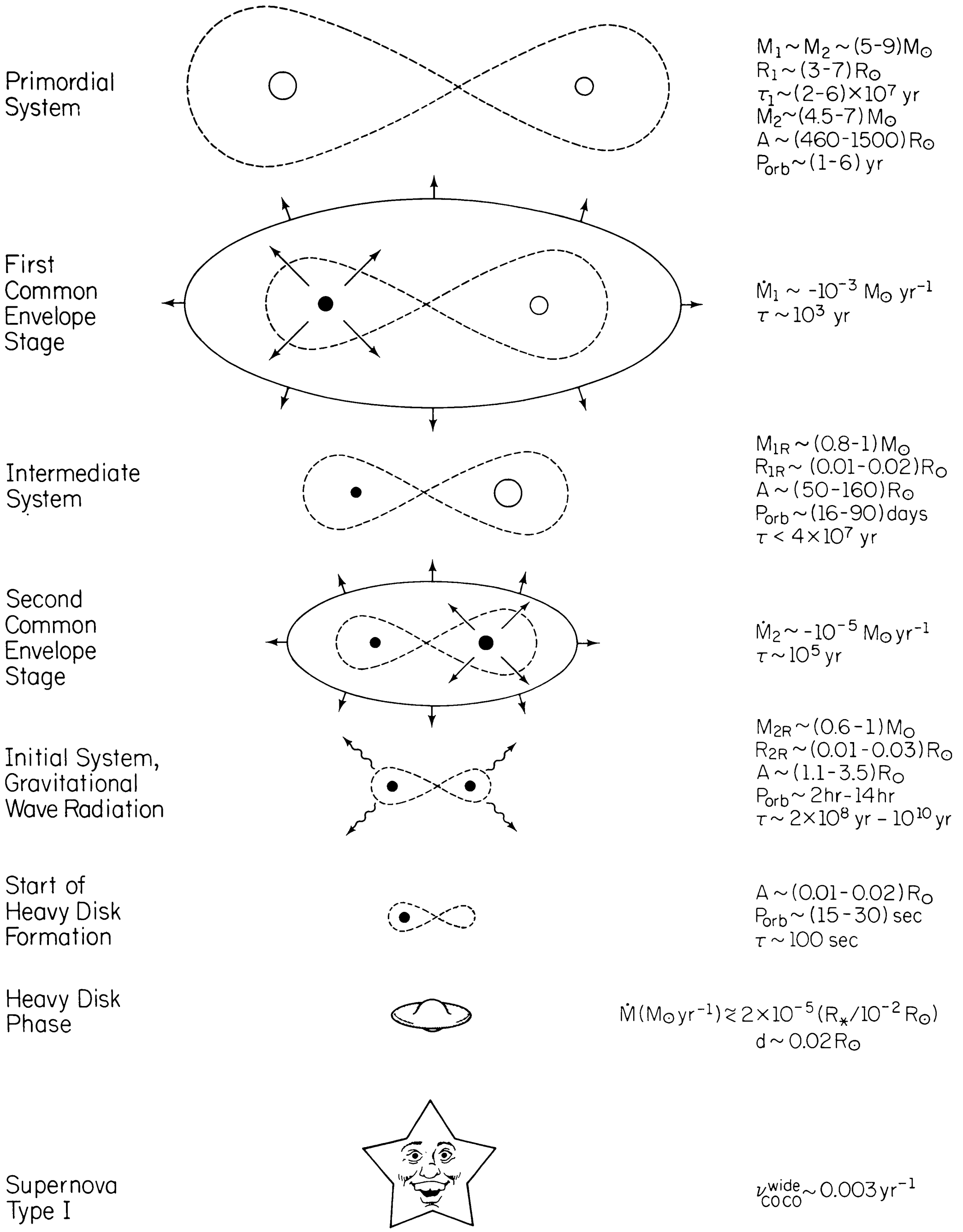

FIG. 2.—Same as Fig. 1 for the scenario which produces an initial system containing two degenerate CO dwarfs from a primordially "wide" binary

occurs in less than 5×10^8 yr, and therefore, in elliptical galaxies, most systems must have already become SNeI.

It is therefore of interest to explore the properties of double helium-degenerate dwarf systems in which the second dwarf is not formed until the contemporary epoch. Stars with $M_1 \lesssim 3.8$ $M_\odot$ evolving on the main sequence develop a helium core of mass smaller than ~ 0.5 $M_\odot$ and then evolve along the Hayashi border during the further growth of the helium core. Following the Whelan and Iben (1973) idea, viz., that the "time bomb" in elliptical galaxies is provided by a secondary of primordial mass $\lesssim M_\odot$, we shall start with a primordial system of mass $M_1 \approx 3.5$ and $M_2 \approx 1$ ($< M_1$). The relationship between the radius of a red giant and the mass of its degenerate helium core, for $Z = 10^{-3}$, may be approximated (using the numerical results of Sweigart and Gross 1978) as

$$\frac{R}{R_\odot} = 10^{3.14} \left(\frac{M_{\rm He}}{M_\odot} \right)^{3.6} . \tag{23}$$

Decreasing the heavy element content still further decreases the radius of red giants somewhat further and makes binaries

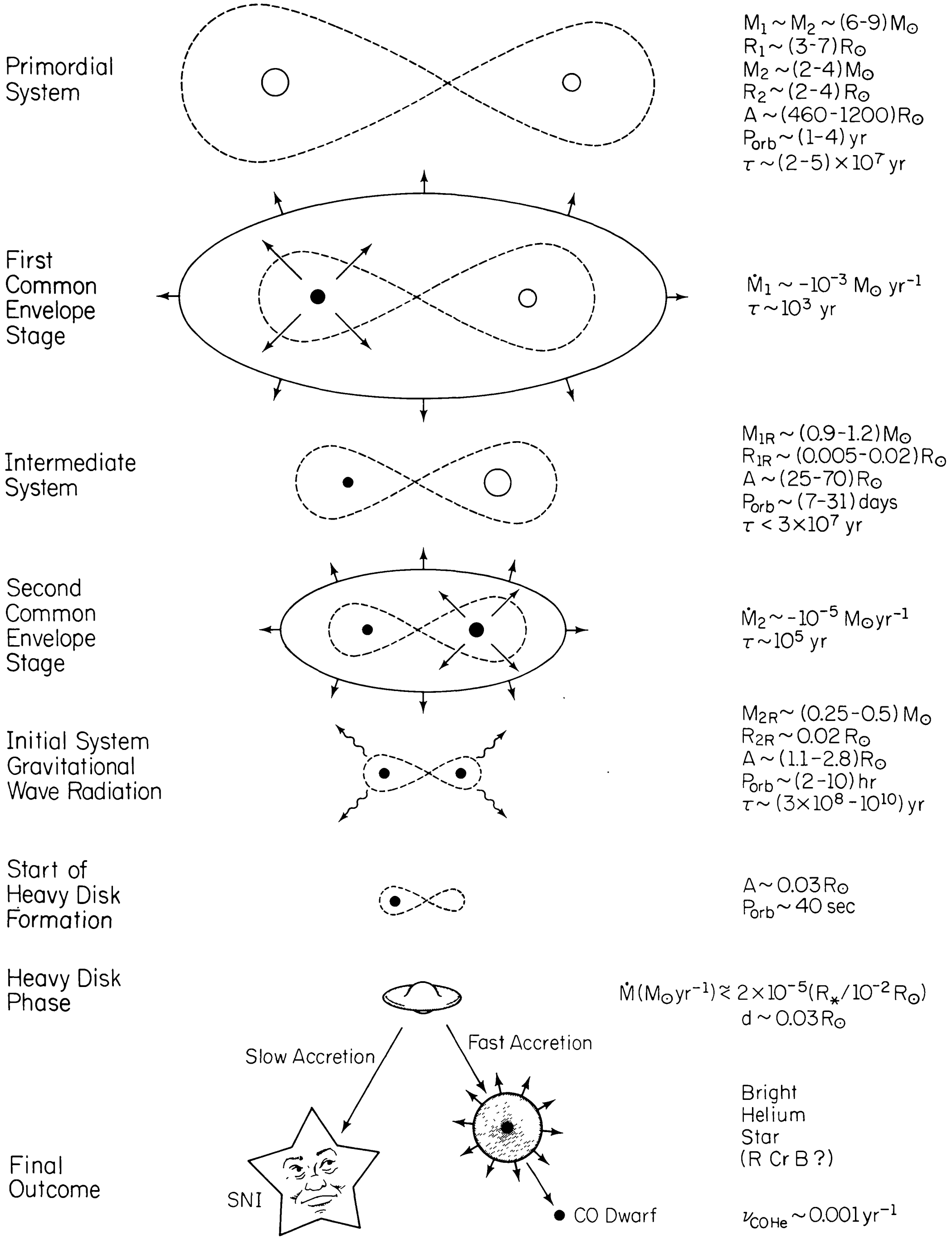

FIG. 3.—Same as Fig. 1 for the scenario which produces an initial system containing a degenerate CO dwarf and a degenerate helium dwarf from a primordially "wide" system.

with red giants more compact, but we shall ignore this complication. The relationship between R and M_{He}, together with equation (6) gives

$$M_{1R} \approx 0.112\, A^{0.28}[M_1/(M_1+1)]^{0.12}$$

and

$$A_f = 0.112\alpha A^{1.28} M_1^{-1.88}(M_1+1)^{-0.12}$$

After the second common envelope stage, the mass of the second helium star will be $M_{2R} \approx 0.061\alpha^{0.28}A^{0.36}M_1^{-0.52}$, and, according to equation (22), the semimajor axis becomes

$$A_{ff} = \frac{7.65\times10^{-4}\alpha^{2.28}A^{1.92}}{M_1^{2.28}(M_1+1)^{0.24}}. \qquad (24)$$

We can estimate from equation (20) that during $\sim 10^{10}$ yr all systems of total mass $\sim 0.5\, M_\odot + 0.5\, M_\odot$ with $A_{ff} \lesssim 2\, R_\odot$

