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A MODEL OF THE GALACTIC X-RAY BINARY POPULATION. II. LOW-MASS X-RAY BINARIES IN THE GALACTIC DISK[1]

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ABSTRACT

A numerical scenario program is used to find those systems which are permitted by the modern theory of binary star evolution and which consist of a low-mass donor filling its Roche lobe and transferring mass to a neutron star or black hole accretor (with mass transfer being driven by evolutionary expansion of the donor, a magnetic stellar wind, or gravitational wave radiation). The birthrate of low-mass X-ray binaries (LMXBs) in the Galactic disk is found to be $\nu \sim (1\text{–}4) \times 10^{-5}$ yr^{-1} when the parameter which measures the efficiency of orbital shrinkage in common envelope events is $\alpha_{CE} = 0.5\text{–}1.0$.

The number of LMXBs of various kinds in the Galactic disk is calculated in the approximation that mass is transferred conservatively. Algol-like LMXBs (subgiant or giant donor and black hole or neutron star accretor) are formed at orbital periods in the range 5–40 days, and their number is $\sim$1200 when $\alpha_{CE} = 0.5$ and is $\sim$6400 when $\alpha_{CE} = 1.0$. Cataclysmic variable-like (CV-like) LMXBs (main-sequence donor) are formed at periods longer than 3.8 hr, evolve at first under the influence of a magnetic stellar wind, and then eventually evolve to shorter periods under the influence of gravitational wave radiation. The number of long-period CV-like LMXBs is $\sim$190 when $\alpha_{CE} = 0.5$ and is $\sim$1900 when $\alpha_{CE} = 1.0$. The number of short-period (and dim) CV-like LMXBs is $\sim$3600 when $\alpha_{CE} = 0.5$ and is $\sim$30,000 when $\alpha_{CE} = 1.0$.

Since only $\sim$100 bright disk LMXBs are observed, possibilities for reducing the theoretical birthrate, the theoretical lifetime, or both are explored. It has been suggested before that an irradiation-induced wind can carry away from the system an order of magnitude more mass than is transferred to the accretor. We use a simple model, which fits to within a factor of 2 the results of detailed calculations in the literature, to show that the lifetime of the LMXB phase is reduced by a factor of 6–60 relative to that given in the mass-conservative approximation, and that the number of theoretically estimated LMXBs is reduced by a similar factor.

In the irradiation-induced wind scenario, the mass transferred (0.01–0.1 $M_\odot$) is sufficient for the neutron star to achieve rotation periods in the range of observed millisecond pulsars (MSPs). For Algol-like systems in which the donor evolves into a helium white dwarf, the final relationship between rotation period and orbital period is consistent with the relationship defined by evolved binary MSPs in the Galactic disk. The predicted ratio of single to binary MSPs in the disk is consistent with the observed ratio. We interpret these consistencies to mean that LMXBs are probably the major precursors of MSPs.

If the relativistic component in an LMXB typically accretes about 10% of the mass lost by the donor, the semiempirical birthrate of bright LMXBs is $\sim(3\text{–}30) \times 10^{-6}$ yr^{-1}, which is consistent with the theoretical estimates produced by the scenario program. Semiempirical estimates of the birthrate of MSPs give $\nu \sim (4\text{–}20) \times 10^{-6}$ yr^{-1}, close enough to the estimates of the birthrate of LMXBs to suggest a causal connection between LMXBs and MSPs. All birthrate estimates are now within a factor of $\sim$3 of 10^{-5} yr^{-1}, but large uncertainties in all estimates remain.

The scenario program produces a bimodal distribution of LMXBs with respect to their peculiar space velocities. Systems in which a neutron star has been formed in consequence of an accretion-induced collapse have low peculiar space velocities ($\sim$10–16 km s^{-1}), and systems arising from initially massive binaries with a high initial component mass ratio achieve peculiar space velocities of $\sim$40–100 km s^{-1} as a result of recoil in response to mass loss during the supernova explosion which produces a neutron star or black hole. The supernova explosion is assumed to be spherically symmetric, with the remnant relativistic star having the same instantaneous orbital velocity as its precursor. The facts that agreement between observed and model LMXBs with regard to space distribution in the z-coordinate is achieved and that agreement between observed MSPs and model LMXBs with regard to peculiar space velocities is also achieved demonstrate that it is not necessary to invoke extra, ad hoc "kicks" associated with an asymmetric supernova explosion in order to achieve consistency with the observations.

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According to the model, the birthrate of neutron stars with envelopes at the base of which a nuclear fuel is burning (Thorne-Żytkow objects [TŻOs]) is much larger ($\sim$0.0015 yr^{-1}) than the birthrate of MSPs, and one may infer that TŻOs do not, as a rule, evolve into MSPs, but possibly evolve into typical radio pulsars. A consideration of angular momentum transfer supports this inference.

The predicted birthrate of systems which consist of a neutron star and a Roche lobe filling helium star is in the range $(1.1\text{–}2.3) \times 10^{-4}$ yr^{-1}, but an initial spike in the mass-transfer rate probably causes these systems to become TŻOs rather than LMXBs. Further study of these systems is necessary to clarify their properties.

Subject headings: accretion, accretion disks — binaries: general — stars: formation — stars: neutron — stars: statistics — X-rays: stars

1. INTRODUCTION

This is the second of two papers exploring the origin and properties of binary X-ray sources in the Galactic disk. Paper I (Iben, Tutukov, & Yungelson 1995) describes some aspects of a numerical scenario program used to estimate birthrates of evolved binary systems and demonstrates that theoretical expectations regarding the number and evolutionary significant characteristics of bright high-mass X-ray binaries (HMXBs) are, in many respects, consistent with the observations. This paper (Paper II) focuses on low-mass X-ray binaries (LMXBs) belonging to the Galactic disk population.

The major distinction between HMXBs and LMXBs has to do with the nature of mass transfer, and this in turn depends mainly on the mass of the optical companion. In HMXBs, mass transfer is a consequence of accretion by the relativistic component (neutron star or black hole) of matter from a wind which the optical component emits by a mechanism (e.g., a radiatively driven wind) which does not depend on its being in a binary. However, the rate of mass transfer is very sensitive to how close the donor is to filling its Roche lobe. In most LMXBs, the mass donor appears to fill, or nearly fill, its Roche lobe, and the mechanism for mass transfer has traditionally been associated with one of several processes which force the donor to transfer matter through the Lagrangian point L1. In the plane of donor mass versus orbital period (Fig. 1), we display where LMXBs might be expected on theoretical grounds (see the figure legend). The locations of some X-ray binaries from the catalog of Aslanov et al. (1989) are superimposed.

The physical properties and origin of LMXBs have been extensively studied and discussed over the past three decades (e.g., Shklovsky 1967; McCluskey & Kondo 1971; Fabian, Pringle, & Rees 1975; Taam, Flannery, & Faulkner 1980; Tutukov 1981; Bradt & McClintock 1983; Joss & Rappaport 1984; van den Heuvel 1987a,b; Verbunt & Rappaport 1988; Cowley et al. 1990; Bhattacharya & van den Heuvel 1991; Webbink 1992; Naylor & Podsiadlowski 1993; Verbunt 1993; Webbink & Kalogera 1994). The Galactic distribution of LMXBs, as defined by entries in the van Paradijs (1995) catalog, is shown in Figure 2. A significant fraction of the sources is concentrated toward the Galactic center, with approximately half of the total of $\sim$100 sources being within 10° in latitude and 15° in longitude of the Galactic center. Most of these "bulge" sources are not seen at optical wavelengths, thus reinforcing the impression from their angular distribution that they are at a large distance; their apparent luminosity is reduced because of the fall-off in flux, which decreases as the inverse square of the distance, and because of obscuration by intervening matter, the amount of which increases roughly linearly with the distance.

As seen in Figure 2, LMXBs which have observed optical radiation (*filled circles and crosses*) are more widely dispersed about the center and above and below the plane of the Galaxy than those which are not seen at optical wavelengths (*small dots*). This suggests that the majority of the optically detected sources belong to a Galactic disk population. The optically relatively bright sources which are at low latitude are relatively evenly distributed in longitude, consistent with their being fairly uniformly distributed about the Sun in the Galactic plane. The "horizon" for viewing LMXBs in the disk at optical wavelengths is about 2–3 kpc (see Paper I). The fact that some of the bright sources are at fairly high latitudes is an indication that, as a group, LMXBs have peculiar space velocities significantly larger than the 10–16 km s^{-1} which is characteristic of the unevolved massive progenitors of neutron stars in the Galactic disk.

In summary, observed LMXBs can be divided into two populations (see also Naylor & Podsiadlowski 1993): a bulge population and a disk population. Many of the known members of the disk population are within a few kiloparsecs of the Sun (e.g., Sco X-1, Nova Mon). If it is assumed that the volume inside the visibility horizon is 5% of the Galactic volume, there are a few hundred disk LMXBs in the Galaxy (Fig. 2). As in the case of HMXBs, only about 100 of them are intrinsically bright, persistent X-ray sources, with a luminosity not infrequently approaching the Eddington limit for a neutron star accretor. The bulge population consists of systems within about 1.5 kpc of the Galactic center, and the disk population consists of systems that can be at any radial distance from the center and at average distances above the Galactic plane which (although poorly known because of uncertainties in distances) can be as large as 0.7 kpc (Naylor & Podsiadlowski 1993). The rather large thickness of the disk defined by the disk population is probably due to a high space velocity ($\sim$40–100 km s^{-1}) imparted during the formation of the neutron star or black hole component (see, e.g., Fig. 6 in Paper I). The relatively long lifetimes of the precursors of LMXBs (before the low-mass donor makes Roche lobe contact) and the large space velocity are responsible for the fact that the disk LMXBs have "forgotten" where they were born and are now rather uniformly distributed throughout a thick disk.

It has been suggested that field LMXBs may have been formed in globular clusters by two- and three-body interactions, and then ejected from these clusters (Fabian et al. 1975). However, globular clusters are highly concentrated toward the center of the Galaxy (half of them are within 1.5 kpc of the

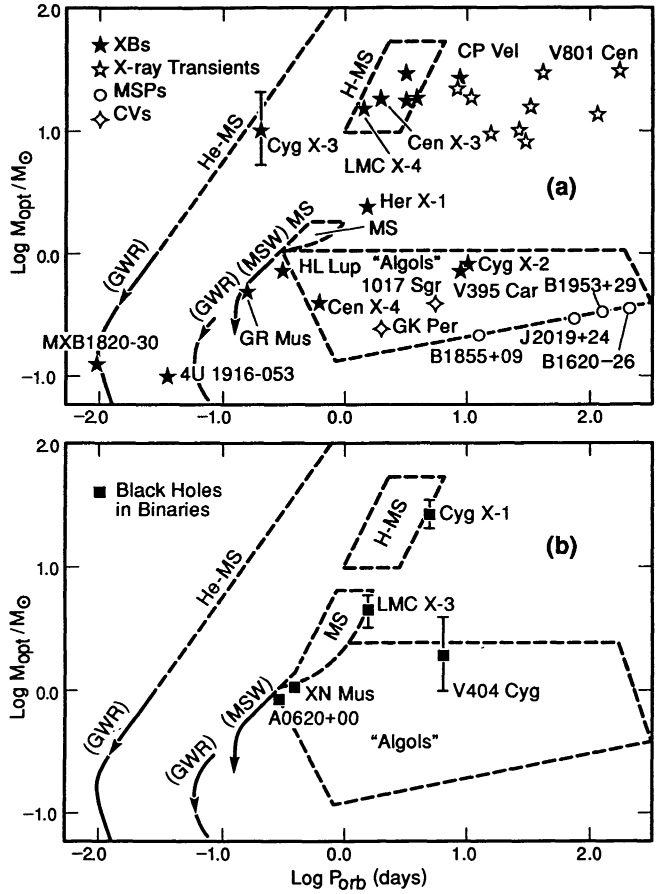

FIG. 1.—Mass of the optical component vs. orbital period for X-ray binaries in which the accretor is a neutron star (*upper panel*) or a black hole (*lower panel*); binary parameters are taken from Aslanov et al. (1989) and, for some HMXBs with black holes, from Tutukov & Cherepashchuk (1992). In the upper panel, stationary (persistent) sources are shown as filled five-rayed stars, and hard transients are shown as open five-rayed stars. Also shown in the upper panel are characteristics of two long-period cataclysmic binary variables (open four-rayed stars) and characteristics of binary radio pulsars with eccentricities smaller than $e = 0.03$ which are probable successors of "Algol-like" LMXBs (*open circles*). Solid lines show theoretical evolutionary tracks (Iben & Tutukov 1984b; Tutukov & Fedorova 1989) for LMXBs driven by GWR (donor a helium main-sequence star [He-MS] or a hydrogen main-sequence star [MS] of mass less than 0.3 $M_\odot$) or by a magnetic stellar wind (MSW) (donor a hydrogen main-sequence star of mass in the range 0.3–1.0 $M_\odot$). Closed polygons outline theoretical positions of binaries with different types of donors: a hydrogen main-sequence star (MS) transferring matter on a nuclear evolution timescale; an evolved hydrogen main-sequence star (H-MS) transferring matter with a wind; and a subgiant or giant with a degenerate helium core (Algol-like donor) transferring matter on a timescale determined by the rate of shell hydrogen burning and (frequently) by the operation of a MSW. The position of Algol-like donors is shown according to evolutionary computations of Iben & Tutukov (1984b). Note that the locations of low-eccentricity binaries containing millisecond radio pulsars are near the lower boundary of the region for post-Algols (Iben & Tutukov 1994b); orbital periods are given by Taylor, Manchester, & Lyne (1993).

center), and ejected LMXBs would tend to have the same space and velocity distributions as the clusters. Thus, formation in and ejection from clusters is unlikely to be a mode for forming disk LMXBs, but it could account for a significant fraction of bulge LMXBs.

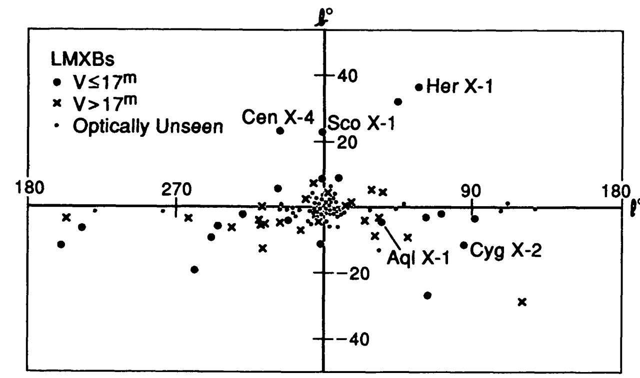

FIG. 2.—Positions of LMXBs on the sky in Galactic coordinates, as given by the van Paradijs (1995) catalog. To distinguish bright and therefore relatively close binaries from dim and distant ones, the apparent visual brightness is indicated by symbols described the upper left-hand corner of the figure.

In Figure 3, visual magnitudes of LMXBs are plotted against their maximum X-ray fluxes as given by van Paradijs (1995). The visual magnitudes V_{obs} have been "dereddened" using $A_V = 3E_{B-V}$. For systems with $V_{obs} < 17$ mag, the average A_V is 1.4 mag, and for systems with $V_{obs} > 17$ mag, the average A_V is 2.1 mag. In the dereddened coordinate, there are approxi-

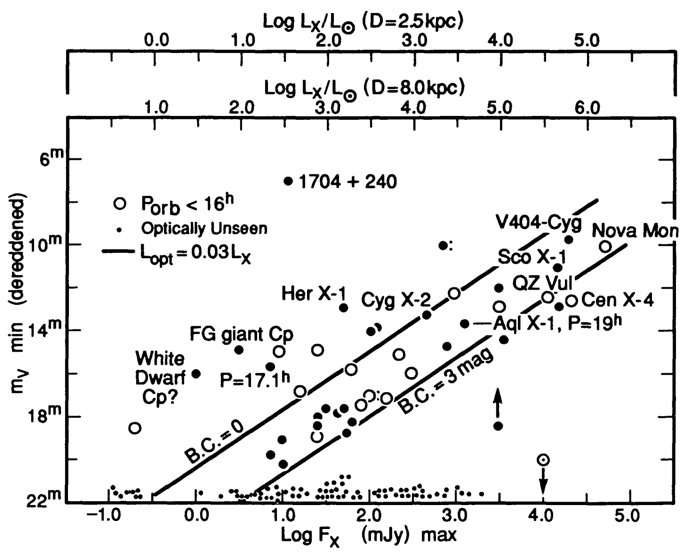

FIG. 3.—Apparent visual magnitude of LMXBs vs. their X-ray flux, as given by the van Paradijs (1995) catalog. Optical magnitudes have been corrected for reddening. For variable sources, F_X denotes the maximum flux and m_V the minimum magnitude. Open symbols denote LMXBs with orbital period smaller than 16 hr; filled symbols denote those with unknown orbital period or with orbital period larger than 16 hr. Dots located at the bottom of the figure represent LMXBs which are not seen at optical wavelengths. The Sun symbol (*circled dot*) indicates the apparent brightness of the Sun if it were placed at the Galactic center and if the line of sight to the Sun were dust free. The lines labeled "$L_{opt} = 0.03L_X$" gives the location of hypothetical LMXBs in dust-free space when the mass donor and the accretion disk transform 3% of the emitted X-ray flux into optical (and UV) radiation; for the upper line, the bolometric correction is B.C. = 0.0, and for the lower one it is B.C. = 3 mag.

mately as many brighter than 15 mag as there are dimmer than this. Open symbols in Figure 3 identify those LMXBs which are known to have an orbital period less than 16 hr. In these systems, the semimajor axis is less than $\sim 4.2\ R_{\odot}$ and the radius of the Roche lobe about the optical component is less than $\sim 1.8\ R_{\odot}$, implying that the donor is either a main-sequence star or is very close to the main sequence.

Although the dispersion is large, there is some indication of a correlation between the optical brightness and the X-ray flux. This could mean that the optical luminosity of these stars is a result of reprocessing of X-ray emission by an accretion disk (e.g., van Paradijs & McClintock 1994) or by the donor, or both. The two lines drawn in Figure 3 have the slope $dm_V/d\log F_X = -2.5$, a slope which would be expected if the optical bolometric flux was proportional to the first power of the X-ray flux and if the bolometric correction was the same for all sources. The normalization of the two lines is discussed in § 6.2.

Many of the X-ray–brightest sources plotted in Figure 3 are actually "transients," or X-ray novae which brighten simultaneously at optical and at X-ray wavelengths by many orders of magnitude, with the duty cycle being typically quite small. In these cases, the visual magnitude at maximum brightness is plotted against maximum X-ray flux. Nova Mon is a good example of an X-ray nova, having undergone optical outbursts in 1917 and 1985, with the 1985 X-ray outburst lasting approximately 15 months (Cowley 1992 and references therein). Of the 22 systems for which $V_{\rm obs} < 17$ mag, 14 are transients. Of the six X-ray–brightest sources, all but one, Sco X-1, are transients. Of the 21 systems with $V_{\rm obs} > 17$ mag, only four have shown pronounced variability.

According to the estimates of many authors, one of the darkest corners remaining in our understanding of binary star evolution is the quantitative aspect of the relationship between LMXBs and millisecond pulsars (MSPs). Estimates of numbers and lifetimes of MSPs and LMXBs in the Galactic disk have suggested a birthrate for MSPs of $\nu_{\rm MSP} \sim (3\text{–}12) \times 10^{-5}$ yr^{-1} (e.g., Kulkarni & Narayan 1988; Narayan et al. 1989; Lyne 1994) and a birthrate for LMXBs of $\nu_{\rm LMXB} \sim (1\text{–}5) \times 10^{-6}$ yr^{-1} (Kulkarni & Narayan 1988). The birthrate of LMXBs would appear to be an order of magnitude smaller than the birthrate of MSPs, an inference which has been drawn many times by others (e.g., Tavani 1991a, b). The same discrepancy arises also in globular clusters (e.g., Bailyn & Grindlay 1990; Chen, Middleditch, & Ruderman 1993). Bailyn & Grindlay (1990) estimate that the birthrate of LMXBs is close to the frequency of inelastic collisions (which result in capture or exchange) between neutron stars and low-mass stars, but that the birthrate of MSPs exceeds both the birthrate of LMXBs and the inelastic collision frequency by two orders of magnitude.

The initial motivation for the work presented in this paper was to use a scenario program to obtain an estimate of the birthrate of LMXBs which does not depend on an estimate of lifetime. The scenario program adopted has already produced models for the birthrates and evolutionary properties of many families of binary star populations, including HMXBs (see Paper I), that are in reasonable agreement with the observed properties of these families. Included in the program are many evolutionary paths that produce LMXBs, which may or may not evolve into MSPs, and many evolutionary paths that do not pass through the LMXB phase but might lead to the spin-up of an old pulsar. It is found that configurations in which the companion of the neutron star or black hole component can transfer mass through the Lagrangian point L1 are born at a rate in the range $(1\text{–}4) \times 10^{-5}$ yr^{-1}, with an uncertainty of a factor of 3 in either direction.

The main problem which arises in a comparison between theory and observation is that three ingredients are required—the number N of members of a given family, the lifetime τ of a typical member of this family, and the birthrate ν of a typical member—but observation (after consideration of selection effects) can give a reliable estimate only of N, and theory (at present) can give a reliable estimate only of ν. In order to transform numbers into a birthrate or a birthrate into numbers, τ must somehow be estimated. In the case of LMXBs, if it is assumed that all or almost all donor matter is transferred through an accretion disk onto the accretor (the "conservative" approximation), this estimate is relatively straightforward, since accretion rates can be estimated from the observations and the amount of mass transferred can be estimated from the theory.

Multiplying the theoretical birthrate for disk LMXBs by a lifetime estimated in the conservative approximation, the estimated number of LMXBs is 10–100 times larger than the observed number. A nonconservative approach, which takes into account a stellar wind induced by the irradiation of the donor by X-rays from the accretor, produces a theoretical number of disk LMXBs comparable to the observed number. The reduced lifetime given by the nonconservative approximation ameliorates the contradiction between semiempirical estimates of birthrates of LMXBs and MSPs.

In § 2 ways are described in which mass may be transferred by a Roche lobe filling secondary in an LMXB, in the approximation that the energy from the mass accretor which is intercepted by the donor has no effect on the rate of mass loss from the donor. Overall results for birthrates and numbers in the conservative approximation are given in § 3. In §§ 4 and 5 the birthrate of disk LMXBs produced in massive binaries with a large initial mass ratio is estimated; in § 4 the neutron star or black hole component is formed by the direct collapse of the Fe-Ni core of an evolved massive star; in § 5 the neutron star component is produced by the electron-capture–induced collapse of an accreting ONe white dwarf. In § 6 uncertainties in the model estimates of birthrates are described and a model for the wind from the secondary which is induced by the absorption of a fraction of the X-ray radiation from the primary is discussed. In § 7 the connection between LMXBs and MSPs is discussed, and in § 8 a concluding summary is given.

2. MODES OF MASS TRANSFER IN THE CONSERVATIVE APPROXIMATION

There are three channels for forming LMXBs (e.g., Bhattacharya & van den Heuvel 1991): (1) capture of a neutron star (black hole) during two- or three-body collisions in dense stellar systems; (2) evolution of a primordial binary consisting of an initially massive star and a low-mass companion; and (3) formation of a neutron star in a moderate-mass system due to the accretion-induced collapse of an ONe degenerate dwarf.

The first channel explains the presence of LMXBs in globular clusters (Hills 1975; Fabian et al. 1975; Fullerton & Hills 1982; Ray, Kembhavi, & Antia 1989; Hut & Verbunt 1985) and possibly in the Galactic bulge (Iben & Tutukov 1984a). In this paper, attention is focused on some details of the other two channels, since they are the most likely ones for explaining how LMXBs are formed in the Galactic disk.

The numerical scenario program follows the evolution of a large grid of model binary stars, and selects those theoretical configurations in which mass transfer onto a neutron star or black hole occurs at rates which, in the mass-conservative approximation (which is later abrogated), are appropriate for understanding the observed properties of LMXBs. In Figure 4 the mass-transfer rates for different modes of mass transfer are displayed as a function of orbital period for systems in which the accretor is of mass ~ 1.2–$1.4\ M_\odot$ (a massive degenerate dwarf or a neutron star) and the donor fills its Roche lobe. The modes of mass transfer and the theoretical rates have proved successful in understanding the evolution of cataclysmic variables (CVs), which are low-mass systems in which the accretor is a white dwarf, and they have been applied also to discussions of LMXB evolution.

The donor in Figure 4 is (1) a main-sequence star of mass in the range 0.3–1 $M_\odot$ forced into Roche lobe contact by a magnetic stellar wind (MSW); (2) a main-sequence star of mass in the range 0.1–0.3 $M_\odot$ forced into Roche lobe contact by gravitational wave radiation (GWR); (3) a main-sequence star of mass in the range 1–1.7 $M_\odot$ which fills its Roche lobe because its radius attempts to grow in response to nuclear transformations in its interior; (4) a main-sequence or near main-sequence star that, being initially more massive than its companion, transfers matter on the thermal timescale of its envelope; (5) a subgiant or giant with a degenerate helium core which transfers mass either because of the growth in mass of the helium core due to hydrogen burning or because of a MSW, or both; (6) a nondegenerate helium star which fills its Roche lobe and transfers mass because of GWR; (7) the helium remnant of a component of a close binary with a CO or ONe degenerate core and a massive nondegenerate helium mantle which attempts to expand as helium burning adds to the mass of the core; or (8) a degenerate dwarf which transfers matter because of GWR. We call systems in which the donor is a low-mass main-sequence star (type 1 or type 2 mass transfer) "cataclysmic variable–like" (CV-like) and those in which the donor is a subgiant or giant with a helium core (type 5 mass transfer) "Algol-like" LMXBs.

These modes of mass transfer have been studied numerically by many authors. A representative list is given here. Type 1 (MSW, CV): Verbunt & Zwaan (1981), Taam (1983a), Iben & Tutukov (1984b), Pylyser & Savonije (1988, 1989), and Rappaport, Verbunt, & Joss (1983). Type 2 (GWR, CV): Paczyński (1967a), Faulkner (1971), Chau & Lauterborn (1977), Taam et al. (1980), Paczyński (1981), Paczyński & Sienkiewicz (1981), Rappaport, Joss, & Webbink (1982), Taam (1983a), Rappaport et al. (1983), Iben & Tutukov (1984b), Tutukov et al. (1985), Rappaport et al. (1987), and Pylyser & Savonije (1988, 1989). Type 3 (classical case A): Paczyński (1966), Kippenhahn & Weigert (1967), and Paczyński & Ziolkowski (1967). Type 4 (classical early case B): Kippenhahn & Weigert (1967), Paczyński (1967b), and Kippenhahn, Kohl, & Weigert (1967). Type 5 (classical Algol): Webbink, Rappaport, & Savonije (1983), Savonije (1983), Taam (1983b), Iben & Tutukov (1984b), de Kool, van den Heuvel, & Rappaport (1986), and Joss, Rappaport, & Lewis (1987). Type 6 (helium Algol): Savonije, de Kool, & van den Heuvel (1986), de Kool, van den Heuvel, & Pylyser (1987), Iben & Tutukov (1987), Iben et al. (1987), and Tutukov & Fedorova (1989). Type 7 (helium Algol): Iben & Tutukov (1993). Type 8 (GWR, CV with degenerate donor): Paczyński (1967a), Vila (1971), Tutukov & Yungelson (1979a), Kieboom & Verbunt (1981), van den Heuvel & Taam (1984), van den Heuvel & Bonsema (1984), Rappaport & Joss (1984), and Rappaport et al. (1987).

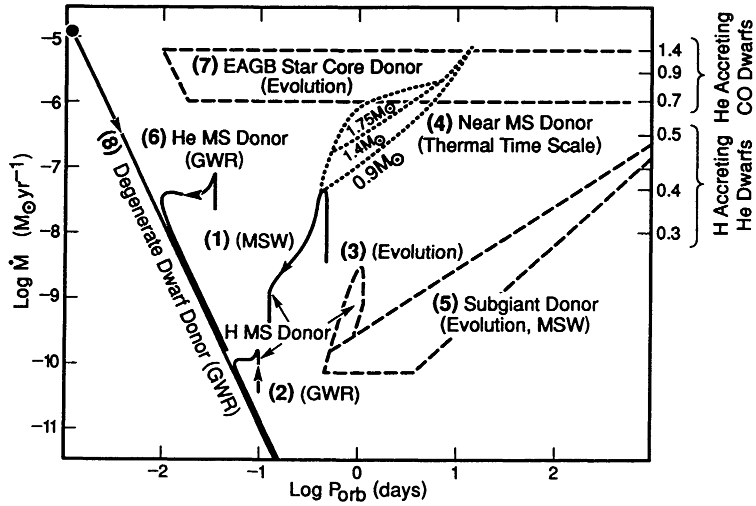

FIG. 4.—Semidetached low-mass binaries in the plane $\dot{M}$ (mass-exchange rate) vs. $P_{\rm orb}$ (orbital period) in the approximation that the donor fills its Roche lobe and mass is transferred conservatively. Numbers describe situations roughly in the order of increasing evolutionary status of the donor: (1) the evolutionary track of a long-period (3–10 hr) CV-like binary in which mass exchange is driven by a MSW (Iben & Tutukov 1984b); (2) the evolutionary track of a short-period CV-like binary in which mass exchange is driven by GWR (Iben & Tutukov 1984b); (3) the positions of binaries in which mass exchange is driven by the evolutionary expansion of a main-sequence donor; (4) mass-exchange rates in semidetached binaries in which mass exchange is driven by the thermal expansion of an evolved main-sequence donor [radii and luminosities along the tracks are from Mengel et al. 1979; $\dot{M} = M/\tau_{\rm KH}$, $P_{\rm orb}$ (days) = $0.4R^{3/2}M_t^{-1/2}$, and $\tau_{\rm KH}({\rm yr}) = 3 \times 10^7 M^2/RL$, where M, L, and R are in solar units]; (5) the positions of binaries in which mass exchange is driven by the evolutionary expansion of a subgiant or giant which contains a degenerate helium core (Iben & Tutukov 1984b) (a MSW can also operate); (6) the evolutionary track of a semidetached binary in which the donor is a nondegenerate helium star and mass exchange is driven by GWR (Tutukov & Federova 1989); (7) the positions of binaries in which the mass donor is the remnant of post-case BB or early case C evolution and now consists of a degenerate CO core and a thick nondegenerate helium mantle (mass transfer is driven by expansion of the helium mantle which burns helium at its base; Iben & Tutukov 1993); (8) the evolutionary track of a binary in which the donor is a helium white dwarf and mass transfer is driven by GWR (the filled circle at the top of this track locates the position where the mass of the donor is $\sim 0.2\ M_\odot$; Tutukov & Yungelson 1979a). The general characteristics of the regions where stable mass transfer may occur are relatively independent of the nature of the accretor. The scales on the right-hand side of the figure show the masses of helium white dwarf accretors and of CO white dwarf accretors if these white dwarfs burn nuclear fuel as rapidly as it is accreted (hydrogen in the case of helium white dwarfs and helium in the case of CO white dwarfs).

In the $\dot{M}$-$P_{\rm orb}$ plane of Figure 5, observed characteristics of some known LMXBs from the catalog of Aslanov et al. (1989) are compared with predicted characteristics. The estimate of the mass accretion rate of MXB 1820−30, a binary with orbital period ∼11 minutes, has been taken from Tutukov et al. (1987) (see also Rappaport et al. 1987). There are many physical effects, such as, e.g., solar-like cyclical variations in the MSW efficiency (e.g., Richman, Applegate, & Patterson 1994), that can lead to large fluctuations about the average rate. Hence an observed mass-exchange rate may not be a reliable measure of the average mass-exchange rate on an evolutionary timescale. In spite of this, there is rough agreement between theoretically predicted characteristics and those derived from the observations for several groups of LMXBs. There are several Algol-like systems (e.g., Sco X-1, Cyg X-2, V395 Car, and V2116 Oph), long-period CV-like systems (e.g., Cyg X-3, GR Mus, V926 Sco, LMC X-2, and 1755−338), and systems in which the donor is possibly a degenerate dwarf or an evolved helium star (e.g., 1820−30, 1916−053, and 2259+586). The proximity of several LMXBs to the curve in Figure 5 labeled "He Dwarfs (GWR)," along which the donor is a degenerate dwarf, does not prove that the donor is a degenerate dwarf; the donor could also be a slightly evolved helium star evolving downward along the curve labeled "He MS Donor" (Tutukov & Fedorova 1989), or even a somewhat evolved hydrogen donor (Iben & Tutukov 1984b; Rappaport & Joss 1984; Tutukov et al. 1985; Nelson, Rappaport, & Joss 1986; Tutukov et al. 1987).

On the negative side, there is only one X-ray–dim system in the van Paradijs (1995) catalog (1603.6+2600 with $P_{\rm orb}$ = 1.85 hr) which has a period in the 1–2 hr range, a range which is well populated by classical CVs, and there are several systems with periods in the 4–10 hr range (e.g., 1728−169, V1727 Cyg, V691 CrA, and 1957+115) for which there are no theoretical Roche lobe filling counterparts. The paucity of short-period CV-like LMXBs may be due to evaporation of the donor caused by absorption of X-rays from the accretor. Such an evaporative process has been invoked to explain the properties of the binary radio pulsar PSR 1957+20 (Kluzniak et al. 1988; Phinney et al. 1988; Bisnovatyi-Kogan 1989). In the case of several of the X-ray–dim sources, the propeller action of a rotating magnetic neutron star may inhibit accretion and the development of strong X-ray emission (Illarionov & Sunyaev 1975; Tutukov et al. 1987). Another possible explanation for the dim sources is that the strength of the MSW undergoes variations associated with variations in the magnetic activity of the donor. Approximately periodic variations in magnetic activity are known to occur in low-mass single stars (Wilson 1978), and might be expected to occur also when the low-mass star is in a close binary. If the rate of angular momentum loss via the MSW depends on the degree of magnetic activity in a highly nonlinear way, this could account for a variation in $\dot{M}$ by several orders of magnitude. Among other possible explanations of the X-ray–dim sources is one involving an interplay between an irradiation-induced bloating of the donor which causes an increase in the mass-exchange rate and a shadowing of the donor by an accretion disk which increases with an increasing mass-exchange rate (Harpaz & Rappaport 1994).

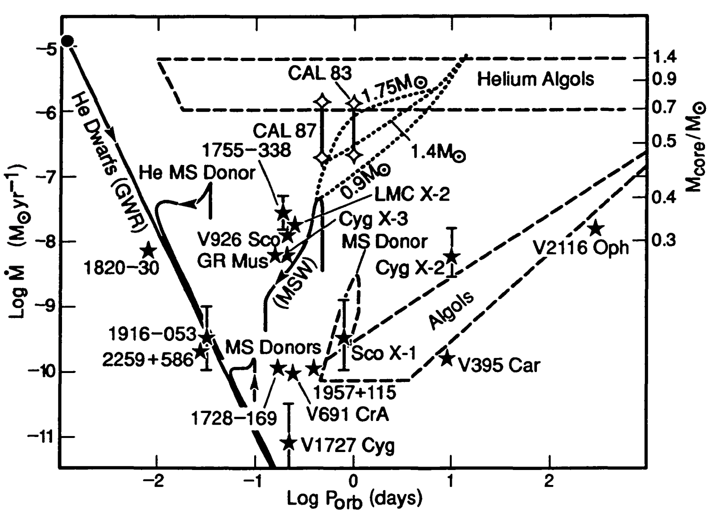

FIG. 5.—Positions of LMXBs in the $\dot{M}$-$P_{\rm orb}$ plane according to the Aslanov et al. (1989) catalog. The two sets of open four-rayed stars connected by a bar mark the positions of the ultrasoft X-ray sources CAL 83 and CAL 87 in the LMC (van den Heuvel et al. 1992); the lower star is appropriate if their luminosity is due to the stationary burning of hydrogen, and the upper star is appropriate if it is due to helium burning. All theoretical lines have the same meanings as in Fig. 4.

3. OVERVIEW OF RESULTS

In §§ 4 and 5, 12 potential ways are described in which LMXBs with a neutron star or black hole accretor can be formed in the course of the evolution of primordial binaries. The main numerical results are summarized in Table 1. These results depend significantly on the choice of a parameter $\alpha_{\rm CE}$ appearing in an algorithm for estimating the degree of orbital shrinkage in common envelope (CE) events (Paper I, eqs. [4]–[6]). This rather strong dependence is due to a common property of the main scenarios which produce LMXBs: a large initial component mass ratio makes even the widest of "close" systems (those which experience a CE stage) become very close binaries. Because of merging during the CE stage or disruption during the supernova explosion, only a relatively small fraction of otherwise appropriate binaries can survive as bound systems. Naturally enough, this fraction depends sensitively on the parameter $\alpha_{\rm CE}$. For $\alpha_{\rm CE} = 0.5$, the theoretical birthrate for disk LMXBs is $\nu \sim 1 \times 10^{-5}$ yr^{-1}, and for $\alpha_{\rm CE} = 1.0$, $\nu \sim 4 \times 10^{-5}$ yr^{-1}. Comparison with observations for other types of binaries favors $\alpha_{\rm CE}$ closer to 1 than to 0.5. Thus, the semiempirical and model estimates of the LMXB birthrate obtained here are larger than the semiempirical estimate of Narayan et al. (1989) but are still several times smaller than their estimate of the MSP birthrate.

Adopting a theoretical scenario for the evolution of an LMXB, a birthrate can be converted into an estimated number of systems. Assuming that all of the mass lost by the donor is accreted by the neutron star or black hole component, and choosing explicit mass-transfer mechanisms, it is found that the number of disk LMXBs is $N_{\rm cons} \sim 5000$ when $\alpha_{\rm CE} = 0.5$ and $N_{\rm cons} \sim 22{,}000$ when $\alpha_{\rm CE} = 1.0$ (Table 1), clearly in conflict with the observations. In § 6, following the work of Tavani (1991a), an attempt is made to resolve this discrepancy by exploring the possibility that, in consequence of absorbing a large

TABLE 1

FORMATION RATES OF LMXBs AND RELATED OBJECTS

Scenario[a]	$\alpha_{CE} = 1.0$			$\alpha_{CE} = 0.5$		
	ν (yr^{-1})	N_{cons}	N	ν (yr^{-1})	N_{cons}	N
1. CV-like LMXBs with a NS (§ 4.1)	3.9×10^{-6}	480	54	0	0	0
2. Algol-like LMXBs with a NS (§ 4.3)	1.6×10^{-5}	4800	350	2.2×10^{-6}	230	28
3. CV-like LMXBs with a BH (§ 4.1)	9.6×10^{-7}	1200	43	3.3×10^{-7}	36	4
4. Algol-like LMXBs with a BH (§ 4.3)	6.2×10^{-6}	1100	100	4.3×10^{-6}	940	79
5. Transition ONe → NS in CVs (§ 5.1)	1.5×10^{-6}	190	21	1.2×10^{-6}	150	17
6. Transition ONe → NS in Algols (§ 5.3)	1.8×10^{-6}	540	46	4.5×10^{-7}	46	6
7. Transition ONe → NS with a He DD (§ 5.6)	9.3×10^{-6}	100	40	2.0×10^{-6}	10	6
8. MS + BH (§ 4.2)	9.0×10^{-7}	45	8	9.0×10^{-7}	45	8
9. He star + BH (GWR) (§ 4.4)	3.9×10^{-6}	3100	140	0	0	0
10. Short-period CV-like LMXBs (NS) (§ 4.1)	5.4×10^{-6}	11000	300	1.2×10^{-6}	2400	68
11. Short-period CV-like LMXBs (BH) (§ 4.1)	9.6×10^{-7}	2800	65	6.1×10^{-7}	1200	33
12. *TŻO*s with a hydrogen-rich envelope (Paper I)	4.0×10^{-4}		2400	9.0×10^{-4}		5300
13. *TŻO*s with a helium envelope (§ 4.6)	3.3×10^{-5}		1	2.4×10^{-5}		1
14. *TŻO*s with a CO or ONe envelope (§ 4.7)	1.1×10^{-3}		41	7.0×10^{-4}		27
15. He star + NS → *TŻO*s (§ 4.4)	2.3×10^{-4}		10	1.1×10^{-4}		5

[a] NS = neutron star; BH = black hole; DD = degenerate dwarf; MS = hydrogen main-sequence star.

flux of X-rays from the accretor, the donor develops a coronal wind which carries mass away from the system. A simple analytical model of the wind (Musilev & Tutukov 1973), which agrees (for overlapping input parameters) within a factor of 2 with the results of the sophisticated models of Tavani & London (1993), suggests that an order of magnitude more mass can be lost from the system than is transferred to the donor, in agreement with the estimates of Tavani (1991a). The net result is that the lifetime of the LMXB phase may be 6–60 times smaller than given in the mass-conservative approximation, thus bringing the theoretical estimate of the number of disk LMXBs more into line with the observations.

The relativistic component can accrete from the irradiation-induced wind, and a "bootstrap" mode of mass exchange (in which the donor does not fill its Roche lobe) is theoretically possible. However, accretion from the wind becomes important only when the donor nearly fills its Roche lobe (Alme & Wilson 1974), so that a MSW and GWR remain the main drivers of mass exchange, even though the rate of accretion from the irradiation-induced wind may rival the mass-exchange rate given in the conservative approximation.

Very little mass need be accreted by a neutron star in order for it to achieve a period in the MSP range. Assuming that 10% of the mass of a low-mass main-sequence donor or of the envelope of a subgiant or giant donor is accreted by a neutron star component, and using a relationship between rotational period and accreted mass that is given by detailed models of spin-up (e.g., Ghosh, Lamb, & Pethick 1977; Alpar et al. 1982; Helfand, Ruderman, & Shaham 1983; Jeffrey 1986; Joss & Rappaport 1983; Savonije 1983; Chen & Ruderman 1993; and see the Appendix to this paper), it is possible to derive a relationship between rotational period and orbital period which is consistent with the observed relationship for binary MSPs of low eccentricity (§ 7). CV-like LMXBs (main-sequence donors) evolve into single neutron stars, and Algol-like LMXBs (subgiant or giant donors) remain binaries with a period that is larger the larger the mass of the degenerate helium core of the donor. The theoretical ratio of Algol-like to CV-like LMXBs is close to the ratio of binary MSPs of low eccentricity to single MSPs in the Galactic disk population. The semiempirical estimates of the birthrate of Galactic disk MSPs which are based on data now available (Lyne 1994; Bailes et al. 1994; Bailes & Lorimer 1995 and references therein; and § 7 below) are consistent with the theoretical estimate of the birthrate of LMXBs presented here.

The scenario program does not invoke the ad hoc "kick" suggested by Shklovsky (1970) and others to take into account a possible asymmetry of the supernova explosion which forms the neutron star or black hole component. Observed radio pulsar space velocities up to ~800 km s^{-1} are completely explained without such an extra kick as the natural consequence of close interacting binaries which dissolve during a second supernova explosion (Gunn & Ostriker 1970; Tutukov, Chugai, & Yungelson 1984; Tutukov & Yungelson 1993a; Paper I). In the absence of an ad hoc kick, noninteracting wide binaries ($P_{orb} > 100$ yr) often remain bound if the orbit is elliptical and if the second neutron star is formed at the apastron phase of the orbit (e.g., Tutukov & Yungelson 1993a). To explain the apparent absence of wide binaries containing radio pulsars, one may suppose that the rotation rate of a newly formed neutron star is intrinsically much smaller than that of observed radio pulsars. A corollary to this supposition is that the observed rotation rates of radio pulsars are a result of spin-up due to mass and angular momentum exchange between components, or of tidal interaction, or of both. In this picture, pulsar-producing supernovae are of the Type Ib and Ic variety (i.e., helium star precursors), whereas supernovae formed from massive stars in wide, noninteracting binaries are of Type II and the remnant neutron star is not a pulsar.

A purely formal way of avoiding the formation of radio pulsars in wide binaries is to introduce an ad hoc "mini-kick" of magnitude ~10–20 km s^{-1}. Such a kick is large enough to disrupt wide binaries but is small enough not to prevent the formation of the well-populated low space velocity component

($v_{sp} < 50$ km s^{-1}) in the bimodal velocity distribution of observed radio pulsars (e.g., Tutukov et al. 1984; Narayan & Ostriker 1990). The scenario program does not include such a kick.

Because they are interesting as potential progenitors of some MSPs, the birthrate of TŻOs has also been estimated. TŻOs are hypothetical single stars which consist of a neutron star core with a hydrogen-rich, helium-rich, or carbon- and oxygen-rich envelope at the base of which a nuclear fuel is being burned (Thorne & Żytkow 1975, 1977; Cannon et al. 1992). They are presumably formed by unstable mass transfer which converts the donor into an envelope about the neutron star. The main source of energy is gravitational potential energy liberated by accretion of matter from the extended envelope through the nuclear burning shell. The scenario program assumes that a black hole can also be the core of a TŻO, in spite of the absence of relevant numerical models. It is possible that TŻOs with black hole cores have properties and origination scenarios which are similar to those with neutron star cores, and the two kinds of TŻOs are lumped together in Table 1.

To estimate the numbers of TŻOs of any given envelope composition, it is assumed, following Thorne & Żytkow (1975, 1977), that TŻOs are stable, long-lived configurations. The luminosity of a TŻO with a hydrogen-rich envelope is taken to be

$$L/L_\odot = 4 \times 10^4 M_{core}/M_\odot + 100(M_{tot}/M_\odot)^2 \,, \qquad (1)$$

where the first term is the Eddington luminosity for a relativistic core of mass M_{core}, and the second term is the nuclear luminosity of the TŻO envelope; M_{tot} is the total mass of the TŻO. An estimate must also be made of the lifetime of a TŻO. Assuming very high temperatures near the neutron star core, Zeldovich, Ivanova, & Nadyozhin (1972) found runaway neutrino losses, accompanied by an accelerating rate of envelope contraction; these effects led to a very short lifetime. The thermal stability and relatively low temperatures found by Thorne & Żytkow (1975, 1977) suggest a much longer lifetime. In the scenario code, it is assumed that the lifetime is limited only by mass loss due to a stellar wind from a red supergiant, which is what a TŻO is expected to look like, and the mass-loss rate given by Nugis (1989) is adopted:

$$\dot{M}(M_\odot \, \mathrm{yr}^{-1}) = a(L/L_\odot)^{1.5} \,, \qquad (2)$$

where $a = 10^{-13}$ for a hydrogen-rich envelope and $a = 10^{-12}$ for a helium- or carbon-oxygen–rich envelope. The lifetime of the envelope is taken to be the ratio of the envelope mass to the mass-loss rate. TŻOs evolve into neutron stars or black holes after most of the matter in their extended envelopes is lost through a stellar wind. TŻOs with hydrogen-rich envelopes are formed mostly by massive binaries with primary mass in the 10–30 $M_\odot$ range. In the relevant following sections, situations are identified which may lead to the formation of TŻOs with helium-rich and carbon-rich envelopes. The total birthrate of TŻOs is $\nu \sim 1.5 \times 10^{-3}$ yr^{-1}, which is significantly larger than the birthrate of MSPs. In the Appendix it is argued that, because of their short lifetimes, TZOs are not spun up into the MSP range of periods, so that the large difference in estimated birthrates of TŻOs and MSPs does not pose a problem.

4. LMXBs RESULTING FROM THE EVOLUTION OF MASSIVE BINARIES WITH A LARGE INITIAL MASS RATIO

Most disk LMXBs are the product of the evolution of a close binary in which the primary component is of initial mass larger than $\sim$11.4 $M_\odot$ (thereby ensuring the formation of a neutron star or a black hole), but less than $\sim$50 $M_\odot$, above which mass expansion of the primary does not occur during nuclear-burning phases (see Paper I and references therein). If it is to develop a degenerate helium core before filling its Roche lobe, the secondary must have an initial mass less than $\sim$2.3 $M_\odot$. Expansion of the primary and a large initial primary-to-secondary mass ratio ensure that a CE is formed and that orbital shrinkage is sufficient to allow the low-mass secondary to eventually fill (or nearly fill) its Roche lobe. It is assumed that LMXBs with neutron-star components arise only from systems in which the primary is initially less massive than $\sim$40 $M_\odot$ (van den Heuvel & Habets 1984) and that LMXBs with black hole components arise from systems in which the primary has an initial mass in the range 40–50 $M_\odot$. If the initial mass of the primary is larger than 50 $M_\odot$, it is assumed that the primary cannot expand to giant dimensions (see, e.g., Humphreys & Davidson 1994) and that a CE phase which brings components close enough for eventual mass exchange from the secondary to the primary is therefore avoided.

Black holes are included in the model because of the existence of LMXBs such as A0620−00, V404 Cyg, and XN Mus in which the mass of the accretor has been shown to be larger than the maximum permissible mass of a neutron star (McClintock & Remillard 1986; Remillard, McClintock, & Bailyn 1992; Casares, Charles, & Naylor 1992; Remillard et al. 1992; Tutukov & Cherepashchuk 1992; Cowley 1992; Marsh, Robinson, & Wood 1994). The scenario code treats the formation of a black hole as an instant reduction (to 10 $M_\odot$) of the mass of the exploding component.

Six channels which lead to a neutron star or black hole receiving matter from a Roche lobe filling companion are described in Figure 6, which is constructed on the basis of the mass-exchange rates shown in Figures 4 and 5. All channels begin with the formation of a CE when the initial primary exhausts hydrogen at the center and expands to fill its Roche lobe. The degree of orbital shrinkage in the CE event is set by a choice of α_{CE}.

As the remnant helium core of the primary burns helium in a convective core, its radius remains nearly constant. After exhausting helium over a finite region about its center, the remnant, if its mass is in the range $\sim$0.75–3.0 $M_\odot$, expands considerably (Divine 1965; Paczyński 1970; Delgado & Thomas 1981; Law & Ritter 1983; Iben & Tutukov 1985; Habets 1985; Avila Reese 1993), and further evolution of the system depends on the orbital separation and on the mass of the remnant. If the orbital separation is large enough, the remnant evolves without further mass loss through a series of nuclear-burning stages until it develops an Fe-Ni core which collapses into a neutron star.

The maximum radius R_{max} attained by a model helium star depends on the mass of the model. In the scenario program,

$$R_{max} = 200 \, R_\odot \quad \text{when} \quad M_{remnant} = 0.78\text{–}2.5 \, M_\odot \qquad (3a)$$

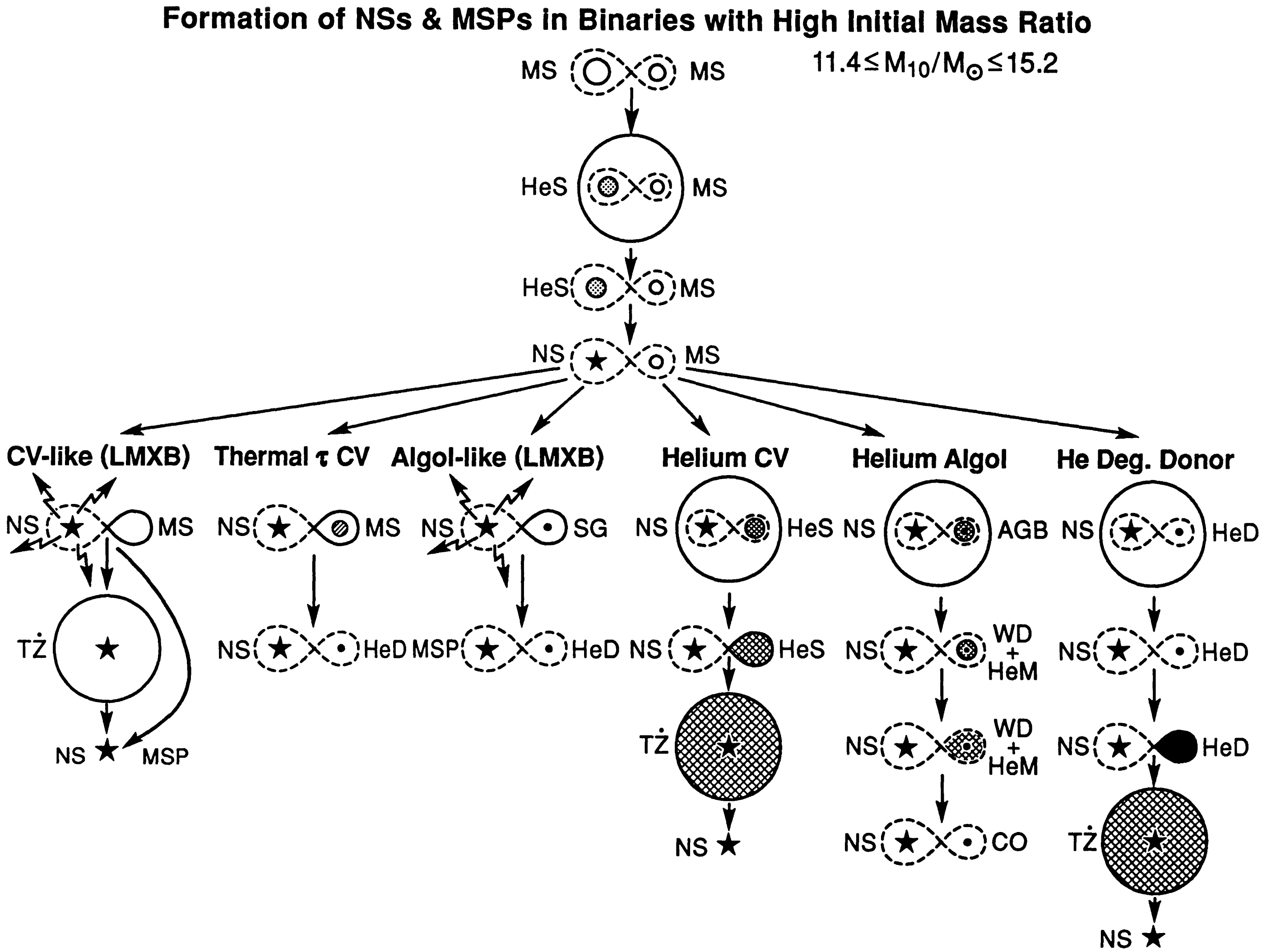

FIG. 6.—Evolutionary scenarios for massive binaries in which the ratio of initial primary mass to secondary mass is large (§ 4). Alphabetical symbols have the following meanings: MS = a main-sequence star; ONe = an ONeMg white dwarf; HeD = a degenerate helium dwarf; NS = a neutron star; MSP = a millisecond pulsar; HeS = a nondegenerate helium main-sequence star; SG = a subgiant (or giant) with a degenerate helium core; AGB = an asymptotic giant branch star; WD+HeM = a degenerate CO or ONeMg core plus a helium mantle which is burning helium at its base; CO = a CO white dwarf; TŻ = a Thorne-Żytkow object (a neutron star with a mantle of hydrogen, helium, CO, or ONe).

and

$$R_{\max} = 7700\, R_\odot \left(\frac{M_{\rm remnant}}{M_\odot}\right)^{-5.5} \quad \text{when} \quad M_{\rm remnant} > 2.5\, M_\odot . \qquad (3b)$$

The limits on $M_{\rm remnant}$ in equation (3a) are based on Iben & Tutukov (1989a), and equation (3b) is a fit to the results of Avila Reese (1993), which are quite close to the results of Habets (1985). If, following the first CE event, the orbital separation is small enough, the helium star remnant fills its Roche lobe prior to core collapse, and a second CE event occurs. In the scenario code it is assumed that, in the second CE event, 60% of the mass of the helium envelope is lost from the system (Habets 1985; Avila Reese 1993), and orbital shrinkage is calculated with the same $\alpha_{\rm CE}$ and the same CE algorithm used to estimate orbital shrinkage in the first CE event. At the conclusion of the second CE event, the mass of the remnant is, in all cases, large enough that the remnant evolves into a neutron star. In many instances, the consequence of orbital shrinkage in the second CE event is a merger, and the merged remnant evolves into a single neutron star.

Orbits are assumed to become circular during the pre-CE phase and to remain circular after the CE phase, so the system remains bound if $M_{\rm lost}$, the mass lost in the supernova explosion which produces the neutron star, is less than $M_{\rm rem}$, the mass remaining after the explosion. After the explosion, evolutionary divergences occur which are related to the mass and evolutionary state of the secondary and to the orbital separation. The birthrate ν and the number $N_{\rm cons}$ of systems of each type, as given by the numerical scenario program (for both $\alpha_{\rm CE} = 0.5$ and $\alpha_{\rm CE} = 1.0$), are displayed in Table 1.