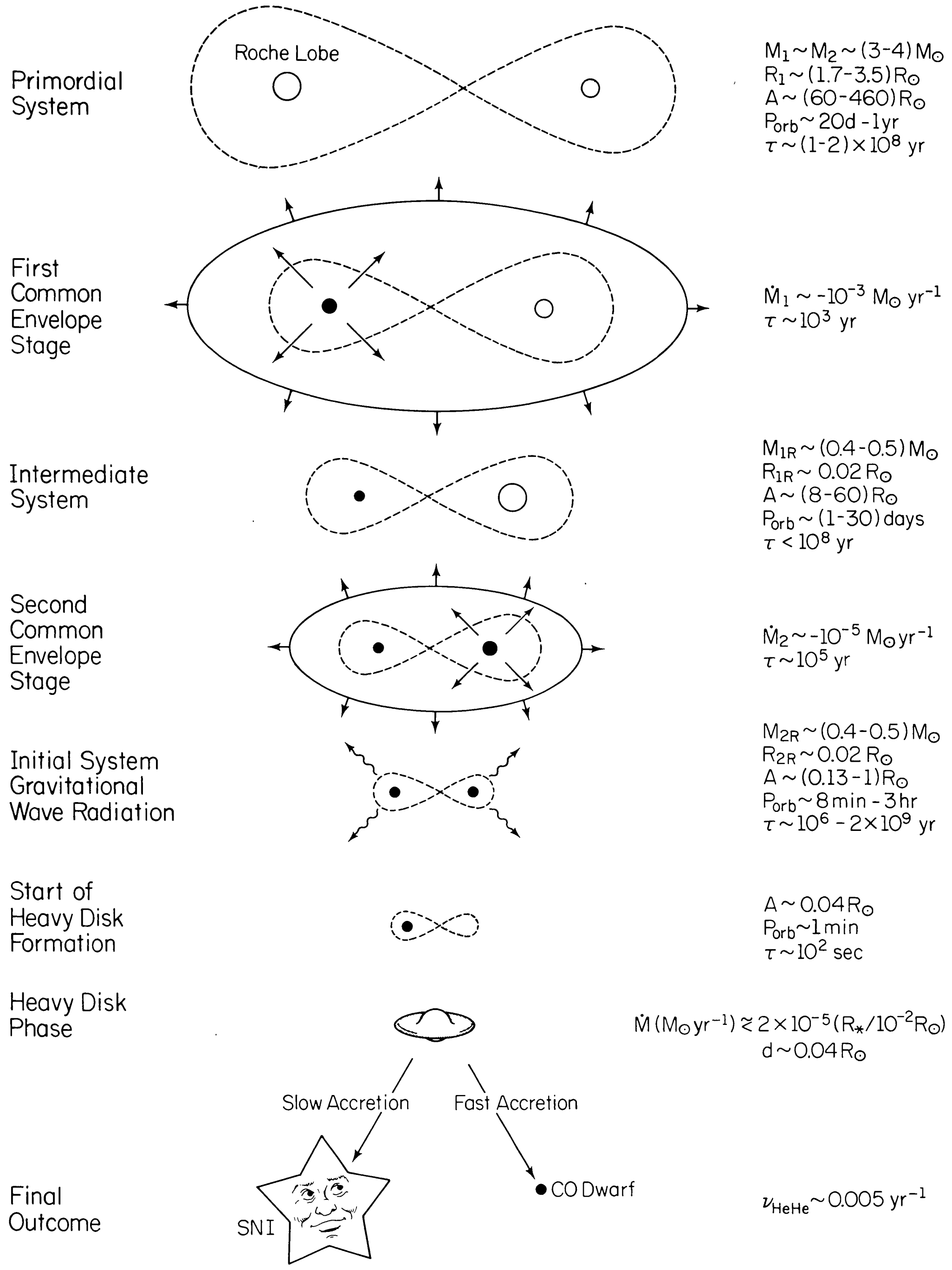

FIG. 4.—Same as Fig. 1 for the scenario which produces an initial system containing two degenerate helium dwarfs

will merge under the influence of the radiation of gravitational waves. This helps us to find a limitation on the primordial semimajor axis A:

$$\frac{A}{R_\odot} \lesssim \frac{60 M_1^{1.19} (M_1 + 1)^{0.12}}{\alpha^{1.19}} . \tag{25}$$

Using this condition, we find the total mass of the final binary to be

$$M_{1R} + M_{2R} \lesssim 0.35 \frac{M_1^{0.45}}{\alpha^{0.33}} + \frac{0.27}{\alpha^{0.15} M_1^{0.03}} . \tag{26}$$

Thus, for $M_1 \approx 3.5\ M_\odot$ and $\alpha \approx 1$, the total mass of the system lies below $0.62\ M_\odot$. Since the minimum mass for achieving an explosion can be estimated from the work of Nomoto and

Sugimoto (1977) to be ~ 0.65–$0.8\ M_\odot$, it is evident that, for most pairs consisting of two degenerate helium dwarfs, merging may not lead to a supernova explosion. Our estimate of $0.62\ M_\odot$ for the maximum mass of a close double-degenerate system in which one component becomes a dwarf only after 10^{10} yr is, of course, highly uncertain; increasing M_1 slightly or decreasing α may possibly increase the total double dwarf mass to the limit necessary for explosion, but in the absence of a predictive common envelope theory, we can do no more than entertain the possibility. There is also uncertainty in the estimate of $0.65\ M_\odot$ as a lower limit for an explosive system. In view of the potential importance of this scenario for understanding the occurrence of SNeI in elliptical galaxies, further exploration of accreting dwarfs (taking the prior thermal history properly into account) is to be encouraged.

In any case, if $\alpha \approx 1$, systems in which one of the degenerate components is produced after $\sim 10^{10}$ yr do not appear to be formed at rates sufficient to account for the frequency of SNeI in ellipticals. We infer that the clock for the delayed time bomb is not, in this case, the time scale for nuclear evolution of the final mass-losing star, but is instead the time scale for angular momentum loss due to the radiation of gravitational waves, this being a function of the semimajor axis A_{ff} of the system at the time when the second degenerate star is formed. Or it could be related to the rate at which matter can be transferred out of a heavy disk. We return to this question in § IX.

iv) *Concluding Comments on Frequency Estimates and Observational Consequences of Double-degenerate Systems*

In the absence of a predictive common envelope theory, our estimates of formation frequency must be viewed as exceedingly preliminary (the unknown parameter α being the major uncertainty). Although, with $\alpha \approx 1$, the total estimated formation frequency ($\sim 1.4\times10^{-2}$ yr^{-1}) of double-degenerate systems of sufficient mass and of appropriate orbital separation to potentially produce SN explosions is remarkably close to the observed frequency of SNI explosions, we must not fall into the trap of believing that we have demonstrated double degenerates to be the major precursors of SNeI. Instead, we must view the proximity of the formal realization frequency to the observed SNI frequency primarily as encouragement to pursue the possible connection much more deeply and to compare carefully properties of observed SNI outbursts with the properties of explosions of degenerate dwarf models accreting at different rates in the hope of ultimately being able to exclude or corroborate one or more of the models.

The frequency of merging double-degenerate dwarfs is expected to far exceed the frequency of SNeI, whether or not these latter are due to the merging of a special subclass of double degenerates, and this fact may have observable consequences. That binaries can evolve into a close system of double degenerates in which mass transfer takes place seems to be well established from analysis of systems such as AM CVn, GP Com, and LB 3459 (see, e.g., Webbink 1979). Choosing $\alpha = 1$ and setting $M_A \approx M_\odot$, $M_B \approx 3.8\ M_\odot$, $q = 1$, and $\Delta \log A \approx 0.8$ in equation (1), we find the frequency of formation of all merging systems consisting of two helium dwarfs to be $\nu_{\mathrm{HeHe}} \approx 0.15$ yr^{-1}. Similarly, we estimate the frequency of merging CO and He dwarfs to be $\nu_{\mathrm{COHe}} \approx 0.02$ yr^{-1} and that of merging CO dwarfs to be $\nu_{\mathrm{COCO}} \approx 0.02$ yr^{-1}, giving a grand total of $\nu_{\mathrm{merge}} \approx 0.19$ yr^{-1}. The rate at which gravitational potential energy is released by all merging systems may be estimated as $L_{\mathrm{merge}} \approx (GM_1M_2/R)\nu_{\mathrm{merge}} \approx 3(\overline{M}_1\overline{M}_2/\overline{R})10^9\ L_\odot\ \nu_{\mathrm{merge}}$, where $\overline{M}_1$ and $\overline{M}_2$ are the average masses (in solar units) of two dwarfs prior to merging and $\overline{R}$ is the average radius of the accreting dwarf in units of $10^{-2}\ R_\odot$. Setting $3\overline{M}_1\overline{M}_2/\overline{R} \approx 1$, we have that $L_{\mathrm{merge}} \sim 2\times10^8\ L_\odot$. This energy flux will be in the far-ultraviolet and may be compared with the flux due to central stars of planetary nebulae ($L_{\mathrm{PN}} \approx 10^8\ L_\odot$) and to the flux due to horizontal branch stars ($L_{\mathrm{HB}} \approx 2\times10^8\ L_\odot$). Horizontal branch stars rarely have surface temperatures in excess of 2×10^4 K, and we thus predict that the dominant contributions to the flux of far-ultraviolet radiation from galaxies is merging degenerate dwarfs.

b) *Evolution to the Explosive Stage*

The key to understanding evolution during mass transfer in a system consisting of two degenerate dwarfs is the fact that the radius of either component will decrease if its mass is increased. This means that the component of lower mass will fill its Roche lobe first. If the initial masses of the components are comparable and if the evolution of the system is initially conservative (that is, if total orbital angular momentum and mass remain constant), then the filling of the Roche lobe leads to mass transfer on a dynamical time scale and disruption of the mass-losing secondary (Tutukov and Yungelson 1979). The gravitational potential energy which could be converted into other forms of energy in such a process may reach $\sim 10^{50}$ ergs.

The radius and mass of a CO degenerate star are related (Iben 1982) by $R/R_\odot \approx 0.0195$–$0.0116\ M_{\mathrm{CO}}/M_\odot$, and, for a helium degenerate star, $d \ln R_{\mathrm{He}}/d \ln M_{\mathrm{He}} \approx -0.4$ (Zapolsky and Salpeter 1969). If dynamical mass transfer is to occur, the logarithmic derivative of the stellar radius in response to mass loss must be smaller (larger in absolute value) than the logarithmic derivative of the Roche lobe

$$\frac{d \ln R_R}{d \ln M_2} \approx 2\frac{M_2}{M_1} - 1.56,$$

where M_2 is the mass of the Roche lobe filling component. This condition is fulfilled for CO white dwarfs with an initial mass ratio larger than $\sim 0.78 - M_{\mathrm{CO}}(3.4 - 2M_{\mathrm{CO}})^{-1}$ and for helium dwarfs with an initial mass ratio larger than ~ 0.61. Practically all systems with two CO degenerate dwarfs have initial mass ratios exceeding this limit. Since, also, overly large mass ratios for helium stars are not excluded in the course of previous evolution, some fraction of all helium double-degenerate systems must evolve into a stage of dynamical mass transfer.

The evolution of systems after the onset of dynamical mass exchange represents a rather complicated problem which has not yet been carefully explored. We might aniticpate that, as a result of the mass flow process, a "heavy" disk may be ultimately formed around the more massive companion, with a bright flash illuminating the event, and mass being lost from the system (Tutukov and Yungelson 1979). The prior formation of a bloated, rapidly rotating envelope may be an initial response to the need for conserving angular momentum. One expects that, even for some systems in which a helium dwarf fills its Roche lobe and the initial mass ratios are less than ~ 0.61, the initial mass exchange rate will be so high ($\dot{M}_{\rm transfer} \gg \dot{M}_{\rm Eddington}$) that the primary cannot immediately accept the transferred matter, forcing it to first build up around the central dwarf a large, rapidly rotating envelope that eventually relaxes into a heavy disk that is then absorbed by the primary over a time scale no smaller than $M/\dot{M}_{\rm Eddington}$.

That is, if matter is transferred onto the central star at a rate larger than the appropriate limit for a cold, compact dwarf, this star will expand until its radius is at least as large as that given by $GM\dot{M}/R_* = L_{\rm Edd}$. In this way, an extended pure helium or even possibly an extended CO envelope may initially be produced around a degenerate helium or CO dwarf (see, also, Iben and Tutukov 1983*a*, *b*). Since the physics of such conjectured objects has not yet been explored, even the coexistence of an extended envelope with a massive disk may be dreamed about.

A supernova-like event associated with mass transfer on a dynamical time scale onto a degenerate CO dwarf is of particular interest. Old degenerate solar mass dwarfs of age $\sim 10^{10}$ yr are very dim: $L \approx (10^{-6}\text{–}10^{-5})\ L_\odot$, $T_e \approx 2500$ K, $M_v \approx 21.3\text{–}23.8$ mag (e.g., Shaviv and Kovetz 1976). An ultraclose binary which is composed of such stars and which has an orbital period of ~ 0.5 minute would hardly be noticed, even if it were placed at a distance of only 1 pc where its apparent magnitude would be only $16.3 \simeq 18.8$ mag. If the orbital plane of the binary coincided with our line of sight, the periodic optical variability could, in principle, help us to identify the system, but its brightness would still be too small to ensure discovery. So, it is conceivable that our first awareness of the existence of such a system could also be our last. However, we may derive some comfort from the fact that the occurrence probability of the envisioned event so close to our Sun is only $\sim 10^{-13}$ yr^{-1}.