In the following subsections we describe each channel, make a rough illustrative estimate of the birthrate when $\alpha_{\rm CE} = 1$, give an explicit estimate of the rate of mass transfer in the mass-conservative approximation, and discuss the results of the scenario program (see Table 1), particularly with regard to the birthrate (ν) and the number of systems ($N_{\rm cons}$) estimated to be present in the Galaxy. Table 1 describes results for the most important scenarios and identifies the subsection where each scenario is discussed most fully. The illustrative estimates often deviate substantially from the more careful estimates produced by the scenario program, demonstrating that, in many instances, detailed numerical calculations are essential for properly estimating birthrates and numbers.

4.1. *CV-like LMXBs*

Channel 1 (the farthest channel to the left in Fig. 6, modes 1 and 2 mass transfer in Fig. 4) leads to a CV-like LMXB in which the secondary is a low-mass main-sequence star of initial mass $M_{20} = 0.3$–$1.0\ M_\odot$ which remains in Roche lobe contact because of angular momentum loss due to a MSW or GWR, or both. In the simplest approximation of a spherically symmetric supernova explosion in an initially circular orbit (Boersma 1961), the condition for survival as a binary after the supernova explosion which produces the neutron star means that the initial mass M_{10} of the primary must be less than 13.4 $M_\odot$ when $M_{20} = 1.0\ M_\odot$ and less than 11.6 $M_\odot$ when $M_{20} = 0.3\ M_\odot$ (see eqs. [2] and [3] in Paper I). The lower limit on M_{10} of 11.4 $M_\odot$ (necessary for the formation of a neutron star) translates into a lower limit of $\sim$0.2 $M_\odot$ on the mass of the secondary. This limit is, fortuitously, almost the same as the lower limit required for a MSW to operate. The mass of the helium nondegenerate remnant of the primary after the first CE event is, according to equation (3) in Paper I, in the range 3.0–3.8 $M_\odot$.

A lower limit on A_0, the initial semimajor axis, is set by the requirement that the secondary does not fill its Roche lobe until after the CE event. If this requirement is not fulfilled, the components are assumed to merge. The mass-dependent relationship between Roche lobe radius and orbital separation, in conjunction with the mass-radius relationship for a zero-age main-sequence star, shows that A_f, the orbital separation after the CE event(s), must be larger than $\sim 1\ R_\odot$ when $M_{20} = 0.3\ M_\odot$ and larger than $\sim 3\ R_\odot$ when $M_{20} = 1\ M_\odot$.

During the supernova explosion, the semimajor axis increases and the orbit becomes eccentric (e.g., Boersma 1961). If components in the resulting binary are close enough, tidal friction acts to recircularize the orbit (Zahn 1977). The scenario code takes these effects into account, but for orientation purposes we assume in our illustration that the orbit remains circular and expands by a factor of $\sim$2 (supposing that the product of the mass of the system and the semimajor axis remains constant).

An upper limit on the orbital separation after the supernova event is set by the requirement that a MSW can drive the secondary into Roche lobe contact in a time less than the age of the Galactic disk, which we assume here to be $\tau_{\rm disk} \sim (10$–$15) \times 10^9$ yr. When $M_{20} = 1$, this limit is $\sim 10\ R_\odot$, and when $M_{20} = 0.3$ it is $\sim 5\ R_\odot$ (e.g., Iben & Tutukov 1984b; Tutukov 1985). This means that only those systems which survive the supernova event with a semimajor axis smaller than $\sim$2.5–5 $R_\odot$ can evolve into LMXBs. Altogether, then, precursors of LMXBs have semimajor axes between $\sim 2\ R_\odot$ (average of 1 and 3 $R_\odot$) and $\sim 3.7\ R_\odot$ (average of 2.5 and 5 $R_\odot$). The algorithm for orbital shrinkage which we use (eqs. [4]–[6] in Paper I) gives a nonlinear transformation between semimajor axes before and after a CE event, but in practice the approximation that the ratio of limiting values of A_0 is the same as the ratio of limiting values of A_f after the CE event is acceptable for rough estimates when $A_f \ll A_0$.

The integral of the birth function (Iben & Tutukov 1984a) may be approximated by

$$\nu = \iiint d^3\nu \sim 0.2\Delta \log A_0 \frac{\Delta M_{10}}{M_{10}^{2.5}} \Delta q_0 \ {\rm yr}^{-1} , \tag{4}$$

where q_0 is the ratio of secondary to primary mass in the initial system, and the symbol Δ implies a suitably chosen range for the following variable. Equation (4) is a keystone of the scenario code. We use it here to demonstrate analytically the origin of the frequencies produced by the code, as well as to point out intrinsic limitations on the initial parameters of binary stars appearing in various scenarios.

In the case at hand, equation (4) gives

$$\nu \sim 0.2\left[\log\left(\frac{3.7}{2}\right)\right]\frac{2}{(12.4)^{2.5}}\frac{0.7}{12.4}\ {\rm yr}^{-1} \sim 1 \times 10^{-5}\ {\rm yr}^{-1} \tag{5}$$

as a rough estimate of the birthrate of CV-like LMXBs. Apart from the error committed by replacing a highly nonlinear integral with a product of finite differences, the most important shortcoming of this estimate is the fact that the expansion of the helium star remnant of the primary after it forms a CO core has not been taken into account. The expansion leads to a second CE event and, in most instances, to a merger, eliminating the low-mass segment of potential contributors with initial primary masses in the 11.4–13.4 $M_\odot$ range. The scenario program adopts a suitably fine grid in the three variables A_0, M_{10}, and q_0 (Paper I); takes into account the second CE event; does a proper job of estimating orbital expansion during the supernova event; and gives $\nu = 3.9 \times 10^{-6}\ {\rm yr}^{-1}$ when $\alpha_{\rm CE} = 1$.

A smaller $\alpha_{\rm CE}$ means a larger degree of orbital shrinkage during each CE event and a consequent increase in the number of systems which experience mergers when the helium star remnant of the primary expands. The scenario program finds that, when $\alpha_{\rm CE} = 0.5$, no CV-like LMXBs are formed at all (Table 1). Thus, in this scenario, as well as in the scenario for Algol-like LMXBs (§ 4.3), the expansion of the helium star remnant of the primary plays a crucial limiting role.

An estimate of the number of systems in the Galactic disk at any given time requires an estimate of the lifetime of the LMXB stage. A heuristic estimate of the rate of angular momentum loss that is based on the Skumanich (1972) relationship between equatorial velocity and age for single stars (e.g., Verbunt & Zwaan 1981; Taam 1983a; Rappaport et al. 1983; Iben & Tutukov 1984b; Tutukov 1985; King 1988; Iben, Fujimoto, & MacDonald 1992a) suggests a mass-loss rate which can be approximated by

$$\dot{M}_{\rm MSW}(M_\odot\ {\rm yr}^{-1}) \sim 10^{-7.5}\left(\frac{M_2}{M_\odot}\right)^{2.5} . \tag{6}$$

In the scenario program, it is assumed that equation (6) is valid as long as the mass M_2 of the secondary is less than 1 $M_\odot$, which is the empirical border between rapidly and slowly rotating main-sequence stars (Tutukov & Fedorova 1994). In order to avoid yet another CE episode and the conversion of the system into a TŻO, it is assumed that the mass-transfer rate must be less than the Eddington limit for a neutron star,

$$\dot{M} < \dot{M}_{\rm Edd}(M_\odot\ {\rm yr}^{-1}) \sim \frac{4.65 \times 10^{-8}}{1+X}\frac{M_{\rm NS}}{M_\odot}, \qquad (7)$$

where X is the abundance by mass of hydrogen in accreted matter and $M_{\rm NS}$ is the mass of the neutron star. For $X = 0.7$ and $M_{\rm NS} = 1.4\ M_\odot$, $\dot{M}_{\rm Edd} \sim 3.8 \times 10^{-8}\ M_\odot\ {\rm yr}^{-1}$. Thus, formally, the donor mass must be less than 1.08 $M_\odot$, a condition already met by the upper limit on donor mass of 1 $M_\odot$, required if a MSW is to operate. Limitation (7) is also applied to systems with a black hole accretor (with $M_{\rm NS}$, of course, replaced by the black hole mass $M_{\rm BH} = 10\ M_\odot$).

In the number-period distribution of CVs, short-period systems are separated from long-period systems by a sparsely populated 2–3 hr "period gap" (Patterson 1984) below which GWR is the main driver of mass transfer and above which a MSW is the main driver (e.g., Iben & Tutukov 1984b). The mass of a donor at the 3 hr end of the period gap is $\sim$0.3 $M_\odot$. Here, in analogy with CVs, we define as "long period" those CV-like LMXBs with $M_2 > 0.3\ M_\odot$, and as "short period" those with $M_2 < 0.3\ M_\odot$. Integration of equation (6) shows that the lifetime of a typical long-period system in the conservative approximation is $\sim 10^8$ yr, suggesting that the number of CV-like LMXBs in long-period systems is $N_{\rm cons} \sim 3.9 \times 10^{-6}\ {\rm yr}^{-1} \times 10^8\ {\rm yr} = 390$ when $\alpha_{\rm CE} = 1$. The scenario program gives $N_{\rm cons} = 480$.

These estimates are 4–5 times larger than the estimated number of $\sim$100 bright LMXBs in the Galactic disk. The discrepancy could be resolved formally by choosing a smaller value of $\alpha_{\rm CE}$ (for $\alpha_{\rm CE} = 0.5$ the scenario code gives $\nu = 0$ and $N_{\rm cons} = 0$), but this would destroy consistency between the observations and theoretical predictions for many other families of binaries. Alternative explanations are discussed in § 6.

Long-period CV-like systems with black holes of mass $\sim$10 $M_\odot$ may be formed if the primary has an initial mass in the range 40–50 $M_\odot$ (see van den Heuvel & Habets 1984; de Kool, van den Heuvel, & Pylyser 1987; Tutukov & Cherepashchuk 1992), and the secondary has a mass in the range 0.3–1 $M_\odot$. The birthrate of such LMXBs, as given by the scenario program, is $\sim 10^{-6}\ {\rm yr}^{-1}$ and their number in the Galaxy, in the mass-conservative approximation, is $\sim$1200 when $\alpha_{\rm CE} = 1$ and $\sim$40 when $\alpha_{\rm CE} = 0.5$.

In principle, short-period CV-like LMXBs with neutron star or black hole accretors can be formed by evolution from long-period systems. When the relativistic component is a neutron star or black hole, short-period systems can also be formed directly when the initial mass of the secondary is less than 0.3 $M_\odot$. In short-period systems, mass transfer, if conservative and driven by GWR, is at rates of $\sim 10^{-10}\ M_\odot\ {\rm yr}^{-1}$, several tens of times smaller than in long-period systems. The correspondingly longer lifetimes imply many more short-period systems than long-period systems (Table 1). For example, for CV-like LMXBs made via scenario 1 (Fig. 6), the short-period systems outnumber the long-period systems by 16 to 1 when $\alpha_{\rm CE} = 1$.

The observations show that the real situation is the reverse of the one predicted by the "conservative" model: there are only two Galactic disk systems in the van Paradijs (1995) catalog (1603.6+2600, $P_{\rm orb} = 1.85$ hr; 1323−619, $P_{\rm orb} = 2.93$ hr) with periods less than 3.7 hr, whereas there are a few tens of long-period CV-like LMXBs. Although the actual numbers of predicted systems (of both sorts) far exceed the observed numbers, indicating a shortcoming in one or more of the assumptions that enter into the theoretical predictions, the discrepancy between the observed and the theoretical *ratios* of short- to long-period systems is of a more fundamental nature. That is, if mass transfer is at all times conservative (so that all long-period CV-like systems evolve into short-period systems), and GWR is the dominant driver of mass transfer in short-period systems (as is known to be the case in short-period CVs), the observable number of short-period systems should be many times larger than the number of precursor long-period systems. With an accretion rate of $\sim 10^{-10}\ M_\odot\ {\rm yr}^{-1}$ and a corresponding X-ray luminosity of $L_{\rm X} \sim 100\ L_\odot$, there should be many short-period (1–2 hr) sources in the 2–3 kpc neighborhood of the Sun and seen at even larger distances (see Fig. 2). Thus, the observed paucity of short-period systems is probably not a consequence of observational selection. Rather, it indicates either (*a*) that if short-period systems can be formed by evolution from long-period sources, the lifetime of short-period systems is much shorter (by a factor of about 100) than given by GWR in the conservative approximation, or (*b*) that potential long-period progenitors are destroyed before evolving into short-period systems, or both. Solutions to the problem have been invoked which involve an influence on the donor of X-ray radiation from the accretor (e.g., London, McCray, & Auer 1981; van den Heuvel & van Paradijs 1988; Ruderman, Shaham, & Tavani 1989a; Krolik & Sincell 1990; Levinson & Eichler 1991; Harpaz & Rappaport 1991) or a magnetic propeller preventing accretion from the secondary (Illarionov & Sunyaev 1975; Lipunov 1987; Tutukov et al. 1987; Kluzniak et al. 1988). In §§ 6.2 and 6.3 we develop the theme that an irradiation-induced stellar wind from the donor may solve the problem, significantly shortening lifetimes of all families of LMXBs relative to lifetimes calculated in the conservative approximation.

4.2. *CV-like Systems with an Evolving Main-Sequence Donor*

In channel 2 (second channel from the left in Fig. 6; mode 3 mass transfer in Fig. 4), the Roche lobe filling star in the conjectured LMXB is a main-sequence star of mass in the range 1 $M_\odot$–$M_{2,\rm max}$, and Roche lobe contact is maintained by the nuclear evolution of the secondary. The lower limit on mass is chosen so as to avoid the operation of a MSW, and the upper limit is chosen so as to prevent unstable mass transfer (see, e.g., Tutukov, Fedorova, & Yungelson 1982; Iben & Tutukov 1994). If the relativistic component of the LMXB is a neutron star, $M_{2,\rm max} = 1.2 M_{\rm NS} \simeq 1.7\ M_\odot$.

To avoid the disruption of the binary system during the supernova explosion, the initial mass of the primary must be smaller than 13.4 $M_\odot$ when $M_2 = 1\ M_\odot$, or 15.2 $M_\odot$ when $M_2 = 1.7\ M_\odot$. Thus, $\Delta M_{10} = 1.8\ M_\odot$ and $\Delta q_0 \sim 0.7/14.3 =$

0.05. We have neglected wind mass loss, since it is expected to be small for stars of low and intermediate mass. An important property of case A mass-exchange systems is that the semimajor axis is confined to a very narrow interval, $\Delta \log A_0 \sim 0.1$. The reason for this is that, during the supernova explosion, the semimajor axis increases. Inserting our estimates for the Δ's in equation (4), we have

$$\nu \sim (0.2)(0.1)\left[\frac{1.8}{(14.3)^{2.5}}\right]0.05 \text{ yr}^{-1} \sim 2 \times 10^{-6} \text{ yr}^{-1} . \quad (8)$$

During the LMXB phase, the mass-exchange rate is given approximately by the initial mass of the main-sequence star divided by its main-sequence lifetime (see the polygon labeled "3" in Fig. 4). Using data from Mengel et al. (1979) for donors with masses 1–3.5 $M_\odot$ and composition $(Z, Y) = (0.01, 0.3)$,

$$\dot{M}(M_\odot \text{ yr}^{-1}) \simeq 10^{-10}\left(\frac{M_2}{M_\odot}\right)^4 . \quad (9)$$

For M_2 in the range 1–1.7 $M_\odot$, the mass-exchange rate is in the range 10^{-10} to 10^{-9} $M_\odot$ yr^{-1}. In the mass-conservative approximation, the lifetime of these systems during the LMXB phase is $\sim 3 \times 10^9$ yr, and their total number in the Galaxy is predicted to be ~6000.

A more careful scrutiny, as conducted by the scenario program, shows that this channel is actually quite improbable if the accretor is a neutron star. Only those systems in which two CE events precede the supernova event develop a semimajor axis small enough that the potential main-sequence donor could eventually fill its Roche lobe in consequence of evolutionary expansion. But, when the supernova explosion occurs, the increase in orbital separation is sufficient to prevent the main-sequence component from filling its Roche lobe as it expands during core hydrogen burning. The scenario program finds that this channel is completely "forbidden." However, this forbiddance is not absolute, since it is based on the details of the expansion of helium star models, and these details depend on the input physics, e.g., on the opacity. Thus, the evolutionary significance and the probability of this scenario remain uncertain; given our analytical estimate, equation (8), this scenario for the formation of some LMXBs is not firmly excluded. On the other hand, clear observational examples of such LMXBs are also absent, with the possible exception of Her X-1, which is probably in a short-lived transitional stage prior to a CE stage. However, it is rather difficult to distinguish such binaries from normal long-period CV-like or short-period Algol-like LMXBs (Figs. 1 and 5).

The situation is much clearer for similar binaries in which the accretor is a black hole of mass ~10 $M_\odot$ deriving from a primary of initial mass in the range ~40–50 $M_\odot$. The immediate progenitors of black holes do not expand, so we do not have to contend with a second CE event, and stable mass transfer can occur even when the mass of the donor is much larger than 1.7 $M_\odot$. The rate of mass exchange during the core hydrogen-burning stage is given by equation (9), and, using the condition that $\dot{M} \leq \dot{M}_{\rm Edd} \sim 2.7 \times 10^{-7}$ $M_\odot$ yr^{-1} (eq. [7] with $X = 0.7$ and $M_{\rm NS} \rightarrow M_{\rm BH} = 10$ $M_\odot$), we have that the initial mass of the donor can be as large as ~7.2 $M_\odot$. The lower limit on the mass of the secondary remains at 1.0 $M_\odot$, so $\Delta q_0 \sim 0.14$. Taking $\Delta \log A_0 \sim 0.1$, the birthrate becomes

$$\nu \sim (0.2)(0.1)\left[\frac{10}{(45)^{2.5}}\right]0.14 \text{ yr}^{-1} \sim 2 \times 10^{-6} \text{ yr}^{-1} . \quad (10)$$

The scenario program gives a birthrate which is about half of this value and finds $N_{\rm cons} = 45$, independent of the parameter $\alpha_{\rm CE}$ (Table 1). The lack of dependence on $\alpha_{\rm CE}$ is a consequence of the absence of a second CE caused by expansion of the immediate precursor of the black hole.

The system LMC X-3 (Fig. 1), consisting of a 3 $M_\odot$ main-sequence star transferring mass to a black hole of mass ~10 $M_\odot$ (e.g., Tutukov & Cherepashchuk 1992; van Paradijs 1995), is possibly an example of this kind of evolution.

4.3. *Algol-like LMXBs*

An important channel for producing an LMXB involves a secondary which can become a subgiant or giant with a degenerate helium core (channel 3 from the left in Fig. 6 and mode 5 mass transfer in Fig. 4). In order for the secondary to evolve off the main sequence with a degenerate helium core in a time less than $\tau_{\rm disk}$, its initial mass must be larger than ~0.8–1 $M_\odot$, the precise value depending on the metallicity at the time of formation of the system and on the value of $\tau_{\rm disk}$. If the metallicity is solar and $\tau_{\rm disk} \sim 10^{10}$ yr, the lower limit is 1 $M_\odot$; if the metallicity is at least 10 times less than solar and $\tau_{\rm disk} \sim 1.5 \times 10^{10}$ yr, the lower limit is 0.8 $M_\odot$. For the donor to develop a degenerate helium core, its mass must be less than ~2.3 $M_\odot$. We suppose that, after the primary has evolved into a neutron star, a CE event can be avoided when the secondary first fills its Roche lobe if the mass of the secondary is less than $1.2 \times M_{\rm NS} \sim 1.7$ $M_\odot$.

If duplicity is to be maintained after the supernova explosion ($M_{\rm lost} < M_{\rm rem}$ when $e = 0$), the initial mass of the primary must satisfy $M_{10} < 12.9$ $M_\odot$ when $M_2 = 0.8$ $M_\odot$, and $M_{10} < 15.2$ $M_\odot$ when $M_2 = 1.7$ $M_\odot$. Since the assumed minimum mass for the formation of a neutron star is 11.4 $M_\odot$, we have that, on average, $\Delta M_{10} \sim 2$ $M_\odot$, and $\Delta q_0 \sim 0.06$.

After the system emerges from the CE phase(s), the secondary will fill its Roche lobe as a main-sequence star unless the semimajor axis at this point is larger than ~6 $R_\odot$ (the Roche lobe radius is approximately one-third of the semimajor axis, and the radius of a terminal main-sequence star in the mass range considered is ~1.5–3 $R_\odot$). The semimajor axis after the first CE phase is less than ~40 $R_\odot$ when $M_2 = 1.7$ $M_\odot$ (eq. [6] in Paper I) and even smaller for smaller M_{10}. Thus, we choose $\Delta \log A_0 \sim \log(40/6) \sim 0.8$, and obtain

$$\nu \sim (0.2)(0.8)\left[\frac{2}{(13.3)^{2.5}}\right]0.06 \text{ yr}^{-1} \sim 3 \times 10^{-5} \text{ yr}^{-1} \quad (11)$$

as a rough estimate of the birthrate of Algol-like LMXBs.

Once again, we have neglected here the effect of a second CE phase brought about by expansion of the primary prior to the supernova event. We have also neglected a third CE phase which occurs after the supernova event if, when it first fills its Roche lobe, the secondary has a deep convective envelope.

In the scenario program, it is assumed that, if the mass $M_{\rm He}$

of the degenerate helium core of the secondary is less than 0.25 $M_\odot$ when the secondary first fills its Roche lobe, the system can avoid a third CE phase as long as $M_2 < 1.2 M_{NS} = 1.7\ M_\odot$. If $M_{He} > 0.25\ M_\odot$, we suppose that, to avoid a CE phase, it is necessary that $M_2 < (2/3) M_{NS} = 0.93\ M_\odot$. The distinction in behavior is based on the fact that giants with $M_{He} > 0.25\ M_\odot$ have very deep convective envelopes, whereas those with smaller values of M_{He} do not. The paucity of classical Algols with donors having $M_{He} > 0.25\ M_\odot$ argues for a difference in physical behavior for systems with M_{He} on either side of the critical value (see, e.g., Iben & Livio 1993). With the choice of $\tau_{disk} = 1.5 \times 10^{10}$ yr and a lower limit on M_2 of 0.8 $M_\odot$ (taking into account that the older the system, the lower its metallicity), the scenario program gives $\nu = 1.6 \times 10^{-5}\ \mathrm{yr}^{-1}$ when $\alpha_{CE} = 1$ and $\nu = 2.2 \times 10^{-6}\ \mathrm{yr}^{-1}$ when $\alpha_{CE} = 0.5$.

The actual age of the disk may be less than 10^{10} yr (Winget et al. 1987; Iben & Laughlin 1989; Wood 1992; Jacoby 1994). A variation in the disk age from 1.5×10^{10} yr to 10^{10} yr corresponds to a change from $\sim 0.8\ M_\odot$ to $\sim 0.9\ M_\odot$ in the minimum mass of a star of low metallicity evolving off the main sequence. Choosing $\tau_{disk} = 10^{10}$ yr and $\alpha_{CE} = 1$, the program gives $\nu \sim 1.2 \times 10^{-5}\ \mathrm{yr}^{-1}$ for Algol-like LMXBs. Since donors of initial mass smaller than 1 $M_\odot$ are involved, the birthrate of CV-like systems (§ 4.1) is less sensitive to the disk age, decreasing (for $\alpha_{CE} = 1$) from 3.9×10^{-6} to $3.4 \times 10^{-6}\ \mathrm{yr}^{-1}$ when τ_{disk} is decreased from 1.5×10^{10} to 10^{10} yr.

A theoretical estimate of the number of Algol-like LMXBs requires a knowledge of how the mass-exchange rate depends on system parameters. There have been many theoretical studies of evolution when the donor is a subgiant or giant with a helium degenerate core and mass transfer is driven on an evolutionary timescale (see references in § 2). The mass-exchange rate can be approximated by (Iben & Tutukov 1984a, b)

$$\dot{M}(M_\odot\ \mathrm{yr}^{-1}) \sim \frac{10^{-10}}{1.56 - 2q}\left(\frac{A}{R_\odot}\right)^{1.4}\frac{M_2^{1.6}}{M_{tot}^{0.6}}, \qquad (12)$$

where $q = M_2/M_{NS}$, and M_2, M_{NS}, and M_{tot} are, respectively, the mass of the donor, the mass of the neutron star, and the mass of the binary system, all in solar units. Similar approximations are given by Webbink et al. (1983) and by Taam (1983a).

According to equation (12), mass transfer is formally unstable if $M_2 > 0.78 M_{NS} \sim 1.09\ M_\odot$. When Roche lobe filling occurs and $q > 0.78$, one might guess that mass is transferred on the thermal timescale of the subgiant or giant donor. Since this can be significantly shorter than the timescale for mass transfer at the Eddington limiting rate for accretion on a neutron star, one might expect the occurrence of a short-lived CE phase which removes the hydrogen-rich envelope of the donor, thereby precluding an LMXB phase. We have hesitated to accept this simple interpretation because of the properties of classical Algols. Many Algols consist of a main-sequence accretor of intermediate mass and a subgiant donor, and it is clear from their total masses (see Fig. 10 in Iben & Tutukov 1984b) that such systems have survived a stage in which $q > 1$ without catastrophic mass loss from either the donor or the system. In other words, nature overcomes the problem posed by equation (12), in spite of the fact that mass exchange begins in this case on the thermal timescale of the donor, which is certainly shorter than the thermal timescale of the main-sequence accretor.

For this reason, in a first pass with the scenario program, it has been assumed that, in the absence of a deep convective envelope, subgiant donors with $q < 1.2$ can smoothly (avoiding CE formation) follow through a semidetached Algol-like phase on a timescale determined by replacing the denominator $1.56 - 2q$ in equation (12) by 1. With this prescription, the scenario program finds that the number of Algol-like LMXBs is $N_{cons} \sim 4800$ when $\alpha_{CE} = 1$ (average lifetime $\sim 2 \times 10^8$ yr) and $N_{cons} \sim 230$ when $\alpha_{CE} = 0.5$ (average lifetime $\sim 10^8$ yr). Given the facts that a neutron star accretor and a main-sequence accretor are two quite different "animals" and that the mass-exchange rate when $q > 0.78$ is usually greater than the Eddington limiting value for a neutron star, estimates of birthrate and number have also been made on the assumptions that (*a*) all potential Algol-like systems with initial $q > 0.78$ fail to become LMXBs and (*b*) the mass-transfer rate in successful systems is as given by equation (12). Then, for $\alpha_{CE} = 1$, $\nu \sim 4.4 \times 10^{-6}\ \mathrm{yr}^{-1}$ (3.6 times less than before) and $N_{cons} \sim 1000$ (5 times less than before). When $\alpha_{CE} = 0.5$, $\nu \sim 6.1 \times 10^{-7}\ \mathrm{yr}^{-1}$ and $N_{cons} \sim 65$. We remain concerned that, in the case of classical Algols, systems with initial $q \gtrsim 1$ apparently evolve smoothly into semidetached systems despite the theoretical expectation of the formation of a CE. The truth is probably somewhere between our two limits. To find this truth, a more detailed model for early stages of evolution in the mass-exchange phase will have to be constructed. Whatever the outcome, it is clear even now that the explanation for an excessive number of LMXBs cannot be blamed entirely on the *q*-factor in equation (12) (see § 6.1).

In some instances, the difficulty with respect to the *q*-factor may be avoided because of wind mass loss from the donor before it fills its Roche lobe. It is known that single stars lose mass along the giant branch. In metal-poor globular clusters, giants lose typically $\sim 0.2\ M_\odot$ (e.g., Iben & Rood 1970; Rood 1973), and the Oosterhoff effect in clusters suggests that the degree of mass loss increases with metallicity. The existence of horizontal-branch stars with masses close to the mass of the degenerate helium core of a star near the tip of the first giant branch in globular clusters suggests that mass loss as large as 0.3 $M_\odot$ can occur. The extended horizontal branch defined by field subdwarf O and B stars suggests that mass loss as large as 0.5 $M_\odot$ can occur for low-mass Population I stars (e.g., Caloi 1990; Heber 1991). The mass-loss rate presumably increases as the star evolves along the giant branch. Thus, the low-mass companion of the neutron star in the precursor of a very wide Algol-like LMXB may have lost up to 0.3–0.5 $M_\odot$ before filling its Roche lobe, thereby enlarging the interval in initial donor masses that avoid the $q > 0.78$ problem (as well as the $q > \frac{2}{3}$ problem when $M_{He} > 0.25\ M_\odot$).

Tout & Eggleton (1988) have proposed that, during the precontact stage, a stellar wind enhanced by duplicity can significantly reduce the mass of the potential donor in Algols. However, their argument is based on only one reliable system, Z Her, for which a mass-radius paradox certainly exists. London & Flannery (1982) discuss Z Her as evidence for an irradiation-induced wind. In point of fact, there is almost a complete absence of observational and theoretical arguments for estimating the wind mass-loss rate in close binaries, so one cannot as yet rely on an enhanced wind mass-loss rate prior to Roche lobe filling for definitive estimates.

The predicted number-period distribution $dN/d \log P_{\rm orb}$ of Algol-like LMXBs given by our standard model ($\alpha_{\rm CE} = 1$, $q \leq 1.2$, and $1.56 - 2q \rightarrow 1$ in eq. [12]) is shown in Figure 7, where "period" refers to the orbital period when the secondary first fills its Roche lobe. The major reason for the decline in $dN/d \log P_{\rm orb}$ at periods larger than 3 days is the effect of CE-induced orbital shrinkage. Because of the small-number statistics (e.g., Figs. 1 and 5), one cannot infer either agreement or disagreement with the observed distribution.

Examples of systems with periods which fit within the model period distribution are Cyg X-2 ($P_{\rm orb} = 9.84$ days), V395 Car ($P_{\rm orb} = 9.01$ days), and Cir X-1 ($P_{\rm orb} = 16.6$ days), but the orbital periods (van Paradijs 1995) are near the upper end of the model distribution. In the conservative approximation, the orbital period can grow during evolution by up to a factor of 6 in some cases (see, e.g., Fig. 8 in Iben & Tutukov 1984b). Hence, before they became LMXBs, precursors of the observed systems presumably had periods closer to the peak in the theoretical distribution of Figure 7, which gives the orbital period prior to the formation of the neutron star component (see legend to Fig. 9 in § 7).

It has been suggested many times (e.g., Paczyński 1983; Savonije 1983; Iben & Tutukov 1984c) that some binary MSPs with almost circular orbits (e.g., PSR 0437−4715 [$P_{\rm orb} = 5.74$ days], PSR 1855+09 [$P_{\rm orb} = 12.3$ days], PSR 2019+2425 [$P_{\rm orb} = 76.5$ days]) and possibly some others (see § 7) are descendants of Algol-like LMXBs. One could argue that the period of even the longest period member of these systems (allowing for orbital period growth during evolution) may be consistent with evolution from an LMXB with an orbital period initially lying within the model distribution (Fig. 6).

The ~300 day estimated period for V2116 Oph places it outside the predicted distribution, even allowing for evolution. But, according to van Paradijs (1995), this period is uncertain.

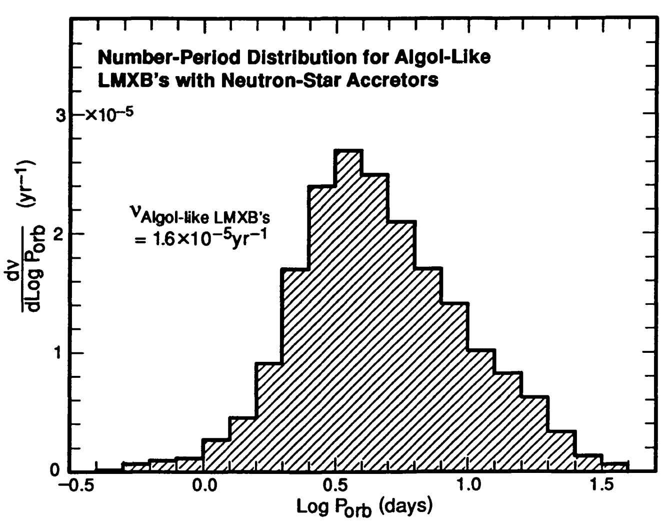

FIG. 7.—Distribution in number vs. (post-CE) orbital period of Algol-like LMXBs with neutron star accretors. It has been assumed that $\alpha_{\rm CE} = 1$, $q \leq 1.2$, and $1.56 - 2q \rightarrow 1$ in eq. (12) in the text.

If there are a number of systems in the Galactic disk (V2116 Oph may be one) with periods larger than, say, 150 days, this could be interpreted as evidence for the operation of a bootstrap mode of mass transfer which does not require the donor to fill its Roche lobe (§ 6.3) or of an enhanced wind from a red giant or asymptotic giant branch (AGB) donor (Tout & Eggleton 1988). A possible successor of evolution of this type may be the binary pulsar 0820+02 (see § 7). On the other hand, to avoid the CE limitation, one could also invoke the sort of two- and/or three-body interactions that are responsible for the formation of LMXBs in globular clusters, or even interactions in triple systems.

Algol-like systems in which the accreting star is a 10 $M_\odot$ black hole and the donor is a subgiant or giant with a degenerate helium core can also be formed. The limits on the initial donor mass in this case are $1 \lesssim M_{20}/M_\odot \lesssim 2.3$, and limits on M_{10} are still $40 \lesssim M_{10}/M_\odot \lesssim 50$, so $\Delta q_0 \sim 0.03$. Limits on the semimajor axis after the CE phase are 5–40 $R_\odot$, giving $\Delta \log A_0 \sim 0.9$. With these choices, the birthrate of Algol-like LMXBs with black hole components is

$$\nu \sim 0.2\left[\log\left(\frac{40}{5}\right)\right]\left[\frac{10}{(45)^{2.5}}\right]0.03\ {\rm yr}^{-1} \sim 4\times10^{-6}\ {\rm yr}^{-1}\,. \quad (13)$$

The scenario program gives a similar birthrate for both $\alpha_{\rm CE} = 0.5$ and $\alpha_{\rm CE} = 1.0$ and predicts a total number of Algol-like binaries with a black hole component of ~1000 (Table 1), which again exceeds the observed number of bright disk LMXBs by a large factor. An example of such a system may be V404 Cyg with $P_{\rm orb} = 6.45$ days (van Paradijs 1995).

4.4. *Helium CV-like LMXBs*

Another potential way of achieving an X-ray stage involves the formation of a system consisting of a neutron star or black hole and a nondegenerate helium star which fills its Roche lobe (channel 4 from the left in Fig. 6; mode 6 mass transfer in Fig. 4).

Mass exchange is driven by GWR, typically at a rate $\sim 3 \times 10^{-8}\ M_\odot\ {\rm yr}^{-1}$, which is smaller than the Eddington limit of $6.5 \times 10^{-8}\ M_\odot\ {\rm yr}^{-1}$ for a neutron star of mass 1.4 $M_\odot$ accreting helium (eq. [7] with $X = 0$). However, when the donor first fills its Roche lobe, there is a strong spike in the mass-transfer rate which considerably exceeds the Eddington limiting accretion rate (Tutukov & Fedorova 1989; see Fig. 4). This spike is especially high if the helium star is even only slightly evolved. Since the core helium-burning timescale and the timescale for merging under the influence of GWR are always comparable in these systems, an evolved donor is a common occurrence.

The mass-exchange rate depends on the mass-radius relationship for a helium star, and this relationship is sensitive to the opacity. Better models might show that the mass-exchange rate driven by GWR is always larger than the Eddington limiting value.

In any case, it currently seems likely that, when the accretor is a neutron star, a CE is formed and a merger occurs, thus preventing the development of an LMXB. If this course of evolution is followed, the system becomes a TŻO with an extended helium envelope, most of which is lost in the form of a wind and a small part of which is converted into neutron star matter. The final product of this evolutionary branch is a single neutron star, probably not spinning rapidly enough to be an MSP (see the Appendix). The birthrate of TŻOs by this scenario, as given by the scenario program, is $\nu \sim 2.3 \times 10^{-4}$ yr^{-1} when $\alpha_{CE} = 1$ and $\nu \sim 1.1 \times 10^{-4}$ yr^{-1} when $\alpha_{CE} = 0.5$.

The absence of known LMXBs with periods in the range $P_{orb} = 13$–50 minutes and with a mass-exchange rate $\dot{M} \sim 3 \times 10^{-8}\, M_\odot$ yr^{-1} (van Paradijs 1995; see Fig. 5) is a possible argument for the formation of TŻOs rather than bright LMXBs by this scenario. If systems could somehow pass through the spike phase, the number of LMXBs expected in the Galactic disk accreting at the rate $\dot{M} \sim 3 \times 10^{-8}\, M_\odot$ yr^{-1} is $N_{cons} \sim 7700$ when $\alpha_{CE} = 1$ and $N_{cons} \sim 3700$ when $\alpha_{CE} = 0.5$. These numbers are clearly at variance with the observations, and one might interpret this as a strong argument against the formation of bright LMXBs by this route. However, an evaporative wind which has been invoked in connection with LMXBs (e.g., Tavani & London 1993 and additional references given in § 7 below) might also operate in this situation to drastically shorten the lifetime of the bright X-ray phase. Thus, the argument against survival through the spike phase is not airtight.

If some LMXBs are formed by this scenario, they will remain bright until the mass of the donor decreases below ~0.18 $M_\odot$ (Tutukov & Fedorova 1989). During subsequent evolution, the mass-transfer rate decreases steadily to very low values. In the scenario program, the calculation of this dim phase is continued until the mass of the donor drops below 0.01 $M_\odot$, giving $N_{cons} \sim 2.3 \times 10^5$ when $\alpha_{CE} = 1$ and $N_{cons} \sim 1.1 \times 10^5$ when $\alpha_{CE} = 0.5$. Several LMXBs, 1820−30, 1916−053, and 2259+586 (van Paradijs 1995), have periods and mass-exchange rates appropriate for this kind of evolution. But, once again, this channel will not be activated unless some systems can somehow avoid merger at the beginning of the bright X-ray phase.

If the accretor is a black hole of mass ~10 $M_\odot$, the mass-exchange rate is much less than the Eddington limit, and a smooth transition of the detached precursor system into an LMXB is possible. The orbital period of such a binary is ~13–50 minutes. The requirement that a nondegenerate helium star be formed places a lower limit of 2.3 $M_\odot$ on the initial mass of the secondary. An upper limit on the initial mass of the secondary follows, in part, from the requirement that evolution driven by GWR will lead to mass transfer from the helium star to the black hole after a time which is short compared with the lifetime of a single helium star [Iben & Tutukov 1985 estimate $\tau_{He}(\mathrm{yr}) \sim 10^{10.7}(M_\odot/M_{MS})^{4.3}$, where τ_{He} is the lifetime of a single helium star and M_{MS} is the mass of the main-sequence precursor]. Successive applications of equation (6) in Paper I for CEs lead to an estimate of the quantity $\Delta \log A_0$ required for an application of the birth function. The scenario program finds 2.8–4 $M_\odot$ as the range in the initial mass of the secondary (so $\Delta q_0 \sim 0.044$), and $\Delta \log A_0 \sim 0.6$. Altogether, $\nu = 3.9 \times 10^{-6}$ yr^{-1} when $\alpha_{CE} = 1.0$ and $\nu = 0$ when $\alpha_{CE} = 0.5$ (Table 1). The number of bright LMXBs of this type predicted to be in the Galaxy is $N_{cons} \sim \nu \times 0.5\, M_\odot/(3 \times 10^{-8}\, M_\odot\ \mathrm{yr}^{-1}) \sim 30$. The large formal number of $N_{cons} = 3100$ in Table 1 is made up primarily of dim systems evolving during the phase of steadily declining mass-transfer rate.

4.5. *Helium Algol-like LMXBs*

In the helium Algol-like scenario (channel 5 from the left in Fig. 6; mode 7 mass transfer in Fig. 4), the donor in the conjectured LMXB has a thick nondegenerate helium mantle above a degenerate CO or ONe core (Iben & Tutukov 1993). The donor is the consequence either of early case C evolution (the secondary first fills its Roche lobe after exhausting helium at the center) or of case B followed by case BB evolution (the secondary first fills its Roche lobe after exhausting hydrogen in its core, shrinks within this lobe during core helium burning, and then fills it again after exhausting helium at its center). Mass transfer is driven by the tendency of the helium mantle to expand as the mass of the degenerate core increases. The mass-exchange rate for such binaries (in the mass-conservative approximation) varies from $3 \times 10^{-8}\, M_\odot$ yr^{-1} ($M_{20} = 4\, M_\odot$) to $4 \times 10^{-7}\, M_\odot$ yr^{-1} ($M_{20} = 6\, M_\odot$) for the closest systems ($A \sim R_\odot$). It becomes even larger for initially more massive secondaries.

For such accretion rates, probably only systems in which the accretor is a black hole of mass ~10 $M_\odot$ can avoid the formation of a CE and evolve as LMXBs. Taking $M_{10} = 40$–50 $M_\odot$, $\Delta \log A_0 \sim 1.0$, and $\Delta q_0 \sim \frac{2}{45}$, we have $\nu \sim 6 \times 10^{-6}$. With a lifetime of ~$10^6$ yr, as given in the mass-conservative approximation, only a few such binaries are expected in the entire Galaxy. Their orbital periods after two CE phases are in the range 1–40 minutes (e.g., Tutukov & Fedorova 1989). There are as yet no observational counterparts of LMXBs such as these (van Paradijs 1995).

4.6. *LMXBs with a Helium Degenerate Dwarf Donor*

The last conjectured LMXB type which we consider consists of a degenerate dwarf transferring matter to a relativistic component (the last scenario on the right in Fig. 6; mode 8 mass transfer in Fig. 4). The primary agent for establishing Roche lobe contact and for mass exchange is GWR. For a degenerate dwarf of mass in the range 0.1–1.4 $M_\odot$, the mass-radius (M_{dd}-R_{dd}) relationship

$$R_{dd}/R_\odot = 0.026 - 0.0167 M_{dd}/M_\odot \qquad (14)$$

is adopted. This is a good approximation for cold degenerate dwarfs (e.g., Iben & Tutukov 1989a). The main uncertainty introduced by the use of equation (14) for low-mass dwarfs is that there is a rather strong dependence of the radius on the age of the dwarf (e.g., Iben & Tutukov 1986; Vennes, Fontaine, & Brassard 1995). Nevertheless, since a dense enough net of evolutionary tracks for low-mass helium dwarfs is not available, it has been assumed in the present study that all dwarfs, independent of their age, have radii given by equation (14). When Roche lobe contact is established, the mass-exchange rate does not exceed the Eddington limiting value if the mass of the degenerate helium dwarf is less than ~0.1 $M_\odot$ (Tutukov & Yungelson 1979a). However, the initial minimum mass of a helium dwarf is ~0.13 $M_\odot$ (e.g., Iben 1967; Iben & Tutukov 1984a, b). When $M_{dd} > 0.1\, M_\odot$, a TŻO with a helium envelope is formed. LMXBs consisting of a low-mass degenerate helium dwarf and a black hole do not, according to the scenario model calculations, form in this way.

To explain the origin of LMXBs of short orbital period like

1820−303 ($P_{\rm orb}$ = 0.19 hr) or 1916−053 ($P_{\rm orb}$ = 0.83 hr) (van Paradijs 1995), a helium nondegenerate donor (see § 4.4) or a slightly evolved hydrogen-rich donor of initial mass $\sim 1\ M_\odot$ (Tutukov et al. 1987) can be proposed.

4.7. *Summary of Direct Neutron Star and Black Hole Formation Scenarios*

There are two principal ways in which the neutron star or black hole in an LMXB is formed by collapse of the Fe-Ni core of the primary in a precursor system. Binaries which consist initially of a component of mass $12.4 \pm 1\ M_\odot$ and a component of mass 0.3–1 $M_\odot$ at a separation of $\sim 150\ R_\odot$ evolve into CV-like LMXBs in which mass transfer is forced by a MSW. Since the mass donor is a main-sequence star, the LMXB may evolve into a single star when the mass ratio of components becomes of the order of few hundredths of a solar mass (Ruderman & Shaham 1983). Binaries which consist initially of a component of mass $14 \pm 1.1\ M_\odot$ and a component of mass 1–1.7 $M_\odot$ at a separation of $\sim 300\ R_\odot$ evolve into Algol-like LMXBs in which mass transfer is forced by nuclear evolution and possibly also by a MSW (e.g., Iben & Tutukov 1984b). Since the donor consists of a subgiant or giant with a compact degenerate helium core, an Algol-like LMXB evolves into a wide binary consisting of a neutron star and a helium white dwarf. The numbers of LMXBs with a neutron star component in the Galactic disk are predicted to be 4800 (Algol-like) and 480 (long-period CV-like) when $\alpha_{\rm CE}$ = 1 and 230 (Algol-like) and 0 (CV-like) when $\alpha_{\rm CE}$ = 0.5. The corresponding numbers of LMXBs with black hole components are predicted to be 1100 (Algol-like) and 1200 (long-period CV-like) when $\alpha_{\rm CE}$ = 1 and 940 (Algol-like) and 36 (long-period CV-like) when $\alpha_{\rm CE}$ = 0.5.

All numbers presented here have been obtained on the assumption that LMXB evolution is mass-conservative. These statistics are reexamined in §§ 6.2 and 6.3, where a stellar wind from the donor induced by absorption of the X-ray emission from the accretor is taken into account. An estimate of the ratio of LMXBs with neutron star accretors to the number of LMXBs with black hole accretors depends not only on the birthrates of the two types of system but also on the lifetimes of the two types. An estimate of lifetime depends on the intensity of the irradiation-induced wind, and this intensity may well depend on the nature of the accretor. Hence, a reliable estimate of the desired ratio is not at present possible.

Examination of Figure 1*b* is useful in seeing where observed systems with black hole accretors fit into the picture developed so far. In this figure, the dashed lines define the borders for conservative mass transfer of various kinds when the mass of the black hole is chosen as 10 $M_\odot$. Cyg X-1, with $M_{\rm opt} \sim 30\ M_\odot$ and $M_{\rm BH} \sim$ 7–18 $M_\odot$ (Cowley 1992; Tutukov & Cherepashchuk 1992), is one of the best established cases for a black hole more massive than the upper limit on the mass of a stable neutron star. This system is a bright ($\dot{M} \sim 10^{-8}\ M_\odot\ {\rm yr}^{-1}$), persistent HMXB in which mass exchange occurs by accretion onto a black hole from the wind of a massive OB star which is close to filling its Roche lobe (see, e.g., Paper I). Mass transfer in the very bright ($\dot{M} \sim 10^{-7}\ M_\odot\ {\rm yr}^{-1}$), persistent source LMC X-3 is probably driven by the evolutionary expansion of a main-sequence star of mass $\sim$ 3–6 $M_\odot$ (mode 3 mass transfer in Fig. 4), and it is thus appropriate to call LMC X-3 an LMXB (according to the convention adopted in Paper I). The other three systems are transient sources, sometimes called X-ray novae, with estimated recurrence times of 1–60 yr (e.g., Bradt & McClintock 1983). Mass transfer in XN Mus and A0620+00 (V616 Mon) is probably driven by a MSW, and mass transfer in V404 Cyg is probably driven by the evolutionary expansion of a subgiant. The estimated masses of the optical components are consistent with these interpretations, and it seems reasonable to classify XN Mus and A0620+00 as CV-like LMXBs and V404 Cyg as an Algol-like LMXB. The reason for the variability of the three X-ray novae is probably associated with a low accretion rate ($\sim 10^{-9}\ M_\odot\ {\rm yr}^{-1}$), which can lead to the development of an unstable disk and to variations in the mass-loss rate of the donor caused by X-ray irradiation (Chen, Livio, & Gehrels 1993).

The scenario program forms $T\dot{Z}O$s (red supergiants with a neutron star or black hole core) in every evolutionary phase when the following two conditions are simultaneously fulfilled: (1) the accretion rate on the relativistic star is so large that the accretion luminosity exceeds the Eddington limit for that star, causing a CE to be formed; (2) the formal orbital separation after the dispersal of the CE is too small for the secondary to be detached. The program forms $T\dot{Z}O$s from HMXBs when, after the CE event, the helium core plus the hydrogen-rich remnant envelope of the donor overfills its Roche lobe (Paper I). Such $T\dot{Z}O$s have hydrogen-rich envelopes (line 12 in Table 1). Another avenue for $T\dot{Z}O$ formation is the merger of a degenerate dwarf with a relativistic component, which occurs in a close system driven by GWR when the degenerate dwarf fills its Roche lobe and transfers mass at a rate larger than the Eddington limit (§ 4.6). Such $T\dot{Z}O$s can be helium-rich (line 13 of Table 1), CO-rich, or ONe-rich. The last two variants have not been addressed in this paper, since their X-ray phases are extremely short, if they exist at all; the scenario results for them are combined and placed in line 14 of Table 1. A final $T\dot{Z}O$ variant is formed when, in systems consisting of helium nondegenerate donors and neutron stars, the helium star first fills its Roche lobe and transfers mass at a rate larger than the Eddington limit (§ 4.4; line 15 in Table 1). The envelope of this variant is, of course, helium-rich.

5. ACCRETION-INDUCED COLLAPSE OF ONe WHITE DWARFS IN CLOSE BINARIES AND FORMATION OF LMXBs

Neutron star formation by collapse of a massive white dwarf which accretes from a companion has been discussed many times in the literature. One of the earliest papers exploring the physics of white dwarf collapse in connection with LMXBs is one by Canal & Schatzman (1976), who examine the competition between electron-capture–induced collapse and pycnonuclear explosion (e.g., Finzi & Wolf 1967). Their motivation was to achieve a nonexplosive collapse in order to increase the probability that a binary remains bound during the formation of the neutron star component. Accretion-induced collapse (AIC) has been invoked to understand the properties of several observed LMXBs (e.g., Flannery & van den Heuvel 1975; Taam & van den Heuvel 1986; van den Heuvel 1987b; Narayan & Popham 1989; Pylyser & Savonije 1988; Coté & Pylyser 1989; Ruderman 1991; Woosley & Baron 1992), and others have argued against AIC (e.g., Verbunt, Wijers, & Burm 1990).

During collapse, the size of the accretor decreases by a factor of ~ 1000 and, if the rotational period of the precursor dwarf were ~ 100 s, as is the case in some CVs, one might expect the formation of a very rapidly rotating neutron star (e.g., Narayan & Popham 1989).

The most likely white dwarf precursor is a massive oxygen-neon (ONe) rather than a carbon-oxygen (CO) white dwarf, since the latter is expected to explode rather than collapse (Arnett 1969; see also Iben 1982; Khokhlov 1991). An ONe white dwarf of mass larger than $\sim 1.39\ M_\odot$ is unstable because of electron capture on sodium, magnesium, and oxygen, and the energy generated by oxygen burning cannot prevent collapse into a neutron star (Miyaji et al. 1980; Hillebrandt, Nomoto, & Wolf 1984; Nomoto, Thielemann, & Yokoi 1984). At birth, an ONe white dwarf formed in a close binary system can have a mass anywhere in the range 1.1–1.39 $M_\odot$ (Nomoto 1984, 1987; Miyaji & Nomoto 1987; Hashimoto, Iwamoto, & Nomoto 1993), depending on the mass of its progenitor and on the details of the CE event during which it is formed (Iben & Tutukov 1985). If the initial white dwarf is massive enough and cold enough, and if the companion of the white dwarf transfers matter to the white dwarf at an appropriate rate, some of the accreted matter may remain on the white dwarf, despite possible nova-like outbursts, and the mass of the white dwarf may grow to the critical value.

If its initial mass is in the range ~ 10.3–10.6 $M_\odot$, a component in a close binary may evolve into a cold ONe white dwarf in a late case B event (e.g., Iben & Tutukov 1985; Dominguez, Tornambè, & Isern 1993). If, on filling its Roche lobe, the companion transfers matter to the cold white dwarf at an appropriate rate (e.g., $\dot{M}$ large enough to avoid strong flashes, so that nuclear burning processes matter at the same rate that it is accreted, but small enough that the accreted hydrogen-rich layer does not expand to a size larger than the Roche lobe about the cold white dwarf), a layer of electron-degenerate helium builds up above the underlying degenerate core. If the total mass of the degenerate configuration reaches a value of ~ 1.39 $M_\odot$ before the accreted helium layer reaches a critical mass $M_{\rm mantle}^{\rm crit}$ somewhere in the range 0.05–0.25 $M_\odot$ (Taam 1980; Nomoto 1982; Iben & Tutukov 1991; Limongi & Tornambè 1991; Woosley & Weaver 1994), electron captures in the ONe core will initiate a collapse that cannot be halted by the explosive burning of oxygen (Miyaji et al. 1980; Nomoto 1984, 1987; Hashimoto et al. 1993; but see Canal, Isern, & Labay 1992), and the core evolves into a neutron star. Helium burning may be ignited in the helium mantle, some of which may be ejected and some of which may become part of the neutron star.

If the total mass of the degenerate configuration is less than 1.39 $M_\odot$ when the mass of the helium mantle reaches $M_{\rm mantle}^{\rm crit}$, explosive helium burning may expel the entire mantle (the "super"-nova mode) or lead to the disruption of the entire star (the double-detonation supernova mode), depending on the mass of the star at the moment of explosion. The value of $M_{\rm mantle}^{\rm crit}$ appears to be rather sensitive to the input physics adopted, ranging from 0.08 $M_\odot$ (Nomoto 1982) to 0.25 $M_\odot$ (Woosley & Weaver 1994). A value of $M_{\rm mantle}^{\rm crit} = 0.15\ M_\odot$ is adopted in the scenario program. This value is close to the estimate of Iben & Tutukov (1991), who find $M_{\rm mantle}^{\rm crit} \sim 0.13$–0.14 $M_\odot$ for a cold (interior temperature $\lesssim 2 \times 10^7$ K, corresponding to a cooling age $\gtrsim 10^8$ yr) white dwarf of initial mass 0.6–1 $M_\odot$ which accretes helium at a rate $\sim 3 \times 10^{-8}\ M_\odot\ {\rm yr}^{-1}$. The critical mantle mass depends only weakly on the mass of the white dwarf, due to the fact that the entire white dwarf must be heated up to average temperatures close to the ignition temperature of helium in the mantle, and it is the same for an ONe core as it is for a CO core. The accretion rate above which $M_{\rm mantle}^{\rm crit}$ begins to depend strongly on the accretion rate is not known precisely.

These results are also valid if the accreted matter is hydrogen-rich and a helium mantle grows in consequence of hydrogen burning. Spectra of many classical novae (which have as precursors hydrogen-accreting white dwarfs) show large overabundances of CNO elements over solar (Mustel & Boyarchuk 1959, 1960; Pottasch 1959; Williams 1985), some by factors of 10–100, and these enrichments cannot be achieved by hydrogen burning of solar system matter during the nova outburst. They can be achieved only if there is substantial mixing between the matter at the base of the hydrogen-rich accreted layer and matter near the surface of the underlying white dwarf. The inevitable conclusion is that more matter is lost during the nova outburst than has been accreted prior to the outburst (e.g., MacDonald 1983; Fujimoto & Iben 1992). Thus, over time, the average mass of the white dwarf decreases in those systems that eject highly enriched matter during the nova outburst. The recognition that the white dwarf in several CVs, such as V693 CrA and V1370 Aql, is an ONe white dwarf follows from the fact that, in them, neon is enriched in the nova ejecta by orders of magnitude over solar (e.g., Livio & Truran 1994).

This evidence for "erosion" could discourage a pursuit of the possibility that hydrogen-rich matter can be accreted on a degenerate dwarf at a rate which permits the mass of the white dwarf to increase. However, in one-third of all classical novae for which abundance analyses have been made, the ejecta do not exhibit overabundances of heavy elements (e.g., RR Pic, LMC 1990, and U Sco). Further, symbiotic novae are clear examples of systems in which the thermonuclear outburst is not preceded by a substantial mixing which leads to an overproduction of heavy elements (e.g., Iben & Tutukov 1995b). To our knowledge, none of seven known symbiotic novae display an overabundance of heavy elements.