The evolution of a degenerate star which is surrounded by a heavy disk depends on the rate at which disk matter is accreted. For most systems whose total mass exceeds the Chandrasekhar limit, a SN explosion seems inevitable, as long as the disk lifetime is shorter than the cosmological time scale. The physics of the disk requires elucidation, and the outcome of accretion at rates in several ranges still requires numerical investigation. One example is the evolution of a degenerate CO dwarf which accretes at rates exceeding $\sim 10^{-6}\ M_\odot$ yr^{-1} from a disk produced by a secondary which can be either a helium or a CO white dwarf. If the disk material has been produced by a helium white dwarf, one might expect the formation of an extended envelope at the base of the envelope. Such an object would resemble an R CrB star. Since R CrB stars lose mass in the form of "puffs" at a fairly rapid rate, we may anticipate that mass loss from the extended envelope could prevent the underlying CO core from achieving the Chandrasekhar mass, and the most probable final result will be a massive single CO white dwarf. If the disk material has been produced by a CO white dwarf, then the most probable result is a carbon deflagration supernova.

The evolution of accreting helium dwarfs for different accretion rates requires further numerical investigation. Existing computations (Mazurek 1973; Nomoto and Sugimoto 1977) show that, for low accretion rates ($\dot{M} \lesssim 2\times10^{-8}\ M_\odot$ yr^{-1}), a SNI explosion will result only if the total mass of the two degenerate helium dwarfs is large enough ($\gtrsim 0.65\text{–}0.8\ M_\odot$), and this requirement, of course, decreases the probability of formation of potentially explosive systems. Accretion at rates exceeding $2\times10^{-8}\ M_\odot$ yr^{-1} leads to weak helium shell flashes, but the final result in this case is not yet clear because it depends upon the thermal history of the dwarf, and additional numerical exploration is needed before we can answer the question as to whether it is possible to achieve a helium supernova explosion if the total mass of the system is of order $\sim 0.65\text{–}0.8\ M_\odot$.

The fate of the heavy disk after a supernova explosion also deserves further investigation. It is not clear whether the supernova explosion will destroy the disk, or if it can survive as a degenerate disk. The latter possibility can lead to the formation of an X-ray source if a neutron star is produced in the SN event.

Only a small fraction of double-degenerate dwarfs which evolve to the stage of mass exchange can finish their evolution as supernovae. Most systems will evolve into single-degenerate dwarfs rotating at rates much higher than do white dwarfs whose evolution has not been influenced by a companion. These merged degenerate dwarfs will differ further from most noninteracting single white dwarfs by the absence of hydrogen in their atmosphere. Some fraction of all non-DA white dwarfs may be produced in this way.

The supposition that SNeI are the result of merging of close binary degenerate white dwarfs as a consequence of gravitational wave radiation leads to a prediction about the flux of gravitational wave energy impinging on the Earth. The oscillation frequency of these waves is of the order of twice the orbital frequency of a compact presupernova binary, or ~ 0.1 s^{-1}. Every presupernova system of the envisioned type radiates $\sim 3\times10^{50}$ ergs in gravitational waves ($= GM^2/2R$, where R is the radius of a dwarf). The time between supernovae is $\sim 3\times10^{9}$ s, so, if the typical galactic SN is at a distance of 10 kpc, the flux at the Earth of energy in the form of gravitational waves at a frequency of 0.1 s^{-1} is $\sim 10^{-4}$ ergs cm^{-2} s^{-1}. This flux appears to be beyond the current limits of detectability (Weiss 1979), but this may not be the case in the foreseeable future (Weiss 1979; Zimmerman 1979).

The estimate of the gravitational wave flux which we have presented may actually be only a lower limit on the real flux that is produced by close double degenerates. This is because only a small fraction (~ 0.1) of all close double degenerates are expected to have sufficient mass to eventually become SNeI, and yet all of them should radiate gravitational waves. Even though the average frequency of the radiation emitted by systems that do not terminate as SNeI will be somewhat less than that of radiation emitted by those that do, the overall likelihood of a future detection is considerably enhanced, and the prospects for such a detection appear bright.

VII. SOME CONSEQUENCES OF A TYPE I SUPERNOVA EXPLOSION CAUSED BY ACCRETION ON A DEGENERATE DWARF IN A CLOSE BINARY IN WHICH THE COMPANION IS A MAIN-SEQUENCE STAR OR A RED GIANT

Let us assume that, after accreting matter at an appropriate rate and over an appropriate interval, a degenerate dwarf undergoes an SNI event. Whether or not a condensed remnant remains after the explosion, we know from the observations that at least some envelope matter is ejected at high velocities. The kinetic energy E_k of the ejected matter is of the order of 2×10^{51} ergs. Since the mass-donating companion is presumably filling its Roche lobe before the explosion, it will intercept $\sim10\%$ of this ejected matter.

If the Roche lobe filling companion is a red (super)giant, the binding energy of the low-density envelope [$E_b \approx 10^{48}(M/M_\odot)^2(R_\odot/R)$ ergs] is likely to be much, much smaller than 0.1 E_k, and, therefore, the most probable result of the interaction between the SN ejectum and the giant's hydrogen-rich envelope will be the loss of the envelope. This has two consequences. The expanding envelope of the supernova will be contaminated by 0.1–0.6 $M_\odot$ of hydrogen- and helium-rich matter, and this should be enough to be observable. A study by Kirshner and Oke (1975) has suggested as much as 0.1 $M_\odot$ of hydrogen in SN 1972e. However, Oke and Searle (1974) have indicated that hydrogen is absent in the spectra of SNeI, and this argues against those scenarios in which the mass-donating star is a red (super)giant.

A second consequence is that, since the compact degenerate core of the red (super)giant will remain after the extended envelope has been blown off in the SN explosion, the final system may survive as a binary if, and only if, the exploding dwarf is not completely disrupted by the explosion. If complete disruption does not occur, but instead the dwarf transforms into a neutron star, then the orbit of the surviving binary will acquire an eccentricity. The absence of a large hydrogen-rich envelope about the stripped companion and the large orbital period ($P \gtrsim 10$ hr) prevent the system from evolving into an X-ray binary. However, some fraction of the hypothesized neutron star remnants should be detectable as radio pulsars exhibiting radial velocity variations. If one were to suppose that all SNI events follow this scenario, then, given the expected relative frequency of SNI events in our Galaxy, almost half of all neutron stars and, consequently, radiopulsars, should be detectable as members of a binary system. In actuality, the number of all observed radiopulsars that exhibit duplicity is almost two orders of magnitude less than this expectation (Taylor 1981). If, then, all SNeI were the consequence of a transfer of matter from a red (super) giant to a white dwarf, we could infer that total disruption must be the ultimate fate of the exploding star. However, the expected presence of detectable hydrogen in the SNI spectrum still remains as an argument against this scenario and prevents us from drawing this inference.

We next examine some consequences of SN explosions in close cataclysmic-like systems consisting of an accreting degenerate He, CO, or ONeMg dwarf and a Roche lobe filling main-sequence component. According to Patterson (1983), mass exchange can supply 10^{-9}–10^{-7} $M_\odot$ of hydrogen-rich matter every year. Let us assume, just as an example, that both the SN ejectum and the main-sequence star are of mass 0.3 $M_\odot$ and that the velocity of the ejected matter is $\sim10^9$ cm s (Pskovsky 1977*a*, *b*). The radius of the Roche lobe filling main-sequence star is $\sim$ 0.3 $R_\odot$, and the mass of the neutron-star remnant (supposing one is formed) is 1.1 $M_\odot$. About 5% (or 1.5×10^{48} ergs) of the kinetic energy of the SN envelope is available in this case to interact with the main-sequence component. The impulse supplied to the main-sequence star by ram pressure is less than 3×10^{39} g cm s^{-1}, and this corresponds to a velocity increment in the direction of the line of centers of only $\sim$ 25 km s^{-1}. Since the orbital velocity of the main-sequence star filling its Roche lobe is $\sim$ 400 km s^{-1}, it is clear that the SN explosion cannot as a rule unbind the system.

The impact of an expanding supernova on a main-sequence companion has been investigated numerically by Fryxell and Arnett (1981). These authors assume that the main-sequence star is completely radiative and that its structure may be approximated by an $n=3$ polytrope. In all of the cases investigated, the kinetic energy of the impacting mass is restricted to values substantially less than the binding energy of the main-sequence star; the ratio of the transferred momentum to the momentum of the impacting mass is on the order of 0.5, and $\sim10\%$ of the kinetic energy of the impacting matter is used up in stripping mass from the main-sequence star.

Unfortunately, in the more realistic situation in which the main-sequence star with mass lower than $\sim M_\odot$ fills its Roche lobe, the energy of the impacting mass is expected to exceed the binding energy of the main-sequence star, so the numerical results just described are not strictly applicable. Assuming, nevertheless, that the Fryxell-Arnett result (10% of the kinetic energy of the impacting matter goes into ejecting mass from the main-sequence companion) carries over to the more realistic situation, we have that the amount of mass lost from main-sequence stars of initial mass 0.14, 0.3, and 1 $M_\odot$ is, respectively, 0.03, 0.05, and 0.07 $M_\odot$. Mass loss of such a magnitude probably cannot disturb the star strongly enough to disrupt it.

But, if the kinetic energy of the supernova ejectum were $\sim$ 4 times larger than we have assumed (for example, if the exploding star is completely disrupted), then the impact of the explosion may also disrupt the main-sequence companion. The possibility of complete disruption of the main-sequence star becomes even more likely if this star is of small enough mass ($\lesssim 0.3$ $M_\odot$) to be completely convective. For, then, the star is better described by an index $n=3/2$ polytrope, which is much less centrally condensed than is an index $n=3$ polytrope, and which therefore has a higher concentration of matter near the surface where ablation takes place. Clearly, additional stripping calculations should be performed, not only with larger impacting kinetic energies, but also with completely convective main-sequence stars.

Should such calculations reveal that, in the case of an ONeMg dwarf undergoing an SN explosion followed by collapse to form a neutron star, the energy of the matter ejected in the explosion is sufficient to disrupt the low-mass companion (which does not seem improbable if $M_{\rm MS} \lesssim 0.3\ M_\odot$), then a most interesting possibility opens up for understanding the very short-period ($P \approx 1.56 \times 10^{-3}$ s) pulsar PS 1937 214 (Baker *et al.* 1982). In the process of accreting through a Keplerian disk a mass of $\delta M_{n\alpha}$, the ONeMg ($= n\alpha$) dwarf will, in the conservative case, acquire a rotational period of $P_{n\alpha} \approx 0.15(M_{n\alpha}/\delta M_{n\alpha})P_{\rm K}$, where $P_{\rm K}$ is the Keplerian rotational period at the surface of the $n\alpha$ dwarf. For an $n\alpha$ dwarf with $M_{n\alpha} \approx 1.4\ M_\odot$ and a radius $R_{n\alpha} \approx 2.1 \times 10^8$ cm (Iben 1982), $P_{\rm K} \approx 1.42$ s, giving $P_{n\alpha} \approx 0.3\ M_\odot/\delta M_{n\alpha}$ s. In an angular momentum conserving collapse, this period will be reduced to $P_{\rm NS} \approx P_{n\alpha}(R_{\rm NS}/R_{n\alpha})^2$, where $R_{\rm NS}$ is the radius of the neutron star. For a star of mass $\approx 1.4\ M_\odot$, $R_{\rm NS} \approx 10^6$ cm and $P_{\rm NS} \approx 0.7 \times 10^{-5}\ M_\odot/\delta M_{n\alpha}$ s. The observed period of $\sim 1.56 \times 10^{-3}$ s therefore implies the accretion of $\sim 4.5 \times 10^{-3}$ $M_\odot$ by the $n\alpha$ dwarf, which achieves a period $P_{n\alpha} \approx 67$ s. Such a period is comparable with the observed rotational periods of several white dwarfs in cataclysmic binaries, e.g., DQ Her (71 s) and AE Aqr (33 s) (Webbink 1982). The ease with which the accretion and collapse scenario can account for the period of PS 1937 214 is an intriguing indication that, sometimes perhaps, accreting ONeMg dwarfs are supernova precursors.