Even more encouragement for the possibility of a secular increase in mass during a long accretion phase is provided by the properties of the binary radio pulsar PSR 1718−19 in the globular cluster NGC 6342 (Lyne et al. 1993). The eclipses in the system are presumably due to obscuration by coronal matter of a rapidly rotating optical companion of mass ~ 0.2 $M_\odot$ (Burderli & King 1994). The orbital period is 6.2 hr, and the pulsar has a 1 s spin period, a characteristic spin-down timescale of $\sim 10^7$ yr, and a magnetic field strength of 1.5×10^{12} G. The fact that the neutron star has a strong magnetic field suggests that it was formed very recently. The facts that, in projection, the system is within 10 core radii of the center of the globular cluster and that (if it is indeed in the cluster) this places it well within the tidal radius of the cluster are strong evidence that the system is bound to the cluster.

If the pulsar really belongs to the cluster, the possibility that it arose in consequence of the collapse of an ONe white dwarf in a CV-like system within the globular cluster is compelling. A possible scenario is that the white dwarf either tidally captured a low-mass main-sequence star (Fabian et al. 1975) or ejected and replaced the lighter component of a tight binary (Hills 1975). Eventually, due to a MSW, the main-sequence companion of the white dwarf filled its Roche lobe and transferred matter at a rate which the white dwarf could accrete. On achieving the critical mass, the white dwarf then collapsed to form a neutron star.

This scenario is invoked by Wijers & Paczyński (1993) to show that the orbital period of the current system is within the narrow limits 5.6–8.2 hr expected if, at the moment of the collapse of the white dwarf, the precursor system was a CV with a donor of mass $(M_2/M_{\odot}) \sim (P_{\rm orb}/8.9\ {\rm hr})$. This M_2-$P_{\rm orb}$ relationship predicts that the donor mass at the moment of the formation of the neutron star was $\sim 0.7\ M_{\odot}$, which is consistent with the fact that the maximum mass of a (primordially) single nuclear-burning star in a globular cluster is less than $\sim 0.8\ M_{\odot}$. A possible answer for why the current mass of the secondary is so much smaller than the mass of the secondary when the neutron star was first formed is that, being bright and having a strong magnetic field, the young neutron star succeeded in evaporating a significant part of the secondary (e.g., Ruderman et al. 1989a, b; see also § 6.2); the energy required for evaporation presumably comes from the rotational kinetic energy of the neutron star.

In the scenario program, it is assumed that, if the initial mass of the cold ONe white dwarf is in the range 1.24–1.39 $M_{\odot}$ (corresponding to the choice $M^{\rm crit}_{\rm mantle} = 0.15\ M_{\odot}$) and if the accretion rate is $\lesssim 3 \times 10^{-8}\ M_{\odot}\ {\rm yr}^{-1}$ (otherwise a CE will be formed after neutron star formation), the white dwarf will implode while accreting hydrogen, before helium is ignited in the mantle. The net result may be a low-mass neutron star, low-mass both because the gravitational mass of a neutron star is $\sim 0.15\ M_{\odot}$ smaller than its baryonic mass and because the helium mantle may "lift" off from the imploding core in consequence of an explosion initiated by helium burning (see, e.g., Nomoto & Kondo 1991). Most important of all, in order to take into account formation of ONe white dwarfs in early case CE events, the range of masses of primaries which can become ONe white dwarfs is widened from the 10.3–10.6 $M_{\odot}$ range given by Iben & Tutukov (1985) to 9.6–11.4 $M_{\odot}$. This optimistic choice may considerably overestimate the probability of realization of this scenario, but the real volume in parameter space for appropriate initial binaries is unknown.

In any case, estimates of the number of massive degenerate dwarfs with masses close to the Chandrasekhar limit remain uncertain, since they are not as yet supported directly by the white dwarf mass distribution inferred from the observations. Zhang & Kwok (1993) find no central stars of planetary nebulae in the Galaxy more massive than 1.1 $M_{\odot}$, and Bergeron, Saffer, & Liebert (1992) find no cooling white dwarfs with masses larger than 1.2 $M_{\odot}$. Kaler & Jacoby (1991) find that the distribution of masses of central stars in the Magellanic Clouds extends up to $\sim 1.2\ M_{\odot}$. Without a special analysis, which properly takes into account observational selection effects, it is not clear whether these are results of selection against very massive dwarfs or whether there are really some problems with their formation (e.g., Han, Podsiadlowski, & Eggleton 1994). The situation with respect to massive dwarfs may become clearer when appropriate statistics for ultrasoft X-ray sources become available, since the very high surface temperatures of these stars indicate that they are probably mostly massive accreting white dwarfs (van den Heuvel et al. 1992; Iben & Tutukov 1994, 1995b).

To complete the story, it is necessary to explain why many white dwarfs in CVs (those which eject nova shells with large overabundances of heavy elements) are decreasing in mass. We propose that the white dwarfs in these CVs are initially less massive than 1.39 $M_{\odot} - M^{\rm crit}_{\rm mantle}$ and that evolution in them proceeds exactly as we have outlined, with the differences that (a) the helium mantle achieves its maximum mass before the mass of degenerate matter in the white dwarf reaches the Chandrasekhar limit and (b), in the ensuing explosion triggered by helium burning, essentially the entire mantle is removed, leaving a "bare" CO or ONe core. During the next accretion episode, with no helium mantle to block it (see Iben, Fujimoto, & MacDonald 1992b), hydrogen penetrates into the CO or ONe interior in consequence of diffusion or rotationally induced mixing or both, and during the next hydrogen-burning runaway, heavy elements are mixed into the hydrogen-rich envelope, most of which is ejected in the next outburst. Thereafter, since $\Delta M_{\rm He}$ (the mass of the helium-rich layer which remains at the surface of the white dwarf after nuclear burning ceases) decreases with an increasing abundance of heavy elements (Fujimoto & Iben 1992), it is possible that $\Delta M_{\rm dg}$ (the amount of white dwarf material which is incorporated into the convective shell at the start of a nova outburst) is larger than $\Delta M_{\rm He}$, even though the reverse was true before the removal of the large helium mantle. Because many more white dwarfs are produced with initial mass less than 1.24 $M_{\odot}$ than with mass greater than 1.24 $M_{\odot}$, one may infer that many more CVs evolve into systems which produce heavy-element–enriched novae than evolve into systems which produce neutron stars.

Whatever the truth of the matter, it is assumed in the scenario program that accretion onto an ONe white dwarf of hydrogen-rich matter at a rate in the range $\sim (1\text{–}3) \times 10^{-8}\ M_{\odot}\ {\rm yr}^{-1}$ or of hydrogen-free matter at a rate less than $3 \times 10^{-8}\ M_{\odot}\ {\rm yr}^{-1}$ can lead to the formation of a Chandrasekhar-mass configuration which implodes into a neutron star. Inspection of Figure 4 shows that such rates are predicted for the longest period systems of both the case 1 and the case 5 varieties. A 100% efficiency of accretion is assumed, and wind mass loss is completely neglected.

Six evolutionary channels are described in Figure 8. The birthrates ν and the number $N_{\rm cons}$ of systems of each type given by the numerical scenario program (for both $\alpha_{\rm CE} = 1$ and $\alpha_{\rm CE} = 0.5$) are displayed in Table 1. In the following sections the ingredients that enter into making each estimate are outlined. The scenario variants in Figure 8 are numbered from left to right.

5.1. *Systems with Main-Sequence Donors: CV-like LMXBs*

To form an ONe white dwarf of mass in the range 1.24–1.39 $M_{\odot}$, the initial mass of the primary must be in the interval 9.6–11.4 $M_{\odot}$. In the scenario program it is assumed that stars of initial mass 9 $M_{\odot}$ produce an ONe white dwarf of mass 1.2 $M_{\odot}$, and those of initial mass 11.4 $M_{\odot}$ produce an ONe white dwarf

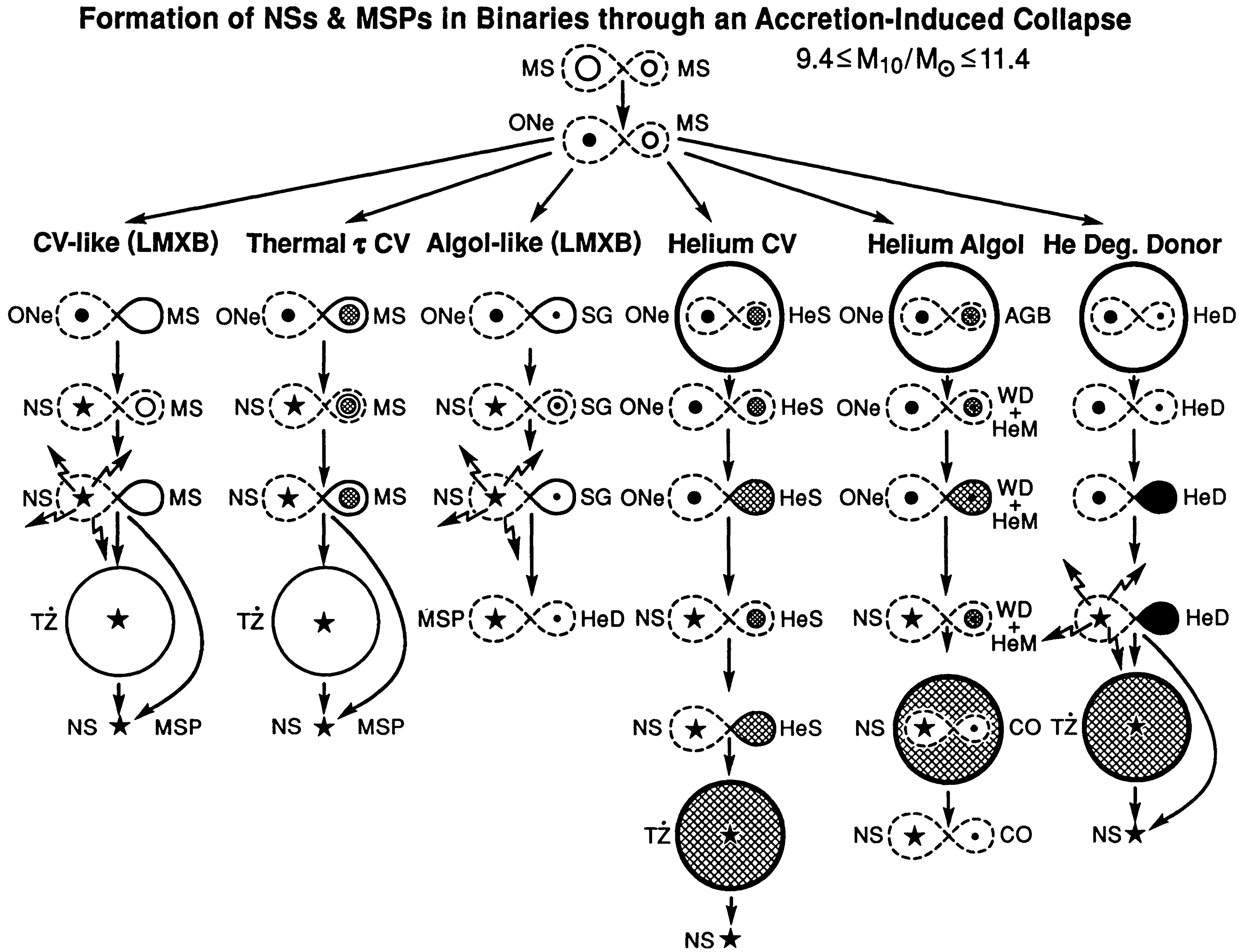

FIG. 8.—Evolutionary scenarios for binaries in which a neutron star is the consequence of the collapse of an ONe white dwarf (§ 5). Alphabetical symbols have the same meanings as in Fig. 6.

of mass 1.39 $M_\odot$. To achieve type 1 mass exchange (Fig. 4), the mass of the progenitor of the donor must be $\lesssim 1$ $M_\odot$, and this ensures that the ratio of primary mass to secondary mass in the precursor system is large enough that a CE is formed and that considerable orbital shrinkage occurs. It is evident from equations (4)–(6) in Paper I that orbital shrinkage by a factor of the order of 100 is to be expected during the CE event.

For M_2 = 0.63–1.0 $M_\odot$, equation (6) gives $\dot{M}_{\rm MSW}$ = (1.0–3.2) $\times$ 10^{-8} $M_\odot$ yr^{-1}. Thus, there is overlap between the available mass-exchange rates and the "gate" for accretion at the desired rate $\sim$(1.0–3.4) $\times$ 10^{-8} $M_\odot$ yr^{-1} (the upper limit is from eq. [7] when $X = 0.7$ and $M_{\rm NS}$ = 1.25 $M_\odot$) onto the neutron star eventually formed by collapse. Since $\sim$(0.15/2) $M_\odot$ must be permanently added to the white dwarf before collapse, only donors with initial mass in the range 0.7–1.0 $M_\odot$ contribute. Thus, for the purpose of estimating a birthrate using equation (4), $\Delta q_0 \sim 0.03$.

There are several ways of estimating the range in the semimajor axis of appropriate primordial binaries. The scenario program uses the relationship between initial and final semimajor axes given by a variant of equations (4)–(6) in Paper I and chooses those final systems in which the secondary will fill its Roche lobe in less than the age of the Galactic disk. The minimum semimajor axis for a system in which the components have masses of, say, 0.9 and 1.4 $M_\odot$ is $\sim$2.3 $R_\odot$. The maximum semimajor axis which the system can have initially and still achieve mass transfer by Roche lobe filling within the age of the disk is $\sim$10 $R_\odot$ (Tutukov 1985). Hence, one might guess $\Delta \log A_0 \sim \log(10/2.3) \sim 0.63$. However, the more fine-tuned exploration of the scenario program shows that this is a considerable overestimate: many systems can be rejected because of orbital shrinkage and merger during the second CE stage after expansion of the helium remnant. Since, in a case BB scenario, two CE events occur, equations (4)–(6) in Paper I must be applied twice. The net result is that the phase space for A_0 is reduced to $\Delta \log A_0 \sim 0.1$.

Inserting estimates for the Δ's in equation (4), the birthrate of LMXBs in which mass transfer is of the type 1 variety is

$$\nu \sim (0.2)(0.1)\left[\frac{1.8}{(10.5)^{2.5}}\right]0.03 \text{ yr}^{-1} \sim 3 \times 10^{-6} \text{ yr}^{-1} . \quad (15)$$

The scenario program produces $\nu \sim 1.5 \times 10^{-6}$ yr^{-1} when $\alpha_{CE} = 1.0$ and $\nu \sim 1.2 \times 10^{-6}$ yr^{-1} when $\alpha_{CE} = 0.5$ (Table 1). The number of long-period disk LMXBs produced by this route (see § 4.1 for a description) is estimated by the scenario program to be $N_{cons} \sim 190$ when $\alpha_{CE} = 1.0$ and $N_{cons} \sim 150$ when $\alpha_{CE} = 0.5$ (Table 1). It is interesting that when $\alpha_{CE} = 0.5$ the ONe WD → NS scenario is the dominant source of short-period CV-like LMXBs, producing $N_{cons} = 2400$ such systems.

5.2. *Systems with Evolving Main-Sequence Secondaries: Mass Transfer on a Nuclear or Thermal Timescale*

When mass transfer is of the case 3 or case 4 variety (Fig. 4), once again a large initial mass ratio is involved and a CE event brings about considerable orbital shrinkage. After the CE event which leads to case 3 mass transfer, the main-sequence secondary fills its Roche lobe and transfers mass because of radius expansion due to nuclear transformations in its interior. The mass-exchange rate is given by equation (9). In order to avoid runaway mass transfer, the mass ratio $q = M_2/M_{ONe}$ must be less than ~ 1.2. Since $M_{ONe} \sim 1.3\ M_\odot$, it follows that $M_2 \lesssim 1.56\ M_\odot$. However, the mass-exchange rate must also be larger than $\sim 1 \times 10^{-8}\ M_\odot$ yr^{-1}. According to equation (9), this requires $M_2 > 3\ M_\odot$. One may conclude from this contradiction that a viable case 3 mass-exchange scenario does not exist.

In the scenario involving mass exchange of the type 4 variety, the secondary is a main-sequence or near–main-sequence star which is initially more massive than the ONe white dwarf and which transfers mass on the thermal timescale of its envelope. If the donor is on the zero-age main sequence, its mass must be initially at least as large as $1\ M_\odot$ and at least 1.2 times the mass of the accretor (Tutukov et al. 1982). Since M_{ONe} must typically be $\sim 1.3\ M_\odot$ initially, it follows that M_{20} is typically larger than $1.56\ M_\odot$. Because of a sharp decrease of donor radius in response to mass loss (Tutukov et al. 1982; Hjellming & Webbink 1987), the restriction on the donor-accretor mass ratio q operates as long as the donor has not developed a degenerate helium core and evolved close to the Hayashi border; when the donor has evolved into a full-fledged subgiant, the system becomes an Algol-like system (§ 4.3). Thus, mass transfer on a thermal timescale is possible for systems in which the donor is initially of mass comparable to, but somewhat larger than, the mass of the ONe white dwarf.

To find typical mass-exchange rates, evolutionary tracks for low-mass stars computed by Sweigart & Gross (1978) and by Mengel et al. (1979) have been used. $\dot{M}$-P_{orb} characteristics of systems containing donors of three different masses are shown in Figures 4 and 5. The parameter which varies along each curve of constant mass is the age of the donor when it first fills its Roche lobe; the more evolved the donor when it first fills its Roche lobe, the larger the mass-exchange rate. Mass transfer continues in this case at a nearly constant rate, at least until the mass ratio is inverted (Paczyński 1971), and, at some point, it continues on a nuclear-burning timescale, whereupon the system moves from the region labeled "(4)" to the region labeled "(5)" in Figure 4.

It is evident that, in systems in which mass transfer is of type 4 and in which the donor has an initial mass 1.0–1.4 $M_\odot$, the mass-exchange rate is precisely in the range favorable for the development of a helium mantle of mass $\sim 0.15\ M_\odot$ on a cold ONe white dwarf. If the initial mass of the white dwarf is larger than $1.24\ M_\odot$, collapse to a neutron star occurs. In order to estimate the birthrate of neutron stars via this scenario, we set $M_{10} \sim 10\ M_\odot$, $\Delta M_{10} \sim 1.8\ M_\odot$, $\Delta q_0 \sim 0.04$, and $\Delta \log A_0 \sim 0.2$ to obtain $\nu \sim 0.9 \times 10^{-5}$ yr^{-1}. Once the neutron star is formed, an LMXB will not result because, when the donor again moves into Roche lobe contact, mass transfer resumes on the thermal timescale of the envelope of the donor, a CE is formed, and a merger probably ensues, producing a TZO. For this reason, in spite of a rather large estimated birthrate for neutron stars, this scenario is not included in the numerical program and estimates of frequency are not given in Table 1.

5.3. *Systems with Subgiant or Giant Donors: Algol-like LMXBs*

In the standard model, $\tau_{disk} = 1.5 \times 10^{10}$ yr and it is assumed that mass loss during the main-sequence and giant stages, except at the very top of the giant branch, is negligibly small. In order to become a giant in less than τ_{disk}, the secondary must have an initial mass larger than $\sim 0.8\ M_\odot$. In order to avoid the formation of a CE when it fills its Roche lobe and begins to transfer mass to its ONe white dwarf companion, the subgiant or giant must have a mass less than $\sim 1.2 \times M_{ONe} \sim 1.5\ M_\odot$ (if $M_{He} < 0.25\ M_\odot$), or less than $\sim 0.67 \times M_{ONe} \sim 0.84\ M_\odot$ (if $M_{He} > 0.25\ M_\odot$). Thus, when it fills its Roche lobe, the secondary in the primordial binary must have a mass in the range 0.8–1.5 $M_\odot$. A further restriction is set by a consideration of allowable semimajor axis.

In order to achieve $\dot{M}_{ex} \geq 10^{-8}\ M_\odot$ yr^{-1} when $M_2 \sim 0.8\ M_\odot$ and $M_{tot} \sim 2.1\ M_\odot$, the semimajor axis of the system must, according to equation (12), exceed $\sim 16\ R_\odot$. After the first CE phase, during which the helium star precursor of the ONe white dwarf is formed, the semimajor axis of the system is less than $\sim 30\ R_\odot$ if the initial semimajor axis is less than $\sim 1500\ R_\odot$. Thus, $\Delta \log A_0 \sim 0.3$, and $\Delta q_0 \sim 0.004$. Inserting these estimates in equation (4), a birthrate $\nu \sim 1.2 \times 10^{-6}$ yr^{-1} is obtained. The birthrate given by the scenario code is $\nu = 1.8 \times 10^{-6}$ yr^{-1} when $\alpha_{CE} = 1$ and $\nu = 4.5 \times 10^{-7}$ yr^{-1} when $\alpha_{CE} = 0.5$ (Table 1).

After the formation of the neutron star, a long phase as an LMXB is expected. The scenario program finds $N_{cons} = 540$ when $\alpha_{CE} = 1$ and $N_{cons} = 46$ when $\alpha_{CE} = 0.5$. Cyg X-2 is a possible observational counterpart, although the direct way of forming Algol-like systems is considerably more probable (see § 4.3 and Table 1).

Since only the initially most massive dwarfs can accrete enough matter to collapse, the frequency of transitions of ONe dwarfs into neutron stars is almost independent of the assumed age of the Galactic disk. When the $1.56 - 2q$ factor in equation (12) is included, and it is insisted that initial $q < 0.78$, the frequency of ONe → NS transitions reduces to 1.7×10^{-7} yr^{-1} when $\alpha_{CE} = 1$ (almost 10 times less than given in Table 1).

5.4. *Helium CV-like Systems*

There are three paths which lead to the transfer of helium and other products of hydrogen burning (mostly ^{14}N) at a rate which allows an ONe white dwarf to grow in mass. One path leads to transfer from a core helium-burning "helium star" (type 6 mass transfer in Fig. 4), the second leads to transfer from the helium mantle of a "helium Algol" which has a CO or ONe core (type 7 mass transfer in Fig. 4), and the third leads to transfer from a helium degenerate dwarf (type 8 mass transfer in Fig. 4).

In type 6 mass transfer, the mass-transfer rates are unusually well determined and accretion on an ONe white dwarf of enough matter to cause it to collapse into a neutron star seems assured in many cases. The driving force for mass transfer is GWR, so the average mass-exchange rate is $\sim 3 \times 10^{-8}\ M_{\odot}\ \mathrm{yr}^{-1}$ (see § 4.4). The conditions are perfect for the formation of a degenerate helium mantle of $\sim 0.15\ M_{\odot}$ on a cold white dwarf, and, if the initial mass of the ONe white dwarf is $\gtrsim 1.24\ M_{\odot}$, collapse into a neutron star occurs prior to helium ignition in the mantle.

Because the gravitational mass of the neutron star ($M_{\mathrm{NS}} \sim 1.25\ M_{\odot}$) is smaller than the gravitational mass of its white dwarf precursor, and because some mass may be lost from the system in the course of the collapse, the semimajor axis of the system becomes slightly larger than before the collapse (e.g., Zeldovich & Novikov 1971; Verbunt et al. 1990). The helium star temporarily detaches from its Roche lobe, but reestablishes contact in consequence of GWR. If the mass-exchange rate resumes at $\sim 3 \times 10^{-8}\ M_{\odot}\ \mathrm{yr}^{-1}$, which is comfortably below the Eddington limit of $5.8 \times 10^{-8}\ M_{\odot}\ \mathrm{yr}^{-1}$ for a $1.25\ M_{\odot}$ neutron star accreting helium, a long LMXB phase will follow. On the other hand, another spike in the accretion rate might be expected, aborting the LMXB phase (see § 4.4). The scenario program gives $\nu \sim 10^{-4}\ \mathrm{yr}^{-1}$ as the birthrate of neutron stars by this scenario when $\alpha_{\mathrm{CE}} = 1$, and $\nu \sim 10^{-5}\ \mathrm{yr}^{-1}$ when $\alpha_{\mathrm{CE}} = 0.5$. The birthrate of semidetached systems consisting of a neutron star and a helium star via this scenario is (when $\alpha_{\mathrm{CE}} = 0.5$) comparable to the birthrate of systems by the scenario in § 4.5. Because of the lack of observed counterparts (Fig. 4), details of a possible LMXB phase are not listed in Table 1.

5.5. *Helium Algol-like Systems and Formation of NS + WD Binaries*

In type 7 mass transfer (Fig. 4), the driving force is the evolutionary expansion of the nondegenerate helium mantle of mass ~ 0.1–$0.4\ M_{\odot}$ surrounding a degenerate CO or ONe core of mass $\gtrsim 0.56$–$0.61\ M_{\odot}$. The mass-exchange rate is in the range

$$\dot{M}_{\mathrm{ex}}(M_{\odot}\ \mathrm{yr}^{-1}) = (0.1\text{–}10) \times 10^{-6}\,, \quad (16)$$

the precise value being a function of the donor mass and the orbital separation (Iben & Tutukov 1993). An $\dot{M} \gtrsim 10^{-6}\ M_{\odot}\ \mathrm{yr}^{-1}$ is suitable for the (quasi-)stationary burning of helium at the base of the helium layer accreted by the white dwarf. Systems of this type are aesthetically pleasing: both components are structurally very much the same, and both are burning the same fuel; the less evolved component is contributing to the evolution of the more evolved component into an exotic state it could not have achieved by itself. Accretion at rates given by equation (16) provides for a steady growth in mass of a rather compact ONe dwarf and eventual collapse of the dwarf (Iben & Tutukov 1984a, 1994a). Observed counterparts of these binaries could be ultrasoft X-ray sources represented by systems like CAL 83 and CAL 87, but there are several other good possibilities for explaining these systems (van den Heuvel et al. 1992; Rappaport, DiStefano, & Smith 1994; Iben & Tutukov 1994a).

To illustrate what enters into an estimate of the birthrate of systems of the envisioned sort, we examine closely one specific scenario: the primary becomes an ONe white dwarf after filling its Roche lobe as a thermally pulsing AGB star (late case C event), and the secondary fills its Roche lobe just before it ignites helium at the center (late case B event). In both Roche lobe filling events, a CE is formed. In order to produce an ONe white dwarf of mass in the range 1.24–1.39, the primary must have a mass in the interval 9.6–11.4 $M_{\odot}$. In order for the secondary to be less massive than the white dwarf after the second CE event, the initial mass of the secondary must be less than 6 $M_{\odot}$, but in order for sufficient helium to be transferred to push the white dwarf over the Chandrasekhar brink, the secondary must be massive enough to undergo a case BB event, emerging from this event with a helium mantle of mass in the range 0.01–0.15 $M_{\odot}$. This places a lower limit of 5 $M_{\odot}$ on the initial mass of the secondary. The most uncertain quantity is the range in initial A_0. Choosing $\Delta \log A_0 \sim 1.5$, and $\Delta q_0 \sim 0.2$ (Iben & Tutukov 1994a), it follows that

$$\nu \sim (0.2)(1.5)\left[\frac{1.8}{(10.5)^{2.5}}\right]0.2\ \mathrm{yr}^{-1} \sim 3 \times 10^{-4}\ \mathrm{yr}^{-1}\,. \quad (17)$$

As the neutron star of mass $\sim 1.25\ M_{\odot}$ is formed, the orbit expands. The helium mantle of the donor continues to enlarge and, when Roche lobe contact is made again, mass transfer at a rate given by equation (16) resumes. Since this rate is much larger than the Eddington limit rate of $5.8 \times 10^{-8}\ M_{\odot}\ \mathrm{yr}^{-1}$ for a low-mass neutron star accreting helium, a CE is formed and mass loss from the system continues until very little helium is left above the degenerate core of the donor, leaving a neutron star in orbit with a massive degenerate dwarf. Thus, the LMXB phase is bypassed. Immediately following the CE phase, the radius of the compact component is small enough compared with the orbital separation that a merger does not occur. However, as the scenario program shows, in many instances GWR shrinks the system in less than a Hubble time until the massive CO (ONe) dwarf fills its Roche lobe. Because of a relatively large donor-accretor mass ratio, the ensuing mass-exchange rate is too high for a stable semidetached phase. Instead, a TŻO composed of a neutron star core and a CO (ONe) giant envelope forms.