A second example is the X-ray pulsar 1E2259+586 (Fahlman *et al.* 1982). The theoretically most likely possibility requires that the accreting degenerate star must be of the ONeMg variety. However, even though most theoretical studies suggest that, on igniting carbon, accreting CO white dwarfs will deflagrate and be completely disrupted rather than collapse to form a neutron star, we do not reject that, under very special circumstances, even an accreting CO degenerate may collapse to produce a remnant like PS 1937 214 or 1E2259+586.

If (in what we shall now show must be an uncommon event) the typical SNI explosion were to leave behind a neutron star and, at the same time, the main-sequence companion were not destroyed, then it is highly probable that the compact remnant and the main-sequence star would remain bound. However, some mass would be lost, and therefore the semimajor axis of the system would initially increase. The decrease in mass of the main-sequence star in response to the ablation of its outer envelope might contribute further to the degree of detachment. However, as time progressed, angular momentum loss from the system due to a magnetic stellar wind or to the radiation of gravitational waves would drive the neutron star and its main-sequence component together again until the main-sequence star again filled its Roche lobe. The system would thereupon become a low-mass X-ray binary.

Let us assume that, when Roche lobe contact is resumed, the average mass of the main-sequence star is $\sim 0.5\ M_\odot$. Almost all of this mass would be accreted by the neutron star during 10^8–10^9 yr to produce $\sim 10^{53}$ ergs, most of which energy would be emitted as X-rays. Since the average frequency of SNeI in the Galaxy is $\sim 10^{-2}$ yr^{-1}, or $\sim 3 \times 10^{-10}$ s^{-1}, the X-ray luminosity of our Galaxy would be $\sim 8 \times 10^9\ L_\odot$ if all SNeI were produced according to this scenario. Since this exceeds the observed X-ray luminosity (McClintock and Rappaport 1983) by a factor of $\sim 10^4$, we infer that no more than 10^{-4} of all cataclysmic-like systems can survive as double systems (neutron star plus main-sequence star) after an SNI explosion if all cataclysmic binaries were to evolve to the SN explosion stage. We are therefore led to the conclusion that SNI explosions do not, in general, leave neutron star remnants if most of them are products of close binary evolution. This interpretation is, of course, in agreement with the fact that radio pulsars are not distributed randomly about the Galaxy, as are low-mass stars, but are concentrated in spiral arms (Helfand 1980).

For completeness, it must be mentioned that no contradiction with the observed galactic X-ray flux would arise if all SNeI left behind white dwarf remnants. However, we might then have difficulty in accounting for the galactic abundance of iron, which is now commonly thought to be produced primarily by SNeI.

VIII. ON THE FORMATION OF LOW-MASS X-RAY BINARIES

Despite the unlikelihood that most accreting white dwarfs will collapse to form neutron stars, it has on occasion been proposed that there is an evolutionary connection between bright, low-mass X-ray binaries (LMXBs) and systems consisting of a degenerate dwarf and a Roche lobe filling low-mass companion. However, as we have seen in the previous section, the vast majority of low-mass binaries that evolve to the stage of a SNI explosion cannot become LMXBs. If, further, we can point to ways of producing LMXBs at the observed frequency (10^{-7} yr^{-1} in our Galaxy; see Webbink, Rappaport, and Savonije 1983; Taam 1983) without requiring prior passage through one of the SNI scenarios which we have examined, then we have shown that there is no reason for continuing to believe in a direct connection between most SNI events and LMXBs.

One scenario for LMXB formation in the galactic disk begins with a primordial wide binary consisting of a relatively massive ($M_1 > 8\ M_\odot$) main-sequence component and a lighter component of near-solar mass (see, e.g., van den Heuvel 1981; Webbink *et al.*). The system evolves through a common envelope stage at the end of which the primary emerges as a helium star of mass $M_{\rm He} > 2\ M_\odot$ and proceeds to burn successive nuclear fuels until it has developed an Fe-Ni core of mass $\sim 1.4\ M_\odot$, whereupon it explodes as a classic SNII, leaving behind a neutron star remnant. The presupernova may also be an accreting ONeMg degenerate dwarf with a main-sequence component.

Although these scenarios might account for the so-called bulge LMXBs, which are not obviously associated with globular clusters and of which there are ~ 32 known (McClintock and Rappaport 1983), the very existence of LMXBs in globular clusters suggests that some other mechanisms must be operating to form the LMXBs in the clusters. That is, since there are ~ 10 LMXBs in clusters (Hertz and Grindlay 1983; McClintock and Rappaport 1983), and since the total mass in globular clusters is $\sim 10^4$ times smaller than the (luminous) mass of the Galaxy, the efficiency of production of LMXBs in globular clusters is $\sim 10^4$ times greater than it is among field stars. Since there is no reason to suppose that the frequency of wide binaries of the sort proposed by van den Heuvel (1981) and Webbink *et al.* was 10^4 larger in the early life of surviving globular clusters than elsewhere in the Galaxy, one must clearly invoke a quite different formation mechanism.

Fabian, Pringle, and Rees (1975) have shown that inelastic two-body collisions in the dense cores of globular clusters could, on occasion, lead to the tidal capture of a main-sequence star by a neutron star and that this process might produce globular cluster LMXBs at the observed rate. It is worth pointing out here that the idea of forming binaries by way of dissipative collisions of two single stars is an ancient one. For example, according to Aitken (1964), such collisions were proposed in 1867 by G. J. Stoney to explain the formation of both binaries and planetary systems. Fabian *et al.* estimate that the number of capture systems in a cluster with radius ~1 pc is

$$N \approx 5\times10^{-7}\frac{R_*}{R_\odot}\frac{N_* N_{\rm NS}}{v_6}\tau_{10}\frac{M_t}{M_\odot}, \tag{27}$$

where R_* is the stellar radius, M_t is total mass of the binary system, N_* and $N_{\rm NS}$ are the number of main-sequence stars and of neutron stars, respectively, τ_{10} is the age of a globular cluster in units of 10^{10} yr, and v_6 is the dispersion of star velocities in the core of the cluster in units of 10^6 cm s^{-1}. The observed number of bright X-ray sources in galactic globular clusters agrees well with those predicted by this relationship (Lightman and Grindlay 1982). This agreement makes it interesting to explore other potentially observable properties of tidal capture systems.

It is a relatively simple matter to estimate the distribution over orbital period that tidal capture systems should obey. At the outset, one might expect that, since there are many giants and supergiants in a cluster, and since the probability of tidal capture is proportional to stellar radius, many systems of large semimajor axis, and hence large period, would be formed. However, the fact that the probability of tidal capture increases with radius is offset by the fact that the larger its radius, the more rapidly does a star evolve. To examine this quantitatively, we note that the relative number of capture systems per unit interval of stellar radius may be written as $dN/dR \approx R(dR/dt)^{-1}$, where dR/dt is the rate at which stellar radius increases. To obtain the dependence of dN/dR on R, we have used the evolutionary tracks of Sweigart and Gross (1978) for stars of mass 0.9 and 1.2 $M_\odot$ and for heavy element abundances in the range 10^{-4}–10^{-2}. We find that, in approximately half of all systems formed by capture, the component now filling its Roche lobe has a radius in the range 1–1.3 $R_{\rm MS}$, and in only ~10% of such capture systems does the radius exceed ~2 $R_{\rm MS}$. Here, $R_{\rm MS}$ is the radius of a main-sequence star of the same mass. To within ~10%, these results do not depend on either the mass or the composition. Thus, tidal capture involving a neutron star and a star currently burning hydrogen can produce primarily only short-period ($P_{\rm orb} \lesssim$ few hours) systems with a fairly unevolved component on or near the main sequence.

Long-period systems can be formed during early stages of globular cluster evolution by the capture of a low-mass star by a massive supergiant in a two-body collision involving direct impact (Ergma and Tutukov 1980). The frequency of formation of such systems is almost equal to the frequency of formation of systems consisting of a low-mass star and neutron star as given by equation (27). The fact that, for the supergiant plus low-mass star, the product $R_* N_*$ is about four orders of magnitude larger than $R_* N_*$ for the neutron star and low-mass star is offset by the fact that the lifetime of a massive red supergiant is almost four orders of magnitude shorter than the lifetime of a low-mass star. The product $N_* N_{\rm NS}$ is the same for both cases if we assume that all massive stars produce neutron star remnants. The common envelope will bring the captured main-sequence star close enough to the evolved core of the supergiant that nuclear evolution will eventually lead to Roche lobe overflow and intense X-ray emission.

Long-period systems can also be formed by exchange captures, whereby the neutron star replaces one of the (usually less massive) components of a low-mass wide binary (Hills 1975; Fullerton and Hills 1982). Once again, relation (27) can be used to estimate the number of LMXBs which are products of such exchange collisions (Hut and Verbunt 1983). We assume that a fraction ϕ of all stars in a globular cluster are in "primordial" binaries and are distributed over A according to $dN = 0.2\phi\, d\log A$. Only those binaries which have a semimajor axes in the interval 10–50 $R_\odot$ can transfer mass at a rate lower than that given by the Eddington limit for a neutron star. Secondary components in binaries with $M \approx 0.8$ $M_\odot$ will expand and fill their Roche lobes in the course of normal evolution after ~10^{10} yr. Primordial binaries with $A \lesssim 10$ $R_\odot$ are probably very rarely present due to the action of magnetic braking (Kraicheva *et al.* 1978; Tutukov 1983). Even in the absence of this action, an exchange collision involving such a close binary accelerates the newly formed binary to such a high velocity (several tens of kilometers per second) that it escapes from the globular cluster.

The relative number of binaries that are made up of stars of mass ~0.8 $M_\odot$ and that have a semimajor axis satisfying $10 \lesssim A/R_\odot \lesssim 50$ is ~0.14ϕ. Next we estimate the number of globular cluster stars with a radius between ~4 and 20 $R_\odot$, a necessary condition for Roche lobe filling in a system in which components are of comparable mass. The distribution of globular cluster stars with respect to mass is taken to be

$$dN = 0.5\left(\frac{m}{M_\odot}\right)^{0.5}\frac{M_{\rm GC}}{M_\odot}\left(\frac{M_\odot}{M}\right)^{2.5}\frac{dM}{M_\odot}, \tag{28}$$

where m is the minimum initial mass of a star born in the cluster (~0.1 $M_\odot$), $M_{\rm GC}$ is the total mass of the cluster, and M is the initial mass of an individual star. The number of stars with $M \approx 0.8$ $M_\odot$ ($\tau_{\rm MS} \approx 10^{10}$ yr) and radius exceeding ~4 $R_\odot$ is proportional to the lifetime of a star with radius greater than this. Again, using the data provided by Mengel *et al.* (1979), we find the required number to be $N_* \approx 0.0016$ $M_G/M_\odot$ for $m \approx 0.1$ $M_\odot$. In a similar fashion, if the evolution of all stars with $M \gtrsim 10$ $M_\odot$ terminates in neutron star formation, we estimate the number of neutron stars in a globular cluster to be $N_{\rm NS} \approx 0.0033$ $M_G/M_\odot$.

Putting these estimated numbers into relation (27), together with $R_* \approx A = 10$ $R_\odot$, $M_G = 10^5$ $M_\odot$, $M_t \approx 2.2$ $M_\odot$, and $v_6 \approx 2$, we obtain an estimate of the probability that an LMXB produced by exchange capture will be in a typical cluster:

$N_{\rm LMXB} \approx 0.033\phi$. Since the probability of capture increases with A (see eq. [27]) and since the X-ray luminosity also increases with A (see eq. [11a]), the X-ray radiation will be dominated by (perhaps only a very few of) the brightest sources. These will have characteristic luminosities $L_X \approx 10^4$ $L_\odot$ and periods $P_{\rm orb} \approx 5^{\rm d}$–$25^{\rm d}$ and appear much like Cyg X-2. Our estimate with $\phi \approx 1$ suggests that there should be about seven bright LMXBs formed by exchange collisions in the 200 or so globular clusters in our Galaxy. Since 10 bright X-ray sources have actually been found, one might infer that ϕ, the degree of duplicity in globular clusters is closer to one, which is appropriate for stars in the disk of our Galaxy (Abt 1983), than it is to zero, as might be inferred from the study by Gunn and Griffin (1979). It is worth noting that the recent study by Harris and McClure (1983) suggests that ϕ in the globular cluster M3 could be as large as 0.4.

If a captured neutron star winds up with an unevolved secondary of mass less than ~ 0.8 $M_\odot$, the nuclear evolution of this secondary will not lead to a large enough radius to permit Roche lobe overflow in 10^{10} yr. But, if the semimajor axis is less than ~ 10 $R_\odot$, then a magnetic stellar wind can drive the components together at a rate which will produce mass exchange at a rate $\sim 10^{-9}$–10^{-8} $M_\odot$ yr^{-1}, and this is quite sufficient to produce an X-ray luminosity of $\sim 10^3$–10^4 $L_\odot$ (Tutukov 1983). The total number of such systems formed over 10^{10} yr per cluster is $\sim 35\phi$, if we assume that the distribution of primordial binaries over A may be prolonged to $A \approx 2$–3 $R_\odot$. Using the typical values $\dot{M} \approx 3 \times 10^{-9}$ $M_\odot$ yr^{-1} and $M \approx 0.5$ $M_\odot$, we see that the lifetime of the final accretion phase is only $\sim 1.7 \times 10^8$ yr, so that only $\sim 0.6\phi$ such X-ray binaries will at any one time be in the active stage. This is comparable with the number of observed X-ray systems per cluster if $\phi \approx 1$.

In summary, the exchange capture of single neutron stars by low-mass binaries in the dense cores of globular clusters appears to be a very efficient way to form LMXBs of period longer than $P_{\rm orb} = 1^{\rm d}$, provided only that the primordial duplicity exceeds, say, ~ 0.3; the lower limit on $P_{\rm orb}$ is due to the action of a magnetic stellar wind which significantly reduces the number of primordial systems with $A < 10$ $R_\odot$. On the other hand, tidal capture appears to be an efficient way to produce LMXBs whose orbital periods are less than $P_{\rm orb} \approx 0\overset{\rm d}{.}5$. Finally, direct capture of a low-mass star by a massive one in the early years of a globular cluster's life provides an additional source of globular cluster LMXBs.

We began this section by suggesting that, although a scenario based on the evolution of primordial wide binaries in which one component is massive enough to become a SNII may explain the existence of the bulge LMXBs, it could not account for the frequency of globular cluster LMXBs. We now examine whether or not the converse is true: can the processes which appear to account for cluster LMXBs (tidal or exchange capture, or both) also account for the bulge sources?

Hut and Verbunt (1983) have found that the rate of exchange capture with a compact star agrees within several tens of percent with the rate of binary formation through tidal capture of a compact star if one assumes $A = R_*$. Therefore, formulae (27) and (28) may be used once again to estimate the number of LMXBs formed by exchange collisions in the disk of our Galaxy during the past $\sim 1.5 \times 10^{10}$ yr. We assume that the mass of the disk is $\sim 1.5 \times 10^{11}$ $M_\odot$, the volume is $\sim 10^{11}$ pc^3, the minimum mass of stars which are distributed according to the Salpeter law with $dN/dM \propto M^{-2.5} = -2.5$ is ~ 0.1 $M_\odot$, and the space velocity of a typical neutron star is ~ 100 km s^{-1}. If we assume that all stars with an initial mass greater than 8 $M_\odot$ will form neutron stars of mass ~ 1.4 $M_\odot$, the number of neutron stars in our Galaxy will be $\sim 7 \times 10^8$.

Two types of primordial binaries are of interest as potential targets for exchange reactions that evolve into X-ray sources. The first type consists of binaries with primary masses 0.3–0.8 $M_\odot$ and semimajor axis 2.5–10 $R_\odot$ which, after an exchange collision, may be driven by a magnetic stellar wind and persist as X-ray sources over a period of 10^8–10^9 yr. The number of primordial binaries with appropriate masses and semimajor axes is $\sim 4 \times 10^7$–4×10^8, and we have assumed that almost all stars in the Galaxy are formed in binaries ($\phi = 1$). The second type consists of binaries with primary masses of ~ 0.8 $M_\odot \pm \Delta M$, where $\Delta M = 0.002$–0.02 follows from the fact that the X-ray stage lasts only 10^8–10^9 yr ($\Delta t_{\rm MS} \approx -3.5(\Delta M/M)_{\rm MS} t_{\rm MS}$). The appropriate semimajor axes for these binaries are in the range 10–50 $R_\odot$, and their number can now be estimated with the help of equation (28) as $\sim 0.6 \times 10^7$–0.6×10^8.

Finally, assuming the average A for stars of the first type to be ~ 6 $R_\odot$ and for the second type to be ~ 30 $R_\odot$, we obtain from equation (27) that the number of X-ray sources formed by exchange capture and driven by a magnetic stellar wind is in the range 6–60 and that the number of those formed by exchange capture but driven by the evolutionary expansion of a red giant of mass ~ 0.8 $M_\odot$ is in the range 4–40. The luminosity of these sources will be limited to 10^{36}–10^{38} ergs s^{-1}. The action of a magnetic stellar wind will reduce the number of primordial binaries with $A \lesssim 10$ $R_\odot$ and, consequently, reduce the number of X-ray binaries of the first group. Thus, most X-ray binaries formed in the disk by exchange capture are probably driven by evolutionary expansion, and this may explain the larger fraction of bright sources in the disk compared with those in globular clusters. The high recoil velocities of the system newly formed by the exchange capture of a high velocity neutron star may account for the observed distribution of bulge sources in a direction perpendicular to the disk.

Yet another way of explaining at least some of the bulge LMXBs is that they were actually formed by capture processes in globular clusters and were either ejected from the cluster (high recoil velocity achieved in the formation process) or appeared as field stars as the cluster "dissolved." Dissolution due to dynamical friction and the action of tidal forces appears to be the probable outcome for primordial globular clusters which have orbits passing close to the galactic center (e.g., Tremaine, Ostriker, and Spitzer 1975), and it has been estimated that as many as one-half to two-thirds of all primordial clusters may have been dissolved in this fashion (Surdin 1979; Bisnovatyi-Kogan and Romanova 1983); many of the LMXBs in the central regions of our Galaxy may once have been in these vanished clusters.

If exchange capture and tidal capture (whether in the disk or in globular clusters which ultimately dissolve) can account for the bulge sources, what becomes of the scenario involving a primordial massive binary with a very small initial q value? And what becomes of the scenario involving a CO white dwarf accreting from a low-mass companion? At the very least, since we have quite a sufficient number of other ways to account for the handful of bright X-ray sources in which a low-mass component fills its Roche lobe, the existence of these sources may not be invoked as a demonstration that the precursor system once consisted of a degenerate CO dwarf accreting matter from a low-mass companion.

At the risk of committing overkill, we provide one final demonstration of this last truth. Systems consisting of a degenerate CO dwarf and a low-mass hydrogen-burning companion can also be readily produced by exchange collisions between pre-formed single white dwarfs and primordial low-mass binaries. Since one expects the number of white dwarfs produced over 10^{10} yr to exceed the number of neutron stars by about a factor of 20 (using a Salpeter-like mass function and assuming that all stars initially more massive than ~ 8 $M_\odot$ become neutron stars), the number of systems consisting of a white dwarf and a main-sequence component of comparable mass (say, ~ 0.7 $M_\odot$) also exceeds the number of systems containing neutron stars by about a factor of 20. This estimate is supported by Hertz and Grindlay (1982), who examined such systems at X-ray wavelengths. It is therefore evident that no more than one in 20 of such systems could evolve into LMXBs.

IX. SUMMARY AND CONCLUSION

As we have seen, the theory of stellar evolution makes it possible to understand how close binaries which consist of an accreting massive CO or helium dwarf and a component which supplies matter may be produced. Observations provide examples of most of the envisioned models, including possibly a pair of degenerate dwarfs HZ 29 (Smak 1975; Robinson and Faulkner 1975).

The main results of the analysis presented in this paper are summarized in Table 1 and in Figures 5 and 6. The realization frequency for each examined scenario, relative to the "observed" frequency of SNeI, may be interpreted both (either) as a measure of the probability that systems playing out this scenario are the *primary progenitors* of SNeI which are explicable in terms of only one scenario, and (or) as a measure of the probability that such systems *contribute to the formation* of SNeI which are explicable by means of a variety of scenarios. The traditionally most attractive model, which consists of a CO dwarf and a red supergiant filling its Roche lobe, does not fare at all well as a likely candidate. The most serious handicaps which face the model are the need for a large initial mass for the CO dwarf (~ 1.3 $M_\odot$), a strong limitation on the appropriate primordial separation of the system components, and a strong limitation on the primordial ratio of component masses. All of these limitations lead to an estimated frequency of SN explosions which is between two and three orders of magnitude smaller than the observed one. This frequency could be increased if there were a mechanism which produced many CO dwarfs with mass close to the Chandrasekhar limit, but the distribution of single dwarfs over mass shows no sign of the operation of such a mechanism.

Interestingly, and surprisingly, the variant of the traditional model which relies on mass transfer from a wind emitted by a non–Roche lobe filling red supergiant companion of mass ~ 1–8 $M_\odot$ fares much better. However, the estimation of the relevant parameters of such systems is difficult in the absence of a good theory, both of the emitted wind and of the efficiency of accretion from the wind. It is therefore premature to suggest, as our estimate might be taken to imply, that a large fraction of all SNeI could be products of evolution of this sort.

A substantial fraction of all SNeI explosions may be produced by cataclysmic and cataclysmic-like close binaries consisting of a low-mass unevolved main-sequence star which fills its Roche lobe and a degenerate helium or CO dwarf. A magnetic stellar wind appears to be necessary for the formation of such systems after the common envelope stage. As long as the orbital period is larger than ~ 3 hr the magnetic stellar wind keeps the mass exchange rate at a rather high level $\sim 10^{-9}$–10^{-7} $M_\odot$ yr^{-1}. The total frequency of systems of these types is $\sim 4\times10^{-3}$ yr^{-1}, and such systems could contribute substantially to the observed frequency of SNeI, provided accretion rates in the appropriate range can be maintained. That not all cataclysmics can become SNeI is implied by the indication that, as systems containing a massive CO degenerate dwarf evolve toward smaller orbital periods, many of them pass the $P_{\text{orb}} = 3$ hr threshold, whereupon, instead of growing monotonically in mass toward the Chandrasekhar limit, the degenerate component experiences nova outbursts which may lead to the expulsion of more matter than is accreted between outbursts. Perhaps more promising are those systems consisting of a helium dwarf of mass ~ 0.4 $M_\odot$ and a star close to the main sequence. The formation frequency of such systems is $\sim 2\times10^{-3}$ yr^{-1}. Additional numerical investigation is necessary, since it is not now clear how the accretion rate influences the critical mass of the helium degenerate dwarf for which an explosion can occur.