5.6. *CV-like Binaries with Degenerate Helium Donors*

The final scenario leads to the formation of systems in which a low-mass (~ 0.13–$0.2\ M_{\odot}$) helium degenerate dwarf donates mass to a relativistically degenerate companion (see the last pathway on the right in Fig. 8). Mass exchange in such systems [the leftmost branch in Fig. 4, labeled "(8)"] is driven by GWR (see § 4.6). At the moment of Roche lobe filling, the mass-exchange rate is $\lesssim 3 \times 10^{-6}\ M_{\odot}\ \mathrm{yr}^{-1}$ (Tutukov & Yungelson 1979a), allowing for stationary helium burning and the accumulation of carbon in the envelope of a massive ONe dwarf. For Roche lobe filling to occur in less than the age of the Galactic disk, A_f, the semimajor axis at the time of the formation of the helium white dwarf, must satisfy $A_f \leq 2\ R_{\odot}$. The birthrate given by the scenario program is $\nu \sim 10^{-5}\ \mathrm{yr}^{-1}$. Thus, such binaries appear to form rather frequently in the course of stellar evolution.

Especially important are those cases where ONe dwarfs can accrete the critical mass for collapse to a neutron star, avoiding significant expansion and nova-like helium shell explosions which prevent mass accumulation. If it is assumed that, in order to avoid expansion, the mass-exchange rate must be less than that creating the Eddington luminosity for an accreting ONe dwarf, and that the massive accreting dwarf is cold and can accumulate $\sim 0.15\ M_\odot$, the frequency of such events is $\nu \sim 4.3 \times 10^{-5}\ \mathrm{yr}^{-1}$. After the white dwarf collapses into a neutron star, semidetached evolution as an LMXB can occur only if the mass-exchange rate is less than the Eddington limit for the newly formed neutron star. This condition prevents most systems from becoming LMXBs; they typically become TŻOs after the contact of the helium degenerate dwarf with its Roche lobe. Only systems with very low mass donors, less than $\sim 0.06\ M_\odot$, can continue their evolution as LMXBs. The scenario program estimates the birthrate of LMXBs formed in this manner to be $0.9 \times 10^{-5}\ \mathrm{yr}^{-1}$ when $\alpha_{\mathrm{CE}} = 1$. When $\alpha_{\mathrm{CE}} = 0.5$, the rate of collapses is $\nu = 1.5 \times 10^{-5}\ \mathrm{yr}^{-1}$, and the birthrate of LMXBs is $\nu = 2 \times 10^{-6}\ \mathrm{yr}^{-1}$. These rates are very model-dependent, since they are influenced by several important but not yet studied effects such as (*a*) the appropriate conditions for the accumulation of mass by an ONe dwarf accreting helium at a large and rapidly changing rate, (*b*) the radius-mass relationship for mass-losing degenerate dwarfs, (*c*) the role of an induced stellar wind from the donor, and (*d*) the birthrate of massive white dwarfs.

In the conservative approximation, the lifetime of the LMXB phase for this scenario is larger than τ_{disk}. In 15×10^9 yr, the mass of the donor decreases to $\sim 0.006\ M_\odot$ (e.g., Tutukov & Yungelson 1979a). This mass is smaller than the minimum value of $0.02\ M_\odot$ which Ruderman & Shaham (1983) estimate is required for a donor to withstand tidal disruption, but it is larger than the value of $0.002\ M_\odot$ suggested by Hut & Paczyński (1984) as the minimum donor mass for stable mass transfer. The formal number of LMXBs produced in the Galactic disk is very large, $N_{\mathrm{cons}} \sim 1.3 \times 10^5$ when $\alpha_{\mathrm{CE}} = 1$ and $N_{\mathrm{cons}} \sim 2.6 \times 10^4$ when $\alpha_{\mathrm{CE}} = 0.5$, but most of these slowly evolving LMXBs are very dim in X-rays ($L_X \sim 0.1\ L_\odot$). The number of bright sources in the Galactic disk is many times smaller (see Table 1), and cannot exceed $N_{\mathrm{cons}} \sim 10^{-5}\ \mathrm{yr}^{-1} \times 0.1\ M_\odot/(1 \times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}) = 100$, which is comparable to the observed number of LMXBs.

The existence of several LMXBs with a very short orbital period (e.g., 1820 − 30 with $P_{\mathrm{orb}} \sim 11$ minutes; see Fig. 4) makes it prudent to keep this possibility open, in spite of a rather large uncertainty in the birthrate estimate. It is worth mentioning that there are other possibilities for the explanation of such systems, e.g., CV-like evolution with a slightly evolved hydrogen-rich or low-mass helium donor (Tutukov et al. 1985, 1987; Tutukov & Fedorova 1989). Concluding the discussion of this scenario, we note that it can produce a significant number of LMXBs in our Galaxy, but, for more reliable conclusions, additional numerical studies are necessary.

5.7. *Summary of the WD → NS Scenarios*

Summarizing the analysis of six scenarios for transforming an ONe degenerate dwarf into a neutron star, only the CV-like scenario (§ 5.1), the Algol-like scenario (§ 5.3), and the scenario involving a degenerate helium donor (§ 5.6) produce LMXBs in which the accretion rate, if accretion is due to Roche lobe filling, is less than the Eddington limit. Binaries which consist initially of a component of mass $10.5 \pm 0.9\ M_\odot$ and a component of mass 0.7–$1\ M_\odot$ at a separation of $\sim 200\ R_\odot$ evolve into CV-like LMXBs in which mass transfer is forced by a MSW. Binaries which consist initially of a component of mass $10.5 \pm 0.9\ M_\odot$ and a component of mass 0.8–$1.5\ M_\odot$ at a separation of ~ 200–$1000\ R_\odot$ evolve into Algol-like LMXBs in which mass transfer is forced by nuclear evolution and perhaps also by a MSW. Binaries which consist initially of a component of mass $10.5 \pm 0.9\ M_\odot$ and a component of mass 0.8–$2.3\ M_\odot$ at a separation $\sim 500\ R_\odot$ evolve into semidetached LMXBs consisting of an ONe dwarf and a low-mass helium degenerate dwarf donor. Helium donors are more successful in inducing the collapse of a white dwarf than are hydrogen-rich donors, but, because $\dot{M}$ is usually larger than the Eddington limiting value for a neutron star, they are less successful in producing LMXBs. The number of LMXBs with a neutron star component in the Galactic disk is predicted to be 540 (Algol-like) and 190 (CV-like) when $\alpha_{\mathrm{CE}} = 1$, and 46 (Algol-like) and 150 (CV-like) when $\alpha_{\mathrm{CE}} = 0.5$. The number of bright ($4 \times 10^4\ L_\odot$) LMXBs with helium degenerate donors cannot exceed ~ 100. No LMXBs with black hole components are formed in the framework of the scenarios discussed in this section.

6. POSSIBLE REASONS WHY THEORETICAL LMXBs OUTNUMBER OBSERVED LMXBs

6.1. *Uncertainties in Theoretical Estimates*

The total number of LMXBs which we have estimated to exist in the Galactic disk (in the framework of a model which is complete from the point of view of the evolution of close binaries and in which mass exchange is assumed to be conservative) is many times larger than the observed number of ~ 100 (Table 1). There are at least four uncertainties in the theory which may be responsible for this discrepancy.

An easy target for blame is the choice of α_{CE} as large as 0.5–1. Although several of the first numerical simulations of CE evolution (Taam, Bodenheimer, & Ostriker 1978; Tutukov & Yungelson 1979b) assumed $\alpha_{\mathrm{CE}} = 1$, more recent simulations (e.g., Meyer & Meyer-Hofmeister 1979; Livio & Soker 1988; Soker & Livio 1989; Taam & Bodenheimer 1989; Terman, Taam, & Hernquist 1994; Taam, Bodenheimer, & Różyczka 1994) suggest values of α_{CE} substantially less than unity, but these simulations are generally examinations of the behavior of a small body in a structure which corresponds to the envelope of a giant or supergiant. The relevance of the numerical results to the real CE situation is therefore not as yet completely clear (see Iben & Livio 1993 for a discussion).

The strongest argument against opting for an α_{CE} even as small as 0.5 is the fact that estimates of birthrates for many other systems, such as Wolf-Rayet stars (Yungelson & Tutukov 1991), degenerate dwarfs (Tutukov & Yungelson 1992), neutron stars and black holes (Tutukov & Yungelson 1993b), supernovae (Tutukov, Yungelson, & Iben 1992), binary planetary nebula nuclei (Yungelson & Tutukov 1993), and HMXBs (Paper I), are in reasonable agreement with the observations when $\alpha_{\mathrm{CE}} \sim 1$. Semiempirical estimates of α_{CE} for binary nuclei of planetary nebulae (Iben & Tutukov 1989b) favor $\alpha_{CE} \sim 1$.

The theoretical estimates of birthrates also rely heavily on a birth function (eq. [4]). While this birth function is based on the observed properties of thousands of binaries (Popova, Tutukov, & Yungelson 1982), there are still gaps in the observational base. For example, all methods for estimating the birthrate of binaries as a function of the initial mass ratio q_0 break down for $q_0 \lesssim 0.1$. So one way of forcing the theoretical estimate of the number of LMXBs to match the observed number is to assume that the birthrate of systems with $q_0 \leq 0.1$ is ~ 10 times less than given by the adopted birth function. However, using the adopted birth function, an estimate of the birthrate of CVs, which are also systems in which q_0 is small, agrees well with semiempirical estimates of CV birthrate. This suggests that, despite the uncertainties, the adopted birth function is not too far from the truth for $q_0 \leq 0.1$.

There are also substantial uncertainties in theoretical estimates of the available phase space in A_0 and M_{10} for each scenario.

A third major uncertainty is whether or not the low-mass companion survives the supernova explosion which produces the neutron star. Interaction with the supernova ejecta could destroy the secondary or unbind the binary, thus preventing the eventual formation of either an LMXB or a TŻO. Sofia (1967) assumes that all low-mass close main-sequence companions will acquire enough energy to be destroyed. Colgate (1970) argues that the secondary, if it is at a distance $A \leq 10^{12}$ cm from the exploding component, will acquire enough momentum from the supernova ejecta that the system will become unbound. Both studies assume that all of the flux from the exploding star that flows into the cone defined by the projected area of the secondary divided by the square of the distance to the center of the primary is intercepted and absorbed by the secondary. However, the outer layers of a main-sequence star are relatively weakly bound to the interior and can be stripped from the star, carrying off much of the intercepted momentum and energy, leaving an intact secondary remnant which is still bound to the neutron star remnant of the supernova explosion. In a two-dimensional hydrodynamical study, Fryxell & Arnett (1981) find that the stripped main-sequence secondary acquires less than $\sim$0.35–0.8 of the intercepted momentum from the expanding supernova shell, and this is not enough to destroy the binary. Taam & Fryxell (1984) pursue the stripping process further, finding that the variations in orbital parameters are small if the mass of the neutron star formed in the supernova explosion is larger than $1.2\ M_\odot$.

Until now, all modeling of the stripping process has been done for systems in which the secondary is a main-sequence star. However, in precursor systems of relevance for the formation of LMXBs, the typical mass ratio is $q_0 \lesssim 0.1$, so, when the primary explodes as a supernova, the secondary typically has not had time to reach the main sequence. Consequently, the supernova ejecta interacts with an envelope which is even more weakly bound than the envelope of a main-sequence star, and the efficiency of stripping by the intercepted supernova ejecta will be greater than in the case of a main-sequence companion. Thus, further numerical modeling should focus on situations in which the low-mass companion is a pre–main-sequence star. In the absence of appropriate numerical models, it is not possible to make reliable theoretical estimates of the survivability of all relevant precursor binaries which satisfy the formal survival criterion ($M_{\rm lost} < M_{\rm rem}$ when $e = 0$). Given the existence of LMXBs, some systems obviously do survive, especially those which are relatively wide. Further, the existence of binaries such as PSR 1957+20, which is an eclipsing system consisting of a radio pulsar and a very close ($P_{\rm orb} = 0.38$ days), very low mass ($\sim 0.02\ M_\odot$) companion which is being evaporated by the pulsar (Kluzniak et al. 1988; Phinney et al. 1988; Bisnovatyi-Kogan 1989), opens up the possibility that at least some, if not most, very close precursor systems which satisfy the formal survival criterion remain binaries after the supernova explosion.

Of course, there is still the possibility of formally introducing a velocity kick (Shklovsky 1970), but this could destroy the majority of close binaries (e.g., Flannery & van den Heuvel 1975), leaving almost no progenitors of LMXBs. To disrupt the closest CV-like pre-LMXBs, the kick has to be of the order of ~ 300 km s^{-1}, which is admittedly close to a recent estimate of the mean space velocity of radio pulsars (Lyne & Lorimer 1994). However, such a kick would destroy the agreement between the model formation rates of LMXBs and the observed formation rate of MSPs. Furthermore, radio pulsar space velocities can be understood completely without introducing a kick (Iben & Tutukov 1995b). Therefore, in the absence of a clear physical mechanism for producing a kick, we simply assume that it does not occur.

Even in several of the formally successful scenarios, there are circumstances which can prevent the detached binary which survives the supernova explosion from evolving into an LMXB. The young and rapidly rotating neutron star in the detached system has a rotational energy of $\sim 2 \times 10^{52}[P_{\rm rot}({\rm ms})]^{-2}$ erg. Up to 3% of the rotational energy lost may be intercepted by the secondary, which may, in consequence, be significantly reduced in mass if not evaporated completely. It is possible that PSR 1718−19 (Lyne et al. 1993), with a "windy" secondary, is an example of the WD → NS scenario, with the companion of the pulsar being a star of mass $\sim 0.15\ M_\odot$ (see § 5). The final result of such evolution will be the formation of a single radio pulsar, possibly surrounded by a planetary system which forms in the disrupted and vaporized low-mass companion.

In summary, the theoretically estimated number of LMXBs can formally be made to match the observed number by (1) reducing α_{CE} to a value smaller than ~ 0.5, (2) decreasing by a factor of ~ 10 the contribution to the theoretical birth function of systems with $q_0 \lesssim 0.1$, leading to a corresponding reduction in the estimated number of LMXBs, or (3) supposing that the majority of potential precursor systems (those which satisfy the formal requirement for maintaining binarity) are disrupted in the explosion which produces the relativistic component.

6.2. *A Donor Wind Induced by Absorption of X-Rays from the Accretor*

A fourth significant uncertainty in the theory involves the effect on the donor of radiation from the accreting component; it is possible that the effect is such as to vitiate the mass-conservative approximation, a possibility first explored quantitatively by Tavani (1991a). It has been known for some time that most of the optical light from LMXBs is actually reprocessed X-ray radiation (e.g., van Paradijs 1981), and the correlation apparent in Figure 3 between the dereddened visual magnitude m_V and the X-ray flux F_X for the sources in the van Paradijs (1995) catalog makes this abundantly clear.

The relationship between F_X (in mJy) and m_V is

$$m_V = 15.6 \text{ mag} + \text{B.C.} - 2.5 \log(\alpha F_X) , \quad (18)$$

where α is the fraction of X-ray flux intercepted and reradiated at optical wavelengths by the donor and by the accretion disk, and B.C. is the bolometric correction for the optical light. The upper straight line in Figure 3 follows from the choices α = 0.03 and B.C. = 0.0, the lower one from α = 0.03 and B.C. = 3 mag.

How much of the optical light comes from the disk and how much comes from the donor is not an easy question to answer. It is commonly assumed that most of the optical light comes from the disk (e.g., van Paradijs & McClintock 1994). However, if a disk completely shadows a donor which fills its Roche lobe, then, in the approximation that X-rays are emitted spherically symmetrically, the disk absorbs ~30% of the X-rays emitted by the accretor, making it a challenge to explain the observed paucity of systems in which X-ray eclipses occur (Milgrom 1978; see also van Paradijs 1981, 1983; Mason 1989).

Assuming that an irradiated disk provides all of the optical light, and that B.C. $\propto T_e^{-2}$, van Paradijs & McClintock (1994) demonstrate the existence of a correlation between the absolute visual magnitude M_V of a source and the quantity

$$\Sigma = (L_X/L_{\rm Edd})^{1/2}(P_{\rm orb}/1 \text{ hr})^{2/3} \quad (19)$$

which holds over the magnitude range -4.5 mag $< M_V <$ 5.6 mag.

If, instead, it is supposed that the donor is the dominant optical source and that B.C. = $20 - 5 \log T_e$, a straightforward analysis (setting $R_L/A = 0.28$, $L_{\rm Edd} = 10^{4.5}\ L_\odot$, $\sigma T_e^4 = L_{\rm opt}/4\pi R_L^2$, and $A/R_\odot = 0.56 P_{\rm orb}^{2/3}(\text{hr})\, M_{\rm tot}^{1/3}$) gives

$$M_V = 1.81 \text{ mag} - 2.5 \log \Sigma , \quad (20)$$

which fits the observations quite well (Fig. 2 in van Paradijs & McClintock 1994) with regard to both slope and normalization. Thus, the result of assuming that the donor is responsible for most of the optical light from an LMXB fits the observations as well as the result of assuming that most of it comes from a disk. Among real LMXBs, there are undoubtedly large variations in the fractional contributions of disk and donor to reradiated light.

A careful comparison between model predictions and the data in Figure 3 would require an estimate of the bolometric correction as a function of F_X. The observed dispersion in the m_V-F_X relationship can be attributed to variations from one system to another in the bolometric correction (which depends on the intrinsic X-ray luminosity and the geometry of the system), the inclination of the orbital plane, and the efficiency with which X-ray emission is intercepted. All other things being equal, one might expect the bolometric correction for systems with $T_e > 6000$ K to be larger for larger F_X.

Further evidence that radiation from the accretor significantly impacts the donor in LMXBs comes from a comparison with CVs. Components of CVs with orbital periods larger than 3 hr are driven together by a MSW for about $\sim 3 \times 10^7$ yr, and those with orbital periods smaller than ~2 hr are driven together by GWR for about 2×10^9 years. Mass transfer is inefficient in the "period gap" where the donor, relaxing to a smaller radius, remains temporarily out of contact with its Roche lobe. If the radiation from the accretor had no effect on the donor, one might expect a similar behavior for LMXBs. Since both short-period ($P_{\rm orb} \lesssim 3$ hr) and long-period (3 hr $\lesssim P_{\rm orb} \lesssim 10$ hr) LMXBs exist and can, in principle, be seen throughout the entire volume of the Galaxy, the ratio of their observed numbers should be of the order of the ratio of their lifetimes. Based on the ratio of numbers of CVs in the two period domains, one might expect the ratio of short-period to long-period LMXBs to be ~70–100. The observed ratio is only about 0.3 (e.g., Frank, King, & Lasota 1992; van Paradijs 1995). The conclusion that the donor in a short-period LMXB has been drastically influenced by radiation from the accretor is very difficult to circumvent.

Indeed, explanations for the discrepancy rely on such an influence. Van den Heuvel & van Paradijs (1988) suggest that neutron stars in LMXBs become MSPs during the long-period phase of evolution, and that, during the start of a following short-period phase of evolution, radiation and a relativistic wind from the MSP completely evaporate the companion star. Another possibility has been suggested by Tutukov et al. (1987), who assume that accretion can accelerate the spin of the neutron star to a rate sufficient for a transition into the "propeller" regime. In this regime, the rapidly rotating and magnetized neutron star expels by magnetic pressure all matter proffered by the donor (see Illarionov & Sunyaev 1975; Lipunov 1987). Both mechanisms, which rely on the rotational kinetic energy of the neutron star to supply the energy required for evaporation of the secondary, can in principle prevent most long-period LMXBs from reappearing below the period gap as short-period LMXBs. Many other discussions of the effect on the low-mass companion of radiation from the accretor have been presented (e.g., Ruderman et al. 1989a, b).

Another effect which may play as important a role in the evolution of LMXBs as does an irradiation-induced wind is the alteration in the surface boundary condition for the donor that is brought about by the absorption of X-rays. Tout et al. (1989) have explored the characteristics of a star immersed in an isotropic bath of X-ray radiation (assumed in their case to be provided by a quasar or an active galactic nucleus). They suppose that an appropriate outer surface boundary condition for the star can be obtained in the simplest Eddington approximation, with the inward-directed flux at optical depth zero set equal to the flux in the isotropic bath. When the radiation bath is chosen hot enough ($T_e \sim 10^4$ K), resulting low-mass ($\lesssim 1\ M_\odot$) models of homogeneous composition are significantly larger than isolated main-sequence models of the same mass. Podsiadlowski (1991) and Harpaz & Rappaport (1991) adopt a somewhat different approach (in the context of LMXBs) but achieve equivalent results. They deposit the X-ray energy absorbed by the model donor in a spherical shell below the surface of the donor and find that the star is converted on a thermal timescale into a new state of thermal equilibrium. When the luminosity of reprocessed energy flowing outward from the point of deposition is larger than a critical value near ~ 1–$2\ L_\odot$, the initially convective envelope is converted into a radiative envelope and the star becomes quite large relative to the radius of an isolated main-sequence star of the same mass. If, prior to irradiation, a star fills its Roche lobe, the expanding donor transfers matter at a super-Eddington rate until it reaches its new equilibrium size; the evolution of the donor and of the binary system is followed on the assumption that the accretor liberates X-rays at the Eddington luminosity.

Hameury, King, & Lasota (1993) find similar results when they adopt spherical symmetry, but they argue that the rate of irradiation-induced mass loss is dramatically reduced when it is taken into account that the donor is probably constrained by tidal forces to maintain the same face toward the accretor. Hameury, King, & Lasota (1986) emphasize that an accretion disk will shadow the donor from X-rays (see also Milgrom 1978; van Paradijs 1983; Mason 1989). Supposing that the size of the disk and the degree of shadowing of the donor increase with the X-ray luminosity of the accretor, Harpaz & Rappaport (1994) capitalize on this effect to produce a model of episodic mass transfer in which short phases of super-Eddington mass-transfer rates alternate with long phases of mass transfer at much lower rates. This mechanism could, in principle, help solve the problem of the overproduction of at least some CV-like LMXBs. In this case, most of the donor's matter (say, $\sim 95\%$) could be lost from the system during phases of super-Eddington mass-transfer rates, and only a few percent of the donor's mass would be accreted during the sub-Eddington phases. Presumably, the sub-Eddington phases would correspond to observed CV-like LMXBs. Frank et al. (1992) have argued for a similar kind of episodic mass transfer as a way to solve the LMXB/CV number-period distribution problem.

These studies of the possible effect of absorbed X-rays on the structure of low-mass stars are very imaginative, and further progress may be expected when the relevant radiative transfer problem, which deals with how a flux of hard radiation is converted into an outward flow of optical radiation, is solved exactly. Additional schemes for influencing the donor and the mass-transfer rate include absorption of γ-ray emission from the accretor (Kluzniak et al. 1988), absorption of electron-positron pairs (Krolik & Sincell 1990), and ablation of the donor by the powerful radio emission of the accretor (Ruderman et al. 1989a; Levinson & Eichler 1991). In the following, only the effect of an irradiation-induced wind is considered, but it is recognized that other effects, in particular that of the altered boundary condition, could be just as important.

The quantitative study of the wind induced by the radiation intercepted by the donor in an LMXB has a long history which begins with the work of Arons (1973) and of Basko & Sunyaev (1973), and has continued up to the present (e.g., Davidsen & Ostriker 1974; McCray & Hatchett 1975; Basko et al. 1977; London et al. 1981; London & Flannery 1982; Willingale, King, & Pounds 1985; Phinney et al. 1988; Ruderman et al. 1989a, b; Coté & Pylyser 1989; Tavani, Ruderman, & Shaham 1989; Shaham & Tavani 1991; Tavani 1991a,b; Ergma & Fedorova 1992; Tavani & London 1993; D'Antona 1994).

The maximum intensity of the induced wind can be easily estimated. Every gram of donor matter that is accreted by the neutron star (or black hole) releases $\sim 10^{20}$ ergs in X-ray radiation. Since it fills (or nearly fills) its Roche lobe, the donor can intercept up to 3% of this energy, say about 3×10^{18} ergs for every gram of matter it has transferred to the accretor. The gravitational potential at the surface of a main-sequence star depends very weakly on its mass and is typically only about $\sim 10^{15}$ ergs. So, in principle, if radiative losses are ignored, up to 3000 g of donor mass could be lost by the donor for every gram it has transferred to the accretor. In reality, most of the intercepted energy will be lost by radiative cooling.

It is possible to estimate the rate of mass loss by means of a very simple analytical coronal model developed by Musilev & Tutukov (1973) and by Tutukov & Fedorova (1988). Not only does the model provide insight into the basic physics involved, but it gives results which agree within a factor of 2 with results found by a much more sophisticated treatment (Tavani & London 1993) and can be extended over a wider parameter space than has been explored up to now in detailed numerical models.

Pretending that the X-ray flux from the accretor is independent of angle (and ignoring the shadowing effects of a disk), the rate $L_{\rm int}$ at which the donor intercepts energy from the accretor is

$$L_{\rm int} \sim 0.1\, c^2 \dot{M}_{\rm ex} \frac{\pi R_d^2}{4\pi A^2} \sim 3.75 \times 10^{11}\, L_\odot \left(\frac{R_d}{A}\right)^2 \dot{M}_{\rm ex}\,, \quad (21)$$

where $\dot{M}_{\rm ex}$ is the mass-exchange rate in units of $M_\odot$ yr^{-1} and R_d and A are, respectively, the radius of the donor and the semimajor axis of the binary. It has been assumed that every gram of matter exchanged is converted into 0.1 g c^2 of X-ray energy.

Assuming that the volume of the hot corona is equal to the volume of the donor, the rate of radiative cooling can be approximated by

$$L_{\rm cool} \sim 10^{-22}\ {\rm erg\ cm^3\ s^{-1}} \times n_e^2 \frac{4\pi R_d^3}{3}\,, \quad (22)$$

where n_e is the electron density in the corona in units of cm^{-3} and the dimensioned numerical coefficient is equal to the cooling function for a plasma with solar element abundances at a temperature of 10^6 K (it is also within a factor of $10^{\pm 1}$ of the cooling function for temperatures in the range 10^4–10^8 K; Sutherland & Dopita 1993). Setting $L_{\rm int} \sim L_{\rm cool}$ establishes a relationship between n_e and $\dot{M}_{\rm ex}$.

The induced wind mass-loss rate of the secondary is

$$\dot{M}_{\rm iw} \sim 4\pi R_d^2 n_e \mu_e M_{\rm H} v_{\rm iw}\,, \quad (23)$$

where μ_e is the electron molecular weight, $M_{\rm H}$ is the mass of a nucleon, and $v_{\rm iw}$ is the speed of wind matter. If it is assumed that matter evaporates from the corona with a speed which is roughly equal to the escape velocity, then

$$v_{\rm iw} \sim \alpha_{\rm iw} \left(\frac{2GM_d}{R_d}\right)^{1/2}, \quad (24)$$

where $\alpha_{\rm iw}$ is a parameter of order unity. Setting $L_{\rm int} = L_{\rm cool}$ (eqs. [21] and [22]) and using equations (23) and (24), it follows that

$$\dot{M}_{\rm iw} = 18\alpha_{\rm iw}\frac{3R_d}{A}\left(\frac{M_d}{0.5\,M_\odot}\right)\left(\frac{10^{-9}\,M_\odot\,{\rm yr}^{-1}}{\dot{M}_{\rm ex}}\right)^{1/2}\dot{M}_{\rm ex}\,, \quad (25)$$

showing that the induced wind mass-loss rate is typically an order of magnitude larger than the mass-exchange rate.

For main-sequence donors, equation (25) gives a wind mass-loss rate that agrees, within a factor of 2 when $\alpha_{\rm iw} = 1$, with the detailed numerical results of Tavani & London (1993). Approximating $3R_d/A$ by 1, and choosing $M_d \sim 0.5$ $M_\odot$ and $\dot{M}_{\rm ex} = 10^{-10}$ to 10^{-8} $M_\odot$ yr^{-1}, it follows that $\dot{M}_{\rm iw}/\dot{M}_{\rm ex} \sim$ 6–60. So only 0.017–0.17 of the matter lost by a semidetached donor may be accreted by the neutron star. A low ($\sim$10%) efficiency of accretion is expected also on semiempirical grounds (Iben & Tutukov 1994b). Thus, nonconservative mass transfer during the X-ray phase would appear to be an inherent property not only of HMXBs but of LMXBs as well. An accretor in an HMXB accumulates about 0.1% of the mass lost by the donor, but an accretor of an LMXB accumulates about 10% of the mass lost by the donor.