Systems consisting of a Roche lobe filling star of mass ~ 1–2 $M_\odot$, which is close to the main sequence, and a massive CO dwarf ($M \gtrsim M_\odot$) experience mass transfer on a thermal time scale and may evolve to the explosive stage. The frequency of formation of these systems is $\sim 3\times10^{-3}$ yr^{-1}, and their age is $\sim 10^9$–10^{10} yr. These systems could therefore account for some fraction of all SNeI.

Systems consisting of two degenerate dwarfs driven together by gravitational wave radiation have the appropriate frequency of formation ($\sim 1.4\times10^{-2}$ yr^{-1} for all permutations of dwarf compositions) to account for all SNeI by themselves. Systems of this type in which the components are of comparable mass may pass through a phase in which the lighter companion is transformed completely first into a rapidly rotating envelope and then into a "heavy" disk around the other companion. The transformation releases $\sim 10^{50}$ ergs, which is larger than the light energy released in supernova events. The age of these systems can vary from 10^8 to 10^{10} yr. A fuller understanding of evolution during which the heavy disk is slowly accreted (absorbed) by the surviving degenerate dwarf requires further numerical exploration.

Historically, one of the main arguments for believing that SNeI are the result of binary evolution has been the need to

TABLE 1

Realization Frequencies of Possible Type I Supernova Precursors

Presupernova System	Time between Formation of the Primordial Binary and the SN Explosion (yr)	Realization Frequency (no. per 10^3 yr)
Observed supernovae of type I	10^8–10^{10}	10
CO dwarf ($M_{\rm CO} \gtrsim M_\odot$) accreting matter through the Roche lobe from a $\approx 0.8\ M_\odot$ red giant with a helium core	10^{10}	$\ll 1$
CO dwarf ($M_{\rm CO} \gtrsim 1.1\ M_\odot$) accreting matter from a stellar wind emitted by an AGB star not filling its Roche lobe	10^7–10^{10}	< 4
Cataclysmic binaries: A CO dwarf accreting matter from a low-mass ($M \lesssim 1.2\ M_\odot$) main-sequence star	10^8–10^9	2
Cataclysmic binaries: a helium dwarf accreting matter from a low-mass main-sequence star	10^8–10^9	2
Cataclysmic-like binaries: mass exchange on a thermal time scale onto a CO or helium dwarf from a component of mass 1–2 $M_\odot$ filling its Roche lobe	1–3×10^9	3
A close binary consisting of two CO dwarfs evolving as a consequence of gravitational wave radiation	10^8–10^{10}	8
A close binary consisting of a helium dwarf and of a CO dwarf evolving as a consequence of the radiation of gravitational waves	10^8–10^{10}	1
A close binary consisting of two helium dwarfs with total mass $\sim (0.65$–$0.8)\ M_\odot$ evolving as a consequence of gravitational wave radiation	2–10×10^8	5
A close binary consisting initially of two stars with mass above $\sim 8\ M_\odot$. Explosion of a compact remnant with a helium envelope	3×10^6–2×10^7	8

explain the occurrence of SNI explosions in elliptical galaxies, long ($\gtrsim 10^{10}$ yr) after the major phase of active star formation has ceased. It is appropriate to ask at this point whether or not some representatives of each of the binary scenarios we have discussed are able to delay reaching explosive conditions until more than 10^{10} yr after formation. The simplest cases conceptually are those in which the "clock" determining when a "delayed time bomb" SNI goes off is the main-sequence evolution of a component of sufficiently low mass. To achieve delays as large as $\sim 10^{10}$ yr, the mass of this component must be chosen so low that it cannot, by itself, evolve into an explosive state, but, on reaching the end of its main-sequence lifetime, it evolves rapidly to such a large size that it must transfer matter to its companion until the companion evolves into an explosive state.

The first time bomb of this sort to be considered evolves into a CO degenerate dwarf accreting hydrogen-rich matter from a low-mass giant with a helium core. Since the progenitor of a CO dwarf of large enough initial mass must be a fairly massive star ($M \gtrsim 3\ M_\odot$, say, with $t_{\rm MS} \lesssim$ few$\times 10^8$ yr), the time delay is determined primarily by the time necessary for the low-mass component to evolve through the main sequence.

1984ApJS...54..335I

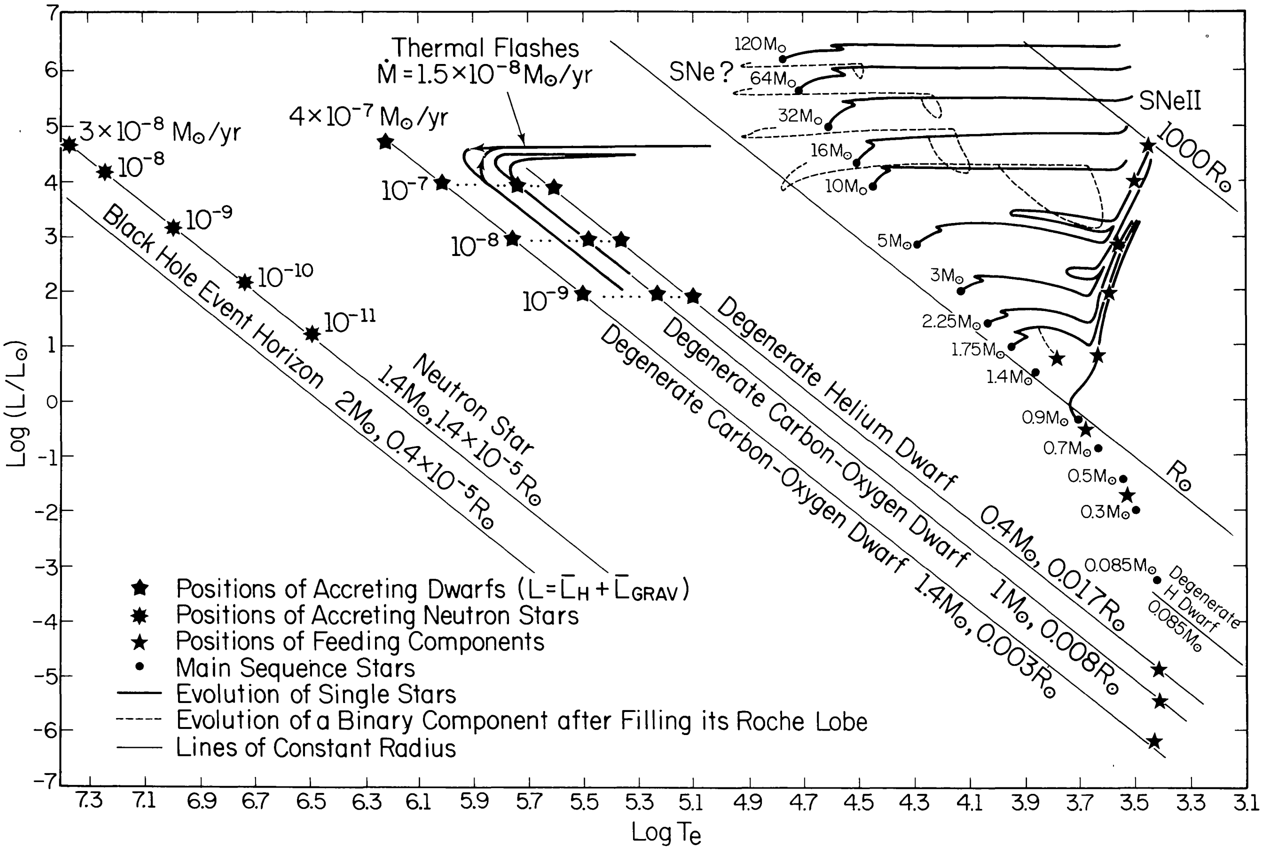

FIG. 5.—Evolution of stars in the H-R diagram. Evolutionary tracks were taken from Stothers and Chin (1979) for a star of mass 120 $M_\odot$, from Tutukov and Yungelson (1973) for stars of mass 64, 32, 16, and 10 $M_\odot$, from Iben (1967) for stars of mass 5, 3, and 2.25 $M_\odot$, and from Mengel *et al.* (1979) for stars of mass 1.75 and 0.9 $M_\odot$. Positions of main-sequence stars with masses 1.4, 0.7, and 0.5 $M_\odot$ have been taken from Mengel *et al.* (1979) and Grossman and Graboske (1971). Evolutionary tracks of accreting carbon-oxygen dwarfs of mass 1.0 and 1.2 $M_\odot$ and accretion rate $\sim 1.5 \times 10^{-8}$ $M_\odot$ yr^{-1} were taken from Iben (1982*b*). "Average" positions of accreting degenerate CO dwarfs with masses 1.4 and 1 $M_\odot$, of an accreting helium degenerate dwarf of mass 0.4 $M_\odot$, and of an accreting neutron star of mass 1.4 $M_\odot$ are shown, as well as the positions of possible mass-donating stars.

Essentially the same mechanism is operating if the accreted matter is abstracted from a wind from a low-mass red supergiant which does not fill its Roche lobe or if the donor is a low-mass main-sequence star in a cataclysmic-like binary. But, in the latter case, the primordial mass of the donor is typically larger than ~ 1.5 $M_\odot$, and thus the clock will run for only $\sim 2 \times 10^9$ yr.

The clock mechanism in the cataclysmic binary scenario is more complicated and consists of three timepieces. The first time scale is the time for the main-sequence evolution of the primordial primary, but, once again, this time is only ~ few × 10^8 yr. The clock is reset after a brief common envelope stage. The second time scale is the time required for the primordial secondary to evolve into Roche lobe contact as a consequence of orbital angular momentum loss due perhaps to a magnetic stellar wind. One may not rely here on the radiation of gravitational waves because the resulting mass accretion rate is too small to lead to an explosive state (see § V*a*). This second time scale depends very much on the orbital period of the system after the common envelope stage, but, on average, it is only on the order of 10^8–10^9 yr (Tutukov 1983*a*). The last time scale is the time which elapses between the onset of mass transfer from the main-sequence star to the degenerate dwarf and is therefore on the order of 0.3 $M_\odot/\dot{M}$. Since $\dot{M}$ must be larger than $\sim 10^{-9}$ $M_\odot$, this means that the third clock may run for only $\sim 3 \times 10^8$ yr, and thus the total time delay between formation and explosion is at most on the order of only 10^9 yr, too short to account for SNeI in elliptical galaxies.

The clock in those scenarios which produce two degenerate dwarfs drawn together by gravitational wave radiation is also threefold. The first time scale is the sum of the main-sequence lifetimes of the two components, and, since each component can begin with a mass typically in the range 0.8–8 $M_\odot$, this time scale can vary from 3×10^7 to greater than 10^{10} yr. The second time scale is determined by the rate at which gravitational radiation drives the degenerate dwarfs closer together, and it can vary from $\sim 10^{10}$ yr for an "initial" semimajor axis of $A_{ff} \approx 3$ $R_\odot$ to $\sim 10^8$ yr for $A_{ff} \approx R_\odot$. The final time scale is determined by the rate at which the heavy disk (which is formed once the less massive of the two degenerates fills its Roche lobe) transfers matter to the remaining degenerate dwarf. This latter time scale is completely unknown at present. In any case, it appears that all scenarios except possibly the cataclysmic and cataclysmic-like ones are able to account for the production of SNeI in elliptical galaxies, even though the

1984ApJS...54..335I

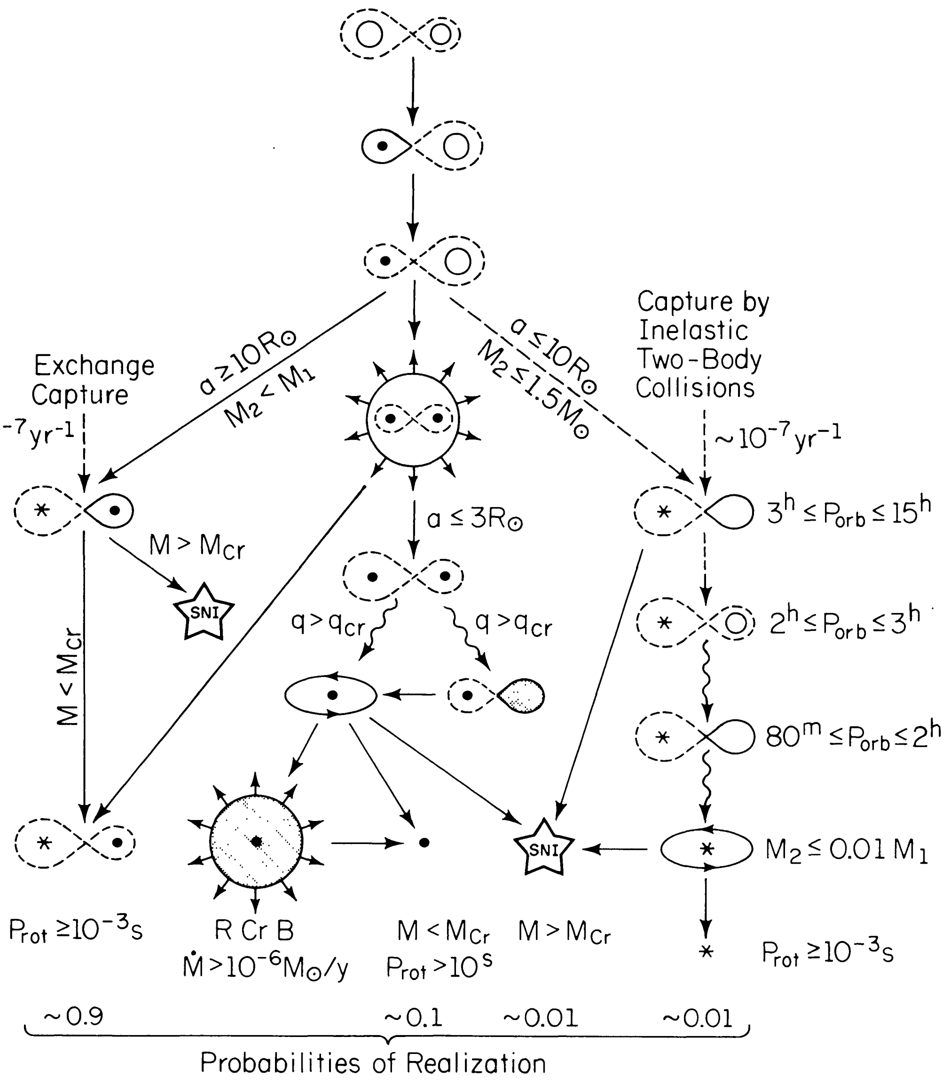

FIG. 6.—Evolutionary scenario for moderate-mass close binaries. Rings are unevolved stars, dots are degenerate helium, carbon-oxygen, or oxygen-neon-magnesium cores, asterisks are degenerate dwarf or neutron stars, wavy lines mark transitions driven by the radiation of gravitational waves, ellipses are heavy disks. Stars represent supernova Type I explosions. Roche lobes are shown by dashed curves (when not filled) or by solid lines (when filled). The probability of realization of different final products is also indicated. The formation rate of moderate mass close binaries is ~1 yr^{-1}.

clock mechanisms are highly variable from one scenario to the next.

Tammann's (1982) data shows, that the frequency of SNeI per unit luminosity increases by between 2 and 4 times in passing from elliptical galaxies to late-type spiral galaxies. This result follows despite statistical indeterminacy and means that some fraction of SNeI (in spiral galaxies, at least half of them) are the product of a relatively recent star-formation process. Double degenerates could explain this result since they provide a time delay from 10^8 to 10^{10} yr, depending on initial orbital period of these systems. The lower frequency of the SNeI in elliptical galaxies could be explained then as a result of the exhaustion of "explosive" binaries according to the law $\nu_{\rm SNI} \approx t_0/t$. This law follows from the assumption that $dN \propto d\log A_{ff}$ and from the fact that the lifetime t of a binary before merging is proportional to A_{ff}^4 (Landau and Lifshitz 1962). We have $dt/t \propto dA/A \propto dN$ or $dN/dt \approx t_0/t$, where t_0 is the characteristic time scale for merging of evolved binaries consisting of two degenerate stars due to the radiation of gravitational waves. From Tammann's (1982) data t_0 would appear to be of the order of the cosmological time scale.

Further insight into the clock properties of double-degenerate scenarios follows from the specifics of our formal transformation relating primordial binary characteristics to initial binary characteristics at the beginning of the evolution controlled by gravitational wave radiation (see Figs. 1–4). We note that, within the framework of our assumed transformations, only the widest primordial systems of the double CO degenerate variety are capable of delaying a SNI explosion as long as ~10^{10} yr. If, for example, we assume that only CO+CO systems produce SNeI, then, within 2×10^8 yr after the cessation of acting star formation, all systems of the primordially "close" variety ($\nu_{\text{COCO}}^{\text{close}} \sim 0.005$ yr^{-1}) will have decayed away, leaving only systems derived from the primordially "wide" variety to become SNeI. Thus, we would guess at least a factor of 2 drop-off in SNI frequency per unit luminosity in passing from spiral to elliptical galaxies, and this is certainly consistent with Tammann's (1982) estimates. We do not wish to place undue emphasis on this apparent numerical agreement, since our formalism is at best a useful guide to qualitative behavior. Note, in particular, that a decrease in assumed α by only a factor of 1.4 permits the longest period representatives of the double helium degenerate scenario to survive for $\sim 10^{10}$ yr, instead of only $\sim 5\times10^{8}$ yr, as derived when $\alpha \approx 1$.

X-ray observations have shown that an accreting degenerate dwarf can radiate X-rays. The supposition that every SN explosion is the result of the accretion of several tenths of a solar mass on a degenerate dwarf therefore has ramifications for the X-ray luminosity of our Galaxy. The efficiency ε of transformation of accretion energy into X-ray radiation depends on many factors. For spherically symmetric accretion and $M_{\text{CO}} \approx 1.4\ M_{\odot}$, Kylafis and Lamb (1982) have found $\varepsilon \approx 0.1$. This estimate is possibly supported by the observed X-ray luminosity of the accreting degenerate dwarf 2505521−63 (Webbink, Rappaport, and Savonije 1983). But Ferland *et al.* (1982) have pointed out that, for many accreting degenerate dwarfs, the observed soft X-ray luminosity is $\sim 10^{2}$–10^{4} times weaker than predicted by a simple accretion model. So, current limits on ε range from 10^{-4} to 10^{-1}. If we assume that the accretion of $\sim 0.3\ M_{\odot}$ by a degenerate dwarf with $R \approx 5\times10^{8}$ cm precedes every SNI explosion, the predicted X-ray luminosity of our Galaxy would be $L_x \approx 2\times10^{7}\varepsilon\ L_{\odot}$ compared with a known luminosity of $\sim 5\times10^{5}\ L_{\odot}$. It is evident that, because of the large uncertainty in ε, it is not yet possible to use this comparison as a limitation on the realization frequency of accretion scenarios for SNeI.

However, X-ray observations place a limit on the probability that the duplicity of a close presupernova binary system is preserved after an SNI explosion. Since the explosion itself does not in general disrupt the binary (see § VII) or a heavy disk around the accreting dwarf (see § VI), we cannot avoid the prolongation of the accretion process after the SN explosion if the exploding star leaves a condensed remnant. If the condensed remnant were a neutron star in every case of a SNI explosion, then the X-ray luminosity of the Galaxy would be 10^{4} times larger than observed. We conclude that essentially all SNI events lead to the complete disruption of the exploding star.

If we were *forced* to choose only one among the scenarios we have explored as the most likely to survive the test of time, we might perhaps settle on the double CO degenerate scenario. Although at the outset we cautioned against taking the similarities among known SNeI as an indication that the immediate precursors of the exploding stars are identical, the list of close similarities is really quite impressive (see, in particular, the similarity between the outburst characteristics of the two SNeI occurring almost simultaneously in NGC 1316: Elias *et al.* 1981), and the deflagrating CO model accounts for essentially all of the common properties. Among the observational features (Branch 1983*a*, *b*) for which the model accounts are the following: (1) light curves can be well accounted for by the decay of ^{56}Ni into ^{56}Co and then into ^{56}Fe; (2) the velocities of ejecta at the maximum-light photosphere are all $\sim 11\times10^{3}$ km s^{-1}, with very little dispersion ($\pm 2\times10^{3}$ km s^{-1})—the measured velocities translate into an energy of $\sim 1.7\times10^{51}$ ergs if the mass of the ejectum is 1.4 $M_{\odot}$, and this corresponds to the energy released by the conversion of $\sim 1.4\ M_{\odot}$ of ^{12}C and ^{16}O into ^{56}Ni; (3) spectra of over 50 examples are practically identical and show evidence for huge abundances of Ni and Fe and for the presence of intermediate mass elements such as Si, S, and Ca; (4) none of the known examples show definitive evidence for hydrogen in their spectra at any time; and (5) there is no direct and incontrovertable evidence for the presence of He in any spectra.

Item 4 is, of course, somewhat damaging for all scenarios which rely on the transfer of hydrogen-rich matter to a degenerate companion; one might expect that the expanding supernova envelope will ablate at least some hydrogen-rich matter from its companion and that the incorporation of only a few hundreths of a solar mass of hydrogen into this envelope is sufficient for detectability. On the other hand, only a small fraction of the expanding envelope will pass over the companion, and the incorporated hydrogen may be confined to this fraction and thus escape detection in most instances.

Item 5 is not of itself a compelling piece of evidence against scenarios involving the accretion of helium-rich matter; however, one might couple this evidence with the theoretical indications that degenerate helium dwarfs which accrete enough helium experience a detonation which produces ejection velocities $\sim 5\times10^{3}$ km s^{-1} larger than the observed mean of 11×10^{3} km s^{-1} and thus have a slightly stronger argument against the double degenerate helium dwarf scenario. An argument that may be leveled against most (double) detonation models resulting from accretion of helium by a degenerate CO dwarf is that the theoretical light curves tend to have a peak half-width that is only about one-half of that defined by typical observed SNeI light curves (Nomoto 1983*b*; see also Weaver, Axelrod, and Woolsey 1980). A final argument against both helium-accretion scenarios is that a detonation converts all of the matter in the exploding star into iron peak elements, and this contradicts the fact that lines of Si, S, and Ca appear in SNeI spectra (item 3 in the list of similarities). However, experiments in progress indicate that, if rotation is taken into account, detonations do not occur (Arnett 1983), and this last objection is overcome. Thus, once again, it is clear that further numerical exploration which takes into account all relevant physical processes must be pursued before any given scenario can be confidently either excluded or accepted.