The induced stellar wind does not alter the theoretical birthrates estimated by the scenario code, but it dramatically affects the duration of the X-ray phase and therefore the predicted number of LMXBs. At this juncture, it seems reasonable to estimate the effect of taking wind mass loss from the donor into account by assuming that the mass-transfer rate is similar to that given in the mass-conservative approximation. Since $\dot{M}_{\rm ex}$ is small compared with $\dot{M}_{\rm iw}$, the new duration $\tau_{\rm new}$ and the new number $N_{\rm new}$ are approximately $\dot{M}_{\rm old}/\dot{M}_{\rm iw}$ times their values $\tau_{\rm old}$ and $N_{\rm old}$ as calculated in the mass-conservative approximation. Setting

$$\tau_{\rm new}\dot{M}_{\rm iw} \sim \tau_{\rm old}\dot{M}_{\rm old} \sim 1\,M_\odot\,, \quad (26)$$

and combining equations (25) and (26) for $3R_d/A(M_d/0.5$ $M_\odot) = 1$, it follows that

$$N_{\rm new} = \nu\tau_{\rm new} \sim \frac{1750}{\alpha_{\rm iw}}\left(\frac{\nu N_{\rm old}}{\beta}\right)^{1/2}, \quad (27)$$

where

$$\dot{M}_{\rm ex} = \beta\dot{M}_{\rm old}\,, \quad (28)$$

and $\dot{M}_{\rm old}$ is the mass-transfer rate given in the mass-conservative approximation. Resulting numbers when $\alpha_{\rm iw} = 1$ and $\beta = 1$ are given in Table 1 in the columns labeled N.

The formal application of equation (28) to LMXBs with helium degenerate donors gives new numbers $N = 1370$ ($\alpha_{\rm CE} = 1$) and $N = 325$ ($\alpha_{\rm CE} = 0.5$). But, as discussed in § 4.6, because of a low mass-exchange rate, most of these X-ray sources will be very weak in any case. The number of bright LMXBs of this sort will not exceed a few tens (see § 4.6).

The predicted number of Algol-like LMXBs is $N = 494$ when $\alpha_{\rm CE} = 1$ and $N = 113$ when $\alpha_{\rm CE} = 0.5$, both estimates being about 10 times smaller than given by the conservative approximation. The number of long-period CV-like LMXBs is $N = 118$ when $\alpha_{\rm CE} = 1$ and $N = 21$ when $\alpha_{\rm CE} = 0.5$, again an order of magnitude smaller than in the conservative approximation. The number of short-period CV-like LMXBs is $N = 368$ when $\alpha_{\rm CE} = 1$ and $N = 101$ when $\alpha_{\rm CE} = 0.5$, both estimates being about 36 times smaller than before. Thus, the induced stellar wind significantly reduces the theoretical estimate of the total number of LMXBs, bringing it much closer to the observed number. Furthermore, a semiempirical estimate of LMXB birthrate which uses the same reduced estimate of LMXB lifetime gives a birthrate much closer to the theoretically estimated one.

It is possible that the optical component of the system containing the pulsar PSR 1718−19, having a mass $\sim$0.2 $M_\odot$ (Burderli & King 1994) is another example of a low-mass main-sequence star being evaporated. It is thought that, at the moment of the AIC of the precursor of the neutron star, the mass of the companion of the accreting white dwarf is $\sim$0.7 $M_\odot$ (Wijers & Paczyński 1993). The characteristic age ($\sim 10^7$ yr) and strong magnetic field ($\sim 10^{12}$ G) of the radio pulsar (Taylor, Manchester, & Lyne 1993) probably exclude the possibility of the system being the result of recent formation by a three-body process.

One might worry that, if only $\sim$10% of the matter from the donor reaches the neutron star, the accreted angular momentum is insufficient to spin the neutron star up to MSP spin frequencies. If matter is accreted at Keplerian velocities at the equator, the total angular momentum added is $M_{\rm acc}Rv_{\rm K}$, where $M_{\rm acc}$ is the total amount of mass accreted and R and $v_{\rm K}$ are, respectively, the radius of the neutron star and the Keplerian velocity at its surface. Assuming that the neutron star is "old" when it enters the LMXB phase, one can equate the added angular momentum to $I(2\pi/P_{\rm spin})$, where I is the moment of inertia of the neutron star and $P_{\rm spin}$ is its spin period. Inserting typical values for I, R, and $v_{\rm K}$ gives $M_{\rm acc} \sim (0.1\ {\rm ms}/P_{\rm spin})\,M_\odot$, where the spin period is in milliseconds. With a typical spin period of 3 ms for an MSP, $M_{\rm acc} \sim 0.03\,M_\odot$ (see also the Appendix), which is about 10 times smaller than what is usually assumed in making semiempirical estimates of LMXB lifetime.

It is now possible to make a semiempirical estimate of the birthrate of LMXBs. Figure 4 suggests an observed mass-exchange rate of $\dot{M}_{\rm ex} \sim 10^{-9}$ to 10^{-8} $M_\odot$ yr^{-1}. The assumption that all MSPs have evolved from LMXBs gives an estimated LMXB lifetime of $\tau_{\rm LMXB} \sim 0.03\,M_\odot/\dot{M}_{\rm ex} \sim 3\times10^6$–$3\times10^7$ yr. Similar lifetime estimates come by dividing a canonical lifetime in the conservative approximation by a factor in the range 6–60. Finally, $\nu_{\rm LMXB} = N_{\rm obs}/\tau_{\rm LMXB} \sim 100/\tau_{\rm LMXB} = 3\times10^{-6}$–$3\times10^{-5}$ yr^{-1}.

6.3. *Accretion from the Induced Wind: The Bootstrap Mode*

Given the rather high induced wind mass-loss rates estimated in § 6.2 for the secondary in an LMXB, it is of interest to examine the rate at which the neutron star or black hole accretes matter from the induced wind. It is immediately clear that, when the secondary fills or nearly fills its Roche lobe, the relativistic component will accrete a few percent of the wind from the donor and that, therefore, a substantial fraction of the mass exchanged, if not most of it, could be contributed by the wind. Since the rate at which the relativistic component accretes matter from the induced wind rivals the mass-exchange rate estimated on the basis of Roche lobe filling, there is a potential for the development of a "bootstrapping" situation in which accretion from the induced wind emitted by a secondary that does not fill its Roche lobe is at a rate sufficient for producing the radiation necessary to maintain the induced wind. The feedback loop is stable and is precisely the mechanism suggested by Arons (1973) and Basko & Sunyaev (1973) to explain the properties of Her X-1. The bootstrap mode of mass transfer has been considered by many authors (e.g., Alme & Wilson 1974; Pringle 1974; Davidsen & Ostriker 1974; Milgrom & Salpeter 1975; McCray & Hatchett 1975; Basko et al. 1977; London & Flannery 1982; Willingale et al. 1985; Molnar 1988; Tavani 1991a; Tavani et al. 1989; Shaham & Tavani 1991).

Adopting the Bondi & Hoyle (1944) formalism used in Paper I, it follows that the rate $\dot{M}_{\rm acc}$ at which the relativistic component accretes from the induced wind (in an approximation that neglects the effect of orbital motion) is

$$\dot{M}_{\rm acc} \sim \frac{1}{4\alpha_{\rm iw}^4}\left(\frac{M_X}{M_d}\frac{R_d}{A}\right)^2 \dot{M}_{\rm iw}\,, \qquad (29)$$

where M_X is the mass of the accretor. Assuming that accretion from the wind is the only mode of mass transfer, the amount of matter $M_{\rm acc}$ accreted by the relativistic component from the wind of the donor is related to the amount of matter $M_{\rm lost}$ lost by the donor by

$$M_{\rm acc} \sim \frac{1}{4\alpha_{\rm iw}^4}\left(\frac{M_X}{M_d}\frac{R_d}{A}\right)^2 M_{\rm lost}\,, \qquad (30)$$

where $M_X = 1.4\ M_\odot$ if the accretor is a neutron star. In this same approximation, one can set $\dot{M}_{\rm ex} = \dot{M}_{\rm acc}$ in equations (25) and (29) to get

$$\dot{M}_{\rm acc}(M_\odot\ {\rm yr}^{-1}) \sim \frac{7.3\times 10^{-7}}{\alpha_{\rm iw}^6}\left(\frac{M_X}{M_\odot}\right)^2\left(\frac{M_X}{M_d}\right)^2\left(\frac{R_d}{A}\right)^6, \quad (31)$$

where masses are now in solar units. Thus, a bootstrapping situation exists. Because the mechanism is stable, only a modest "seed" wind, such as the ordinary wind that every single star emits (e.g., the MSW emitted by a main-sequence star less massive than 1 $M_\odot$ or the gentle wind of a cool low-mass subgiant or giant), is necessary to get it started.

Setting $M_X \sim 1.4\ M_\odot$, $M_d \sim 0.5\ M_\odot$, and $\dot{M}_{\rm acc} \sim 10^{-9}\ M_\odot$ yr^{-1} in equation (29) gives $R_d/A \sim 0.21$. The Roche lobe radius R_L of the donor and the semimajor axis A are related approximately by (Iben & Tutukov 1984a)

$$R_L \sim 0.52\left(\frac{M_d}{M_{\rm tot}}\right)^{0.44} A\,, \qquad (32)$$

where $M_{\rm tot}$ is the total mass of the system. Thus, for the system parameters chosen, $R_d/R_L \sim 0.7$; although the donor does not fill its Roche lobe, it is rather close to filling it.

It has been argued before (e.g., Alme & Wilson 1974; McCray & Hatchett 1975; Hameury et al. 1993) that, in order for a self-induced, irradiation-driven wind to occur in a detached system, the donor must be *very* close to filling its Roche lobe. In the current context, Roche lobe filling and the mass exchange required to produce X-rays is assured by the operation of a MSW and/or GWR (CV-like LMXBs), or by the evolutionary expansion of the donor (Algol-like LMXBs). Thus, the self-induced property of a pure bootstrap mode is not an essential ingredient. The main role of the evaporative wind is to significantly shorten the duration of the X-ray stage compared to usual conservative estimates (by a factor of 6–60; see eq. [25]).

Equation (31) suggests a typical induced wind mass-loss rate of $\sim$(1–2) $\times$ 10^{-7} $M_\odot$ yr^{-1} for bright LMXBs with a mass-exchange rate $\sim 10^{-8}$ $M_\odot$ yr^{-1}, or a typical LMXB lifetime of $\sim$(0.5–1) $\times$ 10^7 yr. Multiplying estimated lifetime by the appropriate theoretical birthrates in Table 1 suggests that the total number of LMXBs in our Galaxy is $\sim$100–200, quite consistent with the observed number of persistent LMXBs, given the uncertainties in both numbers, including the possible existence of MSPs "hidden" by the evaporative wind (Tavani 1991b).

In §§ 4 and 5 mass-conservative evolution has been assumed. Introduction of the rather intense induced stellar wind can influence significantly the evolution of low-mass main-sequence donors in CV-like binaries. This evolution will be driven not only by a MSW but by system mass loss as well. Numerical experiments have shown that the evolution of CV-like binaries is rather sensitive to the braking efficiency of the MSW (e.g., Tutukov et al. 1987). Therefore, for a final conclusion about the role of the irradiation-induced stellar wind, special numerical/evolutionary computations for LMXBs are necessary.

7. ON THE CONNECTION BETWEEN LMXBs AND MSPs

The origin of binary MSPs has usually been associated with accretion during the LMXB stage (e.g., Smarr & Blandford 1976; Alpar et al. 1982; Helfand, Ruderman, & Shaham 1983; Joss & Rappaport 1983; Paczyński 1983; Savonije 1983; Iben & Tutukov 1984c, 1985; Jeffrey 1986; van den Heuvel & van Paradijs 1988; Srinivasan 1989; Lyne 1994), although other possibilities have also been discussed (e.g., van den Heuvel & Bonsema 1984; Ruderman 1991; Chen et al. 1993). Here the accretion model is adopted to study some observed properties of binary radio pulsars.

In Figure 9 are shown the locations of 31 binary radio pulsars; 24 are taken from the Taylor et al. (1993) catalog, and the others are taken from Phinney & Kulkarni (1994), Deich et al. (1993), Foster, Wolszczan, & Camilo (1993), and Camilo, Nice, & Taylor (1993). There are 11 MSPs (defined by having spin periods $P_{\rm rot} \le 10$ ms) in binaries with very low eccentricities ($e \le 0.05$) and with orbital periods in the range $P_{\rm orb}$ = 1–160 days. Their unseen companions are most probably helium degenerate dwarfs. For these 11 systems, $P_{\rm orb}$ and $P_{\rm rot}$ appear to be weakly correlated. Equations (29) and (31), with equation (A4) in the Appendix, may be used to understand the origin of this correlation.

From equations (30) and (32),

$$\frac{M_{\rm acc}}{M_{\rm env}} \sim \frac{\dot{M}_{\rm acc}}{\dot{M}_{\rm iw}} \sim \frac{0.0676}{\alpha_{\rm iw}^4}\frac{M_X^2}{M_{\rm tot}^{0.88}M_d^{1.12}}\left(\frac{R_d}{R_L}\right)^2, \qquad (33)$$

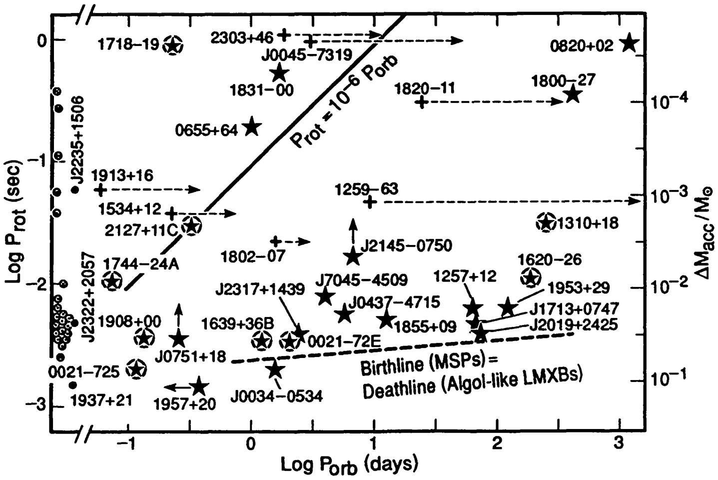

FIG. 9.—Positions of binary radio pulsars in the plane $P_{\rm rot}$ (spin period of the pulsar) vs. $P_{\rm orb}$ (orbital period) and distribution of globular cluster radio pulsars over rotational periods as given by Taylor et al. (1993), Phinney & Kulkarni (1994), Deich et al. (1994), Foster, Wolszczan, & Camilo (1993), Camilo, Nice, & Taylor (1993), and Nice, Taylor, & Fruchter (1993). Uncircled symbols denote Galactic disk radio pulsars, and circled symbols denote those in globular clusters. Five-rayed star symbols mean that the eccentricity of the system is smaller than or equal to $e \sim 0.05$, and plus signs mean that $e > 0.21$. Orbital periods of the latter systems have been reduced to their presupernova values by the transformation $P^0_{\rm orb} = P_{\rm orb}(1-e)^{3/2}/(1+e)^{1/2}$. Dashed arrows point to the actual present orbital period. Spin periods of single radio pulsars given by Taylor et al. (1993) are shown next to the vertical axis, to the left of the break in the horizontal scale. The three small filled circles are single MSPs in the Galactic disk population, and the 21 small circled dots are in globular clusters. The heavy dashed line labeled "Birth line (MSPs) = Death line (Algol-like LMXBs)" is a theoretical estimate of the lower limit on $P_{\rm rot}$ based on a model of LMXBs in which the donor emits a wind induced by the absorption of X-rays from the accreting neutron star component and on a relationship between an equilibrium spin period and the amount of mass accreted by the neutron star. The solid line gives the relationship between the spin period and the orbital period for a radio pulsar which arises as a consequence of the collapse of the core (of white dwarf size and Chandrasekhar-limit mass) of a presupernova which was spinning with a period equal to the orbital period of the binary.

where $M_{\rm env} = M_{\rm lost}$ is the mass of the hydrogen-rich envelope of the donor and masses on the right are in solar units. Taking $\alpha_{\rm iw} \sim 1$, $M_{\rm X} \sim 1.4$, and $M_d \sim 0.8$,

$$\frac{M_{\rm acc}}{M_{\rm env}} \sim 0.085\left(\frac{R_d}{R_L}\right)^2 . \qquad (34)$$

So the neutron star typically accretes less than 10% of the matter lost by the donor.

The radius R_d of the precursor subgiant or giant obeys (Iben & Tutukov 1984a, as modified by Eggen & Iben 1991)

$$R_d \sim 10^{3.38} M_{\rm He}^4 , \qquad (35)$$

where $M_{\rm He}$ is the mass of the helium core in solar units. Setting $R_d = R_L$ in equation (32), Kepler's third law gives, for $M_d = M_{\rm He}$,

$$P_{\rm orb}({\rm days}) \sim 10^{4.56}(M_X + M_{\rm He})^{0.16} M_{\rm He}^{5.34} \qquad (36a)$$

or, taking $M_{\rm He} = 0.3\, M_\odot$,

$$P_{\rm orb}({\rm days}) \sim 10^{4.60} M_{\rm He}^{5.34} . \qquad (36b)$$

For an Algol-like binary with initial donor mass $M_d \sim 0.8\, M_\odot$ and $M_{\rm He} \sim 0.13\, M_\odot$, it follows that $M_{\rm env} \sim 0.67\, M_\odot$, $P_{\rm orb} \sim 0.74$ day, and $M_{\rm acc} \sim 0.088 \times 0.67\, M_\odot \sim 0.06\, M_\odot$. According to equation (A4) in the Appendix, this leads to an equilibrium spin period of $P_{\rm rot} \sim 2.3$ ms. For a binary with initial donor mass $M_d \sim 0.8\, M_\odot$ and $M_{\rm He} \sim 0.46\, M_\odot$, we have $M_{\rm env} \sim 0.34\, M_\odot$, $P_{\rm orb} \sim 630$ days, and $M_{\rm acc} \sim 0.088 \times 0.34\, M_\odot \sim 0.03\, M_\odot$, which, according to equation (A4), gives $P_{\rm rot} \sim 3.9$ ms. A line joining these two estimates is placed in Figure 9 and labeled "Birth line (MSPs) = Death line (Algol-like LMXBs)." The slope of the birth line is consistent with that defined by the 11 binary MSPs of small eccentricity. The birth line is a theoretical lower border for $P_{\rm rot}$, since the braking of rotation with time due to particle emission forces an MSP to move vertically in the diagram. The dispersion of observed MSPs above the birth line is not inconsistent with the ages of the MSPs as gauged from their characteristic times. As they also follow the predicted line, the MSPs in globular clusters which are in binaries with low eccentricity appear to be in the same family as MSPs in the Galactic disk population. To summarize, the observed correlation between the orbital periods and pulsar spin periods in binary MSPs with low eccentricities is predicted to exist because the range in the initial mass of a donor is quite small and because the larger the orbital period, the smaller the amount of mass which can be accreted by the neutron star from the induced wind.

The four binary pulsars in Figure 9 with $P_{\rm rot} > 10$ ms and $P_{\rm orb} > 150$ days and low eccentricity probably constitute another family. For them, spin period is probably much more strongly correlated with the orbital period of the binary than in the case of the Algol-like family. The magnetic field strength of these pulsars grows in step with spin period from $\sim 3 \times 10^9$ G for 1620−26 to $\sim 3 \times 10^{11}$ G for 0820+02 (Taylor et al. 1993). In this interval of orbital periods, the mass-exchange mode possibly changes from one associated with Roche lobe filling to accretion from the self-induced wind of a red (super)giant. This second mode probably does not provide for accretion of enough matter to achieve an equilibrium spin period. Orbits of the four systems were presumably circularized by tidal forces because the (super)giant donor was close to filling its Roche lobe (Zahn 1977; Tassoul 1988). The fact that spin and orbital period are correlated probably reflects a decline with increasing period in the amount of matter accreted during the preceding mass-exchange phase (see the Appendix). The inverse correlation between magnetic field strength and the amount of accreted material is in line with arguments that field strength decays only in consequence of accretion (e.g., Bailes 1989).

Direct support for the inference that a neutron star can accrete from the wind of a giant or supergiant and thereby be spun up comes from the fact that the bright ($\sim 10^4\, L_\odot$) Galactic X-ray source GX 1+4 is a symbiotic star with an M6 III optical component (Davidsen, Malina, & Bowyer 1977). It is well known that the dwarf in a classical symbiotic system accretes enough matter to experience nova outbursts from time to time.

The characteristics of the four longest period radio pulsars suggest that neutron stars may be born with relatively long spin periods and, in very wide binaries, cannot accrete enough matter from their companions to achieve periods short enough (shorter than several seconds) to be transformed into radio pulsars. Arguments for why a neutron star must spin rapidly in order to support an intense radio emission have been given by, e.g., Alpar et al. (1982), Lipunov (1987), and Bhattacharya & van den Heuvel (1991). The inferred slow rotation rate of young neutron stars which are formed after the supernova explosion of single stars or components of wide binary stars implies a slow rotation rate of the precollapse degenerate cores of slowly rotating presupernova supergiants. For example, if the dense nucleus of a supergiant rotates like a solid body with the same period as the rest of the star (at, say, $P_{\rm orb} \sim 1$ yr), when it collapses it will acquire a spin period of about a few hundred seconds, which is too long for the neutron star to be a normal radio pulsar. Only subsequent accretion in a close enough binary, transferring to it sufficient angular momentum, can transform this neutron star into a radio pulsar. The evident absence of radio pulsars in wide binaries with orbital periods exceeding several years is thus perhaps a simple consequence of the inefficiency of the mass and angular momentum exchange between components. Furthermore, the high space velocities of radio pulsars can be naturally explained by their formation in close binaries most of which are disrupted after the second supernova explosion in the system. This restriction of progenitors of radio pulsars to close binaries reduces the expected birthrate of radio pulsars to about one-third of the birthrate of Type II supernovae. Unfortunately, current estimates of supernova rates and of pulsar formation rates are still too uncertain (e.g., van den Bergh, McClure, & Evans 1987; Lorimer et al. 1993) to permit a simple test of this idea.

The fact that the relationship between $M_{\rm acc}$ and $M_{\rm env}$ depends on the fourth power of $\alpha_{\rm iw}$ (eq. [30]), coupled with the fact that the theoretical "birth line" is so close to the observed MSPs when $\alpha_{\rm iw} \sim 1$, suggests that there is probably not as much variation in the induced wind mass-loss rate from one real system to the other as is normally considered in theoretical studies of induced winds (e.g., Tavani & London 1993). This presents a challenge to the theory.

Equations (31) and (A4) can also be used to estimate the initial masses of donors in CV-like systems which produce single MSPs. Setting $M_{\rm env} \sim M_{20}$ (i.e., the entire star), it is evident that, the shorter $P_{\rm rot}$ is now, the larger is the initial mass of the donor in the precursor CV-like LMXB.

On the left-hand side of Figure 9 are four close ($A < 10\ R_\odot$) binary radio pulsars with small orbital eccentricities ($e \lesssim 0.001$) but with spin periods $P_{\rm rot} \gtrsim 0.01$ s. There are two ways of accounting for orbital circularization which has occurred in these systems. Tidal forces acting on a relatively close companion with a deep convective envelope (Zahn 1977; Tassoul 1988) are probably responsible for the almost circular orbit of PSR 1718−19, achieved after the formation of the neutron star. The other three pulsars probably passed through a CE stage after the formation of the neutron star and now have close degenerate dwarf companions. It is important to emphasize that these binaries, which have passed through a CE phase, have very circular orbits ($e \le 0.01$), which is evidence of an efficient dissipation of energy on the CE stage. According to equations (A1) and (A2) in the Appendix, a neutron star with a magnetic field strength of 10^{12} G and an accretion rate of $\sim 3 \times 10^{-8}\ M_\odot\ {\rm yr}^{-1}$ can achieve a spin period of $P_{\rm rot} \sim 0.3$ s after accreting $\sim 10^{-4}\ M_\odot$, and $P_{\rm rot} \sim 0.01$ s after accreting $\sim 0.008\ M_\odot$. Accretion by a neutron star of such quantities of matter during a CE stage, which might last for 10^3–10^5 yr, does not seem unreasonable. According to Taylor et al. (1993), the magnetic field strength of PSR 1744−24A is still unknown, but the small magnetic fields of PSR 0655+64 ($\sim 10^{10}$ G) and of PSR 1831−00 ($\sim 9 \times 10^{10}$ G) are typical for pulsars which have been spun up by accretion (Bisnovatyi-Kogan & Komberg 1975; Shibazaki et al. 1989; Romani 1990; Bhattacharya & van den Heuvel 1991).

The scenario program has also produced distributions over orbital periods of final evolutionary systems consisting of a neutron star and a newly born degenerate dwarf (Tutukov & Yungelson 1993a). These systems are smoothly distributed over the period range 0.0–2 days. Binaries of this sort with orbital periods shorter than ~ 0.1 days are short-lived, since most of them merge (probably into TŻOs) under the braking influence of GWR. Therefore, among long-lived binaries, systems with orbital periods 0.1–2 days predominate. The binary radio pulsars 1744−24A, 0655+64, and 1831−00, and probably J0751+18 and J2145−0750, have periods which place them in this family.

A possible origin of the rotational periods of radio pulsars in close binaries with noncircular orbits was discussed in Paper I, where it was suggested that in precursor binaries which are close enough for tidal forces to act efficiently, the core of the helium star precursor of the neutron star rotates synchronously with a spin period equal to the orbital period. In this picture, the rotation rate of the neutron star in the present system is the consequence of the conservation of spin angular momentum during the collapse of the core of the helium star to neutron star dimensions, with $P_{\rm rot} \rightarrow 10^{-6}\ P_{\rm orb}$. The eccentricity of the current system is a consequence of recoil velocity achieved because of mass loss during the supernova explosion which produces the neutron star. In the simplest approximation (Boersma 1961), the recoil velocity is given by $ev_{\rm orb}$, where $v_{\rm orb}$ is the orbital velocity of the presupernova, and the eccentricity is given by $e = \Delta M / M_R$, where ΔM is the mass lost from the system and M_R is the mass of the remnant system. Thus, eccentricity and peculiar space velocity are expected to be correlated. The orbital periods assigned to the binary radio pulsars with large eccentricity are those of the system prior to the supernova explosion (see the legend to Fig. 8).

Only one high-eccentricity binary pulsar, 2127+11C ($e = 0.68$; Prince et al. 1991), is a member of a globular cluster. This pulsar is far from the center of the parent cluster M15, which is a clear indication of a relatively high space velocity (~ 50 km s^{-1}). This system probably achieved its high velocity in an exchange collision between a single MSP and a binary (Prince et al. 1991), one component of which was a low-mass (a few tenths of 1 $M_\odot$) star. The restriction on the mass of the low-mass component is necessary to prevent the new binary from leaving the globular cluster. Thus, the origin of the orbital eccentricity of this particular binary pulsar is different from that of the other binary radio pulsars of high eccentricity in Figure 8. For this reason, it has been assigned its current orbital period in this figure.

The observed ratio of binary to single MSPs can be used as an additional check on the theory. The theoretical LMXBs have been divided into two families—CV-like (lines 1, 3, and 5 in Table 1) and Algol-like (lines 2 and 4 in Table 1). Members of the first family evolve into single MSPs after the complete evaporation of a main-sequence donor. Members of the second family evolve into MSPs in binaries with low orbital eccentricity and with a degenerate helium dwarf component. The theory produces a value of 2 ($\alpha_{CE} = 0.5$) to 3 ($\alpha_{CE} = 1.0$) for the birthrate of binary MSPs relative to the birthrate of single MSPs (Table 1). If we add single MSPs produced by accretion of the matter of degenerate helium donors (see § 4.6 and Table 1), the ratio becomes close to unity, almost independent of α_{CE}. But, taking into account that numbers presented in Table 1 for LMXBs with degenerate low-mass helium dwarfs are upper limits, the theoretical ratio is in the range 1–3.