Nevertheless, given the results of extant investigations and given the bulk of the circumstantial evidence, the carbon accretion scenario would appear at the present writing to be the most likely survivor among the double-degenerate scenarios. It is possible, however, that it too may suffer from a fatal defect. If accretion of CO-rich matter occurs at rates significantly less than $\dot{M}_{\text{steady}} \approx (M_{\text{dwarf}}/M_{\odot} - 0.5)10^{-6}\ M_{\odot}$ yr^{-1}, deflagration will occur when the central density ρ_c exceeds $\sim 9\times10^{9}$ g cm^{-3} (Iben 1982*a*) and the ratio of ^{58}Fe to ^{56}Fe appearing in the final ejectum exceeds by over a factor of 10 the solar ratio (Woosley 1983). If $\dot{M} \approx 10^{-6}\ M_\odot$ yr^{-1}, the deflagration will begin with $\rho_c \approx 4\times10^9$ g cm^{-3} (Iben 1982*a*), and the ratio of ^{54}Fe to ^{56}Fe in the final ejectum exceeds the solar system ratio by over a factor of 4 (Woosley 1983). Only if deflagration begins at $\rho_c \approx 2\times10^9$ g cm^{-3}, corresponding to $\dot{M}$ significantly larger than $\dot{M}_{\rm steady}$, are the iron isotopes produced in the solar system distribution. There are, surely, many uncertainties in the theoretical calculations, but, if we were to accept the current results at face value, then survival of any of the scenarios which rely on carbon deflagration depends on achieving accretion rates considerably in excess of $10^{-6}\ M_\odot$ yr^{-1}. This immediately excludes all hydrogen accretion scenarios, but not helium or carbon-oxygen accretion scenarios. A possible conclusion is that we have hit upon a property of heavy disks—the rate of outflow from such disks must be greater than $10^{-6}\ M_\odot$ yr^{-1}. Noting that there exist at present no theoretical arguments to gainsay this inference, adding up the arguments against hydrogen and helium accretion scenarios and observing that the formal realization frequency of CO + CO presupernova systems (~ 0.008 yr^{-1}) is close to the observed SNI frequency (~ 0.01 yr^{-1}), we are not excessively uncomfortable with our forced choice of the double CO degenerate scenario.

Although we have stressed the basic uniformity both of observed SNI characteristics and of the characteristics of explosions that terminate "successful" systems of the CO + CO variety, it is important to recognize that there are indications of real departures from uniformity among observed SNI light curves (Barbon, Ciatti, and Rosino 1973; Barbon 1978, 1980) and that, within the framework of the CO + CO scenario, there are several parameters, variations in which could produce some diversity in model explosion characteristics. For example, the central density and the internal temperature distribution in a model degenerate CO dwarf at the time of explosion will vary depending on the total mass and the mass ratio of the two dwarfs when merging begins. The central density and the interior temperature distribution will also depend on the time that elapses betwen the formation of the more massive degenerate dwarf and the beginning of the accretion episode (i.e., on how long the more massive dwarf has been able to cool before accretion heating begins) and on the rotation rate of the dwarf. These variations in interior characteristics at the onset of explosion will introduce variations in the final nucleosynthetic product and possibly also introduce variations in the "speed class" of the model SNI.

Further, the character of the model light curve will depend not only on the initial conditions in the exploding dwarf, but also on the distribution of and total mass in the "remnant" disk and on the line-of-sight aspect angle presented by the plane of the remnant disk at the moment of explosion. Thus, even though the overall behavior of model SNI explosions generated by representatives of the CO + CO scenario may be dominated by the similarities among all immediate precursor systems, variations of the order advocated by some observers (e.g., Barbon 1980; Branch 1983*b*) are not excluded.

There is one more attractive possibility for obtaining a compact presupernova star that has not been discussed in previous sections because it involves fairly massive binaries and thus violates one of the criteria developed at the outset for finding likely precursors of SNeI, namely, that such precursors must not be preferrentially located in spiral arms. We nevertheless discuss it because the immediate presupernova star which it generates has a mass close to the Chandrasekhar limit and is very compact so that, on explosion, it might be expected to produce a light curve similar to that of an SNI (Arnett 1979). Helium stars with masses $M_{\rm He}/M_\odot = 0.1\ (M/M_\odot)^{1.4}$ form in the course of the evolution of massive ($M > 10\ M_\odot$) close binary components (Tutukov and Yungelson 1973). After exhausting the helium fuel in their cores, stars in which the mass of the helium core is initially $M_{\rm He} \lesssim 2.5\ M_\odot$ expand once more until mass exchange reduces their masses below the Chandrasekhar limit (Paczyński 1971*b*; Nomoto and Sugimoto 1977). So, only stars with an initial mass larger than ~ 10 $M_\odot$ create remnants with $M \gtrsim 2.5\ M_\odot$ and therefore remain compact, probably up to a supernova explosion which might follow the SNI pattern (Arnett 1979). About 0.4 of all stars with an initial mass larger than 10 $M_\odot$ can finish their evolution in this way, and, using equation (1) with $\Delta \log A \approx 2$, $q \approx 1$, $M_A \approx 10\ M_\odot$, and $M_B \approx 100\ M_\odot$, we estimate the formation frequency of such systems to be ~ 0.008 yr^{-1}. Since their age is less than ~ 2×10^7 yr, such systems can have nothing to do with SNeI in elliptical galaxies.

However, the frequency estimate of 0.008 yr^{-1} is perhaps the most reliable of all those which we have thus far made, and one wonders if this scenario contributes to SNI explosions in galaxies where active star formation is taking place. Two features of the model argue against this possibility. First, we expect considerable hydrogen to remain in the envelope of the immediate supernova precursor (Tutukov and Yungelson 1973), and, second, the flux of energy in the visible is expected to be one to three orders of magnitude less at maximum than is the case even for observed SNeII (Chevalier 1976*a*, *b*; Litvinoa and Nadyozin 1983). This latter result follows because a neutron star remnant is formed and the photon-diffusion time through the compact helium-rich envelope is less than the expansion time of the envelope (Woosley, Weaver, and Taam 1980). Hence, observational selection will reduce the frequency of observed counterparts to over a factor of 10 less than the predicted one. Some support for this view comes from several properties of the Cas A supernova remnant. Ashworth (1980) has found that in the early 1600s Flamsteed observed a "new" star in the direction of this remnant and compared its brightness with that of well-known stars. Ashworth concludes that, at maximum, the Cas A supernova was ~ 4 mag dimmer than a typical SNII. Further, the abundances in the slow-moving flocculi in the vicinity of the Cas A remnant are reminiscent (N/H > 10, He/H > 2) of abundances at the surfaces of Wolf-Rayet stars of type *N*, and these stars are precisely those which are formed in the scenario under discussion (Lamb 1978).

We emphasize that the distribution of SNeII over maximum luminosity (see Tammann 1982) is so wide (~ 7 mag) and suffers so heavily from incompleteness on the low-luminosity side that all of the compact helium stars which explode as supernovae could define the low-luminosity tail of the observed distribution.

In the course of our exploration, we have commented upon several numerical problems the solution of which would be helpful in understanding late stages of binary evolution both for its intrinsic interest and for better delineating those evolutionary paths that indeed lead to a supernova explosion. We summarize these comments by proposing a concrete set of numerical studies:

1. Evolution in a close binary consisting of components of $\sim 5\text{–}12\ M_\odot$ filling their Roche lobes 2 times and formation of CO dwarfs with mass in the range $1\text{–}1.4\ M_\odot$.

2. Exploration of the process whereby a degenerate CO dwarf accretes stellar wind matter. The objective is to estimate more precisely the realization frequency of such a supernova scenario, which our crude estimates suggest could be quite large.

3. Mass exchange on a thermal time scale in binaries consisting of a degenerate CO (possibly helium) dwarf and 1–2 $M_\odot$ main-sequence primary filling its Roche lobe.

4. The effects of accretion of pure helium on helium and on degenerate CO dwarfs and of the accretion of a carbon-oxygen mixture on a degenerate CO dwarf to find criteria for a supernova-like explosion in such systems for different initial masses, for different initial internal temperatures of the accreting dwarfs, and for a wide range of accretion rates.

5. Evolution of close binaries during the common envelope stage in an attempt to estimate the final semimajor axis which emerges after this stage.

6. Exploration of the possibly SNI-like thermonuclear explosion of a compact massive ($M \gtrsim 1.5\ M_\odot$) Ni-Fe remnant of a star of initial mass exceeding $\sim 10\ M_\odot$, which has been produced by mass transfer and nuclear evolution in a close binary.

7. Exploration of the interaction of the supernova ejectum with a Roche lobe filling red dwarf of mass $\simeq 0.3\text{–}1\ M_\odot$, for the case when the kinetic energy of the impacting portion of the ejectum is comparable to or exceeds the binding energy of the red dwarf.

8. The effect of accretion of hydrogen on an old, cold, degenerate CO or helium dwarf when the accretion rate exceeds the rate at which matter is burned quiescently in an AGB star. The aim is to study the process of common envelope formation.

9. Mass exchange process in a system consisting of two degenerate CO dwarfs of similar mass.

10. Mass exchange in binaries consisting of a degenerate helium dwarf of mass $0.4\text{–}0.5\ M_\odot$ and a Roche lobe filling main sequence star of mass $\sim 1\ M_\odot$.

11. A determination of the mass limits for the formation of degenerate He, CO, and ONeMg cores when these cores are being formed in stars while they are transferring mass to a companion in a close binary system. Of particular importance is the determination of limits when the core-developing star has an initial main-sequence mass in the range $\sim 8\text{–}14\ M_\odot$.

12. A study of the explosion to be expected when an degenerate ONeMG dwarf of a given initial mass accretes H, He, or C and O at various rates. One may expect a wide range of explosion types including collapse to a neutron star, single outward detonation with a white dwarf remnant, double detonation, and central deflagration.

13. A search for the lower limit on the mass of a degenerate helium dwarf which reaches the explosive state. This limit will depend on the initial thermal profile in the dwarf and on the accretion rate.

On the observational side, the most pressing need is a concerted program for determining the primordial distribution over q of binaries with separations in the range $A \approx 10^2\text{–}2\times10^3\ R_\odot$.

In closing, we emphasize once again that our estimates of formation frequencies are exceedingly crude, primarily because of the absence of a theory of common envelope evolution and because of the absence of information about the distribution of wide ($A \approx 10^2\text{–}2\times10^3\ R_\odot$) systems over the ratio of component masses. Table 1 shows that, because of all the uncertainties, we cannot identify one leading scenario for explaining SNeI. Many of our estimations are heavily influenced by our arbitrary assumption that $dN/dq \approx 1$. In truth, we know almost nothing about the relative frequency of close binary systems with an initially low ($\lesssim 0.3$) mass ratio. The one scenario whose frequency is not strongly dependent on an estimated distribution over q is that of a CO degenerate dwarf fed by a red (super)giant which does not fill its Roche lobe. But this scenario has its own unresolved uncertainties. Therefore, in spite of the rather long list of scenarios which we have compiled, we are still not confident that that list is long enough to include the real scenario which describes evolution to the SNI stage. Thus, a vigorous research for new "explosive" stages in the course of the evolution of single as well as of double stars should continue.

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ERRATUM

In the paper by Sidney B. Parsons (*Ap. J. Suppl.*, **53**, 553 [1983]), the first page of Table 2 (p. 559) was incorrectly identified as the continuation of Table 1. The first two pages of Table 2, correctly labeled, follow.