If it is assumed that the lifetime of an MSP is independent of its origin, observed numbers can be used to estimate the birthrate. Grouping PSR 1957+20 with the single MSPs (the donor has a mass of $\sim 0.01\ M_\odot$ and is in the process of being evaporated), the observed ratio of low-eccentricity binary MSPs to the number of single MSPs in the Galactic disk is 10/3 (Fig. 9). This value is similar to the ratio of birthrates of CV- and Algol-like LMXBs given by the scenario program. But the small statistics, the uncertainties as to the lifetimes of different MSPs, and the large dispersions in estimated lifetimes (even if the characteristic lifetime is a reasonable measure of lifetime) prevent a final conclusion. Accumulation of more observational data on MSPs and a further development of the scenario program can eventually make the test more definitive.

The theoretical birthrate for systems consisting of neutron stars and degenerate helium donors is large (§ 4.6). Could these binaries produce single MSPs after evaporation of the donor and significantly modify the statistics just presented? To ensure a mass-exchange rate below the Eddington limit, the donor mass must be less than $\sim 0.07\ M_\odot$. If it accretes only 10% of this mass, the neutron star will not be transformed into a "good" MSP (see eq. [A4] in the Appendix). However, the systems in question evolve from precursor LMXBs in which the secondary is a slightly evolved main-sequence star of mass $\sim 0.8\ M_\odot$ (e.g., Tutukov et al. 1985), and so the neutron star may have already achieved a millisecond spin period before the secondary has evolved into a degenerate helium star.

For globular cluster MSPs, the ratio of observed binary to single MSPs is ~ 0.3 (Fig. 9), almost the inverse of that for field MSPs. Among observed LMXBs in globular clusters (van Paradijs 1995), the ratio of systems with orbital period larger than 16 hr to systems with shorter orbital periods is ~ 0.43. If it is assumed that short-period LMXBs evolve into single MSPs and that long-period ones evolve into binary MSPs of low eccentricity, and it is supposed that MSP lifetime does not depend on origin, the observed ratio of numbers may be used as a measure of the ratio of birthrates. The near-coincidence of the number ratio of observed short- and long-period LMXBs with the observed number ratio of binary to single MSPs supports the expectation of an evolutionary connection between LMXBs and MSPs, but it does not clarify why the ratio for globular cluster MSP types is so different from the ratio for MSP types in the disk population.

A solution is indicated by a consideration of the relative probabilities with which precursor systems of the two types are formed in globular clusters by inelastic collisions between single and binary low-mass stars and neutron stars. Binaries formed in such collisions will preferentially have main-sequence donors (e.g., Iben & Tutukov 1984a) and evolve into single MSPs. In general, only about 10% of all two-body captures will produce a binary with a subgiant component having a radius larger than about twice the radius of a main-sequence star near cluster turnoff. Apart from this, two-body collisions can destroy giant envelopes, preventing mass exchange (Rasio & Shapiro 1991). In reality, the situation is more complicated because of the influence of three-body collisions. With numerical models for capture rates in clusters like ω Cen and 47 Tuc, DiStefano & Rappaport (1992) demonstrate that, after the formation of an MSP in wide systems, the MSP can be ejected in a collision. Hence, the observed ratio of numbers of binary to numbers of single MSPs in different clusters can be used as an additional constraint on models and possibly serve as a check on the theoretical input required for estimating two-body and three-body capture rates.

Assuming that a good case has been established for the thesis that, in the Galactic disk, LMXBs are the precursors of MSPs, it is worth obtaining an independent semiempirical estimate of the MSP birthrate and comparing with other estimates in the literature. Most estimates in the literature depend heavily on sophisticated models of selection effects in pulsar searches, including the accessible volume of the Galaxy and the evolution of the luminosity function. A simpler estimate is possible. From the Taylor et al. (1993) catalog, the average dispersion measure for the seven MSPs which are not in globular clusters is $\sim 34\ \mathrm{cm^{-3}}$ pc, compared with the average dispersion measure of $\sim 92\ \mathrm{cm^{-3}}$ pc for the 558 radio pulsars included in the catalog. The typical observed radio pulsar is at a distance $\lesssim 4$ kpc, and the radio pulsar distribution extends out to a distance of ~ 15 kpc from the center in the Galactic anticenter direction. One may infer that there are $N_P \sim (558/f_P)(15\ \mathrm{kpc}/4\ \mathrm{kpc})^2 \sim 8000/f_P$ radio pulsars in the Galaxy, where f_P is an appropriate beaming factor. The typical MSP, according to emission measure, is at a distance $\lesssim 1.5$ kpc, suggesting the presence in the Galactic disk of $N_{\mathrm{MSP}} \sim (7/f_{\mathrm{MSP}})(15\ \mathrm{kpc}/1.5\ \mathrm{kpc})^2 = 700/f_{\mathrm{MSP}}$ MSPs, where f_{MSP} is the appropriate beaming factor for MSPs. The average spin-down time of the MSPs in the Taylor et al. (1993) catalog is $\tau_{\mathrm{sp}} \sim 2 \times 10^9$ yr, and, if τ_{sp} is identified with lifetime, the birthrate can be estimated as $\nu_{\mathrm{MSP}} \sim (4 \times 10^{-7}/f_{\mathrm{MSP}})\ \mathrm{yr^{-1}}$.

If $f_{\mathrm{MSP}} = 1$, as suggested by Narayan (1984), $\nu_{\mathrm{MSP}} \sim 4 \times 10^{-7}\ \mathrm{yr^{-1}}$. It is not excluded that the beaming factor for MSPs is similar to the beaming factor f_P for ordinary radio pulsars. The latter has been estimated as ~ 0.1–0.2 (Gunn & Ostriker 1970; Narayan 1984). Another estimate of f_P follows from the assumption that the birthrate ν_P of radio pulsars and the combined birthrate $\nu_{\mathrm{SN\,Ib,c}}$ of Types Ib and Ic supernovae in our Galaxy are identical. The birthrate of such supernovae is estimated to be $\sim 0.007\ \mathrm{yr^{-1}}$, both from the observations (van den Bergh et al. 1987; Capellaro et al. 1993) and from the theory (Tutukov et al. 1992). Lorimer et al. (1993) estimate the birthrate of radio pulsars to be $\sim 0.006\ \mathrm{yr^{-1}}$, which is about the same as $\nu_{\mathrm{SN\,Ib,c}}$. Choosing an average lifetime of $\tau_P \sim 4 \times 10^6$ yr (Taylor et al. 1993), the birthrate of ordinary radio pulsars is $\nu_{\mathrm{P}} = N_{\mathrm{P}}/\tau_{\mathrm{P}} \sim (2 \times 10^{-3}/f_{\mathrm{P}})\ \mathrm{yr^{-1}}$. So, if $\nu_{\mathrm{P}} \sim \nu_{\mathrm{SN\,Ib,c}}$, $f_P \sim 0.3$.

As a compromise, we set $f_{MSP} \sim f_P \sim 0.2$, which gives $\nu_{MSP} \sim 2 \times 10^{-6}$ yr^{-1} and $N_{MSP} \sim 4000$. The number of known disk MSPs has been increasing rapidly, and it is reasonable to suppose that our estimates based on the Taylor et al. (1993) sample are lower limits. Noting that there are almost twice as many disk MSPs shown in Figure 9 as there are in the Taylor et al. (1993) catalog, and choosing $f_P \sim 0.2$, we settle on $\nu_{MSP} \sim 4 \times 10^{-6}$ yr^{-1} and $N_{MSP} \sim 8000$. The estimate of N_{MSP} lies between the estimate of $N_{MSP} \sim 2500$ by Johnston & Bailes (1991) and the estimate of $N_{MSP} \sim 4 \times 10^5$ by Thorsett et al. (1993). The latter two estimates involve different treatments of selection effects, and the resolution of the factor of 16 difference between them remains for the future. It is clear that a proper accounting of selection effects is absolutely essential for a determination of N_{MSP}.

From a sample of 17 MSPs, Bailes & Lorimer (1995) estimate $N_{MSP} \sim 3 \times 10^4/f_{MSP}$ and suggest $f_{MSP} = 0.3$–1. Setting $\tau \sim 2 \times 10^9$ yr and $f_{MSP} = 0.65$, this implies that $\nu_{MSP} \sim 2 \times 10^{-5}$ yr^{-1}. Thus, semiempirical arguments suggest that the MSP birthrate is somewhere in the range $(4\text{–}20) \times 10^{-6}$ yr^{-1}. This is compatible with both theoretical [$(1\text{–}4) \times 10^{-5}$ yr^{-1} (Table 1)] and semiempirical [$\sim(3\text{–}30) \times 10^{-6}$ yr^{-1} (at end of § 6.2)] estimates of the birthrate of disk LMXBs.

It may be inferred that, within the still rather large and unavoidable uncertainties, the assumption that LMXBs are the main progenitors of MSPs is not inconsistent with the observations (see also Lorimer et al. 1995). However, the requirement that the theoretically predicted number of LMXBs equals the observed number demands that a stellar wind from the donor, induced by the absorption of a few percent of the X-ray luminosity of the neutron star accretor, carries away from the system most of the matter emitted by the donor. Only $\lesssim$10% of the matter emitted by the donor, which is not required to fill its Roche lobe, is accreted by the neutron star component, but this is sufficient to spin it up to the MSP range.

8. CONCLUDING SUMMARY

The observed population of bright Galactic LMXBs consists of two families distinguished with regard to spatial distribution. One family is certainly associated with the dense concentration of stars within ~1.5 kpc of the Galactic center. The other family defines an extended disk of radius ~15 kpc and thickness ~0.7 kpc, and the ratio of optical luminosity to X-ray luminosity for family members is useful in exploring the role of an irradiation-induced wind in the evolution of LMXBs. Although some members of the centrally located "bulge" family may be formed in inelastic two- and three-body collisions, members of the Galactic disk family are probably all formed in the course of the evolution of primordial binaries in which the ratio of primary to secondary mass is $\gtrsim$10.

In this paper a description has been given of a theoretical model of the Galactic disk population of LMXBs which has been produced by a numerical scenario program. The scenario program is based on a birth function and on a treatment of common envelope evolution that have been used in previous papers to obtain reasonable agreement between observed numbers and birthrates and theoretically estimated numbers and birthrates for a wide variety of binary systems in the Galaxy. The program makes use of a large part of the knowledge concerning stellar evolution that has been accumulated up to now, and its success in modeling every situation to which it has heretofore been applied in a way which is consistent with the observations engenders confidence in its overall correctness and predictive power. In the present instance, the properties of the theoretical population of LMXBs agree reasonably well with the observed properties of disk LMXBs *provided that mass transfer is highly nonconservative.* Many of the theoretical results which have been obtained are well known from the observations, but here they have been obtained from a general program which is founded on an observationally based birth function and on the results of the theory of stellar evolution and which produces a complete model of the binary star population of our Galaxy that is independent of previous scenario modeling.

1. The scenario program produces a bimodal distribution of LMXBs and their products over their space velocity. LMXBs in which the neutron star has been formed in the accretion-induced collapse of a white dwarf have a low dispersion in space velocity (~10–16 km s^{-1}). This velocity is made up of an innate velocity (assumed to be 10 km s^{-1} and isotropic) and a recoil velocity achieved because the gravitational mass of a neutron star formed by collapse of a massive degenerate dwarf is about 10% less than the mass of the white dwarf. These LMXBs and their descendant MSPs constitute a very "flat" component of the Galactic disk population. LMXBs in which the neutron star is formed by the collapse of the helium star remnant of a massive precursor (mass ~12 $M_\odot$) achieve high peculiar space velocities (~40–100 km s^{-1}). The major contributor to this velocity is a recoil velocity which is achieved in consequence of the ejection in a supernova explosion of the envelope of the helium star. In the theoretical model (Table 1), the majority of disk LMXBs belong to this second, "fast" population. The distribution of peculiar space velocities is in agreement with the observed peculiar space-velocity distribution of their descendant MSPs, even though an ad hoc "kick" velocity associated with mass loss in an asymmetric supernova explosion is not introduced. The distribution of LMXBs in the z-coordinate (the scale height perpendicular to the Galactic plane is ~700 pc) agrees well with that estimated from the observations (Naylor & Podsiadlowski 1993).

2. To explain the observed number of LMXBs in the framework of the scenario model, a coronal wind from the donor is invoked; it is supposed that the corona is induced by the absorption of X-ray radiation from the relativistic component. The externally induced stellar wind decreases the estimated lifetime of a typical LMXB by a factor of 6–60 relative to the lifetime estimated in the conservative approximation (wherein all matter lost by the donor is accreted by the neutron star or black hole component). The intensity of the stellar wind can be estimated via a simple model which gives results essentially the same as those produced by detailed numerical modeling of an induced wind (Tavani 1991a, b; Tavani & London 1993). As a result of wind losses, only about 10% of the donor mass in a typical LMXB is accreted by the relativistic component, in spite of the fact that the donor may fill or almost fill its Roche lobe. Thus, the evolution of LMXBs is highly nonconservative, as has been known for many years to be the case in HMXBs. Most of the donor mass is dispersed into space, rather than being accreted to produce X-rays.

3. Simple estimates show that the accretion of $\sim$0.01–0.1 $M_\odot$ of donor matter is adequate to accelerate the rotation rate of neutron star accretors into the MSP range. In the framework of the induced-wind scenario, this means that the amount of donor matter which is actually accreted and the amount of accreted matter that is necessary to spin a neutron star up to several milliseconds are comparable and equal to $\sim$0.01–0.1 $M_\odot$, instead of the usually assumed $\sim$0.3–1 $M_\odot$ predicted by mass-conservative algorithms.

Generally speaking, it is not excluded that some observed MSPs have achieved their high rotation rates in a phase hidden from X-ray observers. However, when the irradiation-induced wind picture is adopted, one achieves agreement between (*a*) observed and theoretically estimated numbers of LMXBs, (*b*) estimated birthrates of LMXBs and of MSPs, and (*c*) the amount of matter exchanged and the amount of accreted matter necessary to acquire millisecond periods. These coincidences make it unnecessary to entertain hidden modes of MSP formation.

4. The model estimate of the birthrate of LMXBs [(1–4) $\times$ 10^{-5} yr^{-1}; see Table 1] and semiempirical estimates of birthrates of MSPs [(0.4–2) $\times$ 10^{-5} yr^{-1}] are comparable and self-consistent. Perhaps one of the most important conclusions of this paper is that all estimates of the birthrates of LMXBs and MSPs do not differ from 10^{-5} yr^{-1} by more than a factor of 3. Significant uncertainties still remain (such as, e.g., the possible existence of "hidden" MSPs; Tavani 1991b) and cannot be removed without further study. It is important to explore (*a*) more sophisticated models of the induced wind with parameters appropriate for LMXBs and (*b*) the effect of the induced wind on the donor, both as regards the mass-radius relationship and as regards the mass-loss rate due to a MSW. It is important also to improve upon semiempirical estimates of the birthrates. For theoretical models, it is important to improve upon the current knowledge of the initial distribution of close binaries for mass ratios in the range 0.05–0.2.

5. The model birthrate of TŻOs is $\sim$0.0015 yr^{-1} (Table 1), which significantly exceeds the birthrates of both LMXBs and MSPs. Most TŻOs are supergiants with a neutron star core, and some of them have a black hole core. The total number of these supergiants in the Galaxy can be of the order of several thousand if their lifetime is determined by a Reimers-like stellar wind (see also Biehle 1994). They can be hidden among usual M and infrared supergiants of similar luminosity. Their very large birthrate relative to the MSP birthrate shows that, as a rule, they evolve into typical single pulsars rather than into MSPs.

6. In the framework of a simple model, the observed correlation between the rotational and orbital periods of MSPs in binaries of low eccentricity is the consequence of the small spread in the initial masses of the donor in Algol-like LMXBs (subgiant or giant donor) and of an inverse correlation between the initial mass of the hydrogen-rich envelope of the donor and the orbital period. The observed ratios of binary to single MSPs in the Galactic disk population and in the globular cluster population can be understood in terms of the relative birthrates of, respectively, the Algol-like LMXB precursors of binary MSPs and the CV-like LMXB precursors of single MSPs. The scenario model predicts a ratio of $\sim$3 for MSPs in the Galactic disk population, where LMXBs are the result of the evolution of primordial binaries. This compares with an observed ratio of $\sim$3.

The observed ratio of $\sim$0.3 in globular clusters, where formation is by inelastic collisions rather by the evolution of primordial binaries, is nearly the same as the ratio of LMXBs which have periods larger than 16 hr to LMXBs which have shorter periods, thus establishing the evolutionary connection between LMXBs and MSPs in globular clusters. The large difference between the number ratios for Galactic disk systems and for globular cluster systems is due to the difference in the manner in which LMXBs are formed in the two populations. If formation by three-body collisions is neglected, the ratio in globular clusters can be understood as a consequence of the fact that, in two-body inelastic collisions, Algol-like LMXBs are produced only 10% as frequently as CV-like LMXBs.

7. Some LMXBs may be systems in which the donor is a very low mass (a few times 0.01 $M_\odot$) degenerate helium dwarf filling its Roche lobe in orbit with a neutron star. The neutron stars in all binaries of this sort that appear in the scenario model are a consequence of the accretion-induced collapse of an ONe dwarf. A final conclusion about the viability and productivity of this scenario requires the construction of numerical models of the evolution of helium-accreting, massive white dwarfs.

8. There is compelling evidence that, in most LMXBs, much more mass is lost from the system than is exchanged. This generates new numerical problems involving the evolution of a low-mass main-sequence, subgiant, or degenerate helium donor and the evolution of a low-mass binary under the influence of a MSW, GWR, and an irradiation-induced wind. Solving these problems is necessary to improve scenarios and to obtain more reliable timescales.

9. Crucial to theoretical estimates of the birthrates of LMXBs of all kinds is the behavior of primaries of initial mass in the range 9.4–15.2 which undergo mass loss in common envelope events. Very few relevant studies have been made, and many more should be undertaken.

10. Because of a mass-exchange rate which exceeds the Eddington limiting value at the start of the mass-transfer phase, semidetached systems consisting of a neutron star and a helium star filling its Roche lobe are not expected to be found, consistent with the absence of any known systems with luminosities and orbital periods in the appropriate ranges ($L_X \sim L_{Edd}$ and $P_{orb} \sim$ 0.01–0.02 days). The scenario code gives $\nu \sim 2 \times 10^{-4}$ yr^{-1} as the formation frequency of TŻOs by this route.

11. The lifetime of an LMXB and the lifetime of its detached precursor can be quite different. Systems which become Algol-like LMXBs spend most of their lives "waiting" for the low-mass (0.8–1.0 $M_\odot$) donor to expand to fill its Roche lobe, and this requires a time of the order of the Hubble time. In contrast, since the main-sequence lifetime of the massive progenitor of a neutron star is short ($\leq 2 \times 10^7$ yr), the lifetime of a system which becomes a CV-like LMXB is comparable to the duration of the LMXB phase itself, and this can be quite short. In the conservative approximation, with a MSW and GWR as drivers of mass transfer, the duration of the CV-like LMXB phase is $\sim 3 \times 10^7$–10^9 yr. In the induced-wind approximation this duration is reduced by an order of magnitude. Therefore, according to the model, whereas Algol-like LMXBs are among

the oldest stars in our Galaxy, a significant fraction of systems which will become or already are CV-like LMXBs are relatively young (age $\sim 2 \times 10^7$–10^8 yr). The presence of relatively young LMXBs in the Galactic disk population has also been deduced by Naylor & Podsiadlowski (1993), but from other considerations.

12. According to the numerical model, the accretor in a significant fraction of LMXBs is a black hole (Table 1). Two members of the CV-like family (A0620+00 and XN Mus) and one member of the Algol-like family (V404 Cyg) are known (Fig. 1*b*). A formal comparison between the ratio of observed LMXBs with a black hole accretor and the number of LMXBs with a neutron star accretor is perhaps premature, since the statistics are still very poor and theoretical lifetimes of systems with black hole accretors (in the absence of relevant evolutionary computations) are not yet available. Model estimates of numbers of LMXBs with black holes depend, e.g., on the assumption that the irradiation-induced wind properties do not depend on the nature of the accretor. But, the X-ray spectra of LMXBs with black hole accretors are quite different from those of LMXBs with accreting neutron stars. This difference could lead to large differences in, e.g., the final velocity of the irradiation-induced wind, and therefore to even larger differences in the efficiency of accretion and in the estimate of LMXB lifetimes (see, e.g., eq. [30]).

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APPENDIX
SPIN-UP OF NEUTRON STARS BY ACCRETION

The analysis of the rotational evolution of accreting neutron stars in close binaries can be conducted in the framework of a simple theoretical model involving the interaction between a dipole magnetic field and matter in an accretion disk (Shakura & Sunyaev 1973). This theory makes several predictions which have been helpful in understanding the observed relationship between spin period and magnetic field strength (e.g., Ghosh, Lamb, & Pethick 1977; Ghosh & Lamb 1978, 1979; Elsner, Ghosh, & Lamb 1980; Wang 1979, 1981). For present purposes, several predictions are particularly useful. The developments by Lipunov & Shakura (1976), Tutukov et al. (1987), and Chen & Ruderman (1993) suggest that an accreting neutron star with a magnetic field of strength B_8 (units 10^8 G), radius $\sim 10^6$ cm, and mass about 1 $M_\odot$ will achieve an equilibrium rotational period of

$$P_{\rm eq}({\rm s}) = 2.9 \times 10^{-3} \dot{M}_{-10}^{-3/7} B_8^{6/7} \quad (A1)$$

after accreting at the rate $\dot{M}_{-10}$ (units $10^{-10}\, M_\odot\, {\rm yr}^{-1}$) for a time $T_{\rm acc}$ given by

$$T_{\rm acc}({\rm yr}) = 4.4 \times 10^8 \dot{M}_{-10}^{-3/7} B_8^{-8/7}\,. \quad (A2)$$

The total amount of accreted mass is

$$M_{\rm acc} = 0.044\, M_\odot \times \dot{M}_{-10}^{4/7} B_8^{-8/7}\,. \quad (A3)$$

Eliminating the field strength from equations (A1) and (A3) gives

$$P_{\rm eq}({\rm s}) = 0.00028 M_{\rm acc}^{-3/4}\,, \quad (A4)$$

where $M_{\rm acc}$ is now in solar units. Once the equilibrium period is reached, the situation becomes more complicated, even to the extent that it is not known what fraction of matter that continues to be provided by the donor is accreted by the neutron star.

For stationary HMXBs, typical parameter values are $\dot{M}_{-10} \sim 1$ and $B_8 \sim 10^4$, giving $T_{\rm acc} \sim 10^4$ yr, $M_{\rm acc} \sim 10^{-6}\, M_\odot$, and $P_{\rm eq} \sim$ 8 s. The duration of the X-ray stage typically exceeds $T_{\rm acc}$, so that the equilibrium period is expected to be achieved in many cases, and this expectation is borne out by the observations (Stella, White, & Rosner 1986). For the smaller accretion rates typical of hard massive transients, spin periods can be much larger, due to the expectation that both $P_{\rm eq}$ and $T_{\rm acc}$ increase with decreasing mass-exchange rate. There is a qualitative trend for spin period to increase with orbital period, consistent with the expectation that the mass-exchange rate decreases with increasing period (Stella et al. 1986), with rotation period $P_{\rm rot}$ ranging, for example, from 7 s for SMC X-1 to 830 s for X Per.

For LMXBs, typical parameters are $\dot{M}_{-10} \sim 10$ and $B_8 \sim 3$, giving $T_{\rm acc} \sim 4.6 \times 10^7$ yr, $M_{\rm acc} \sim 0.05\, M_\odot$, and $P_{\rm eq} \sim 2.7$ ms. Given that, in the induced-wind approximation (see § 5), the lifetime of the X-ray phase is several times shorter than $T_{\rm acc}$, one might expect a typical final spin period to be larger (say, in the range 2.7–10 ms). But, if the field does not decay unless there is accretion (e.g., Phinney & Kulkarni 1994), $P_{\rm rot} \sim P_{\rm eq}$. In any case, it seems clear that, during the X-ray stage, a neutron star in an LMXB can become an MSP by accreting an amount of mass which can be an order of magnitude smaller than the total mass of its low-mass companion.

A neutron star with a massive enough companion can swallow this companion to form (in principle) a TŻO which has the appearance of a red supergiant. If it is determined by a wind from the surface, the lifetime of a TŻO with a hydrogen-rich envelope is typically 6×10^6 yr. The accretion rate is of the order of the Eddington limiting value for a neutron star, namely, $\dot{M}_{-10} \sim 300$.

Since the lifetime of a massive secondary is typically shorter than the spin-down time of a typical radio pulsar, it is reasonable to choose $B_8 \sim 10^4$. Then $T_{acc} \sim 10^3$ yr, $M_{acc} \sim 3 \times 10^{-5}\, M_\odot$, and $P_{eq} \sim 0.7$ s.

The lower limit on the mass of the secondary in a system which can become a TŻO due to merging is $\sim 8\, M_\odot$ (see Tutukov & Yungelson 1979b). If, by the time the secondary fills its Roche lobe, the magnetic field strength decreases to $B_8 \sim 100$, giving $T_{acc} \sim 2 \times 10^5$ yr, $M_{acc} \sim 6 \times 10^{-3}\, M_\odot$, and $P_{eq} \sim 13$ ms. The calculated time to achieve the equilibrium spin period is about 10 times shorter than the estimated timescale of the TŻO phase, so it might appear that the possibility that some TŻOs can evolve into MSPs has been demonstrated. However, the timescale for the TŻO phase has been calculated on the assumption that wind mass loss proceeds on a Reimers-like timescale, and this may be inappropriate. The envelope of a TŻO is essentially the same as that of a massive AGB star, and it is known that massive AGB stars support a superwind with $\dot{M}_{SW} \sim 10^{-4}\, M_\odot\ \mathrm{yr}^{-1}$. It is therefore possible that a TŻO consisting initially of a neutron star core and an $8\, M_\odot$ envelope lives for only $\sim 10^5$ yr, and that, therefore, the final spin period is much larger than 10 ms. If the field has a longer decay time (e.g., Kundt 1981; Kulkarni 1986), the final period will also be longer. It is unfortunate that the uncertainty in estimates of both the lifetime of TŻOs and the magnetic field decay timescale make it impossible at this juncture to either support or exclude the possibility that there is an open channel, however small, between TŻOs and MSPs.

Table 1 shows that the birthrate of TŻOs with carbon envelopes which result from the merger of a degenerate CO dwarf with a neutron star is quite large: $\sim 10^{-3}\ \mathrm{yr}^{-1}$, or a few percent of the birthrate of neutron stars. Suppose that, at the moment of merging, $B_8 \sim 3$. The average lifetime of the TŻO as given by the scenario program is $\sim 4 \times 10^4$ yr (see Table 1). Choosing $\dot{M}_{-10} \sim 500$ (see eq. [A3]) gives $T_{acc} \sim 9 \times 10^6$ yr, $M_{acc} \sim 0.44\, M_\odot$, and $P_{eq} \sim 0.5$ ms. Since the assumed lifetime is ~ 200 times shorter than T_{acc}, the actually attainable spin period is ~ 0.1 s. A similar result follows for TŻOs formed by the merger of a neutron star with a helium star or degenerate helium white dwarf. Thus, "recycled" radio pulsars formed from TŻOs with carbon or helium envelopes are expected to have spin periods which are comparable to the spin periods of originally single radio pulsars that occupy the short-period tail of the number-period relationship, probably producing a few percent of them.

A young neutron star of age less than $\sim 2 \times 10^7$ yr may plunge into the envelope of the secondary if the mass of the secondary exceeds $\sim 10\, M_\odot$. The lifetime during the common envelope stage is about the thermal timescale of the secondary, namely, $\tau_{th} \sim 3 \times 10^7 M^2 A^{-1} L^{-1}$, where all parameters are in solar units. Since the accretion rate is bounded by the Eddington limiting accretion rate $\dot{M}_{Edd} \sim 3 \times 10^{-8}\, M_\odot\ \mathrm{yr}^{-1}$, the amount of accreted matter is $M_{acc} \sim \tau_{th} \times \dot{M}_{Edd} \sim M^2 A^{-1} L^{-1}$. The mass-luminosity relationship for massive stars is $L \sim 10^2 M^2$. Therefore, the equilibrium rotational period obtained using equation (A4) is

$$P_{rot} \sim 0.009 A^{3/4}\ \mathrm{s}\,. \qquad \text{(A5)}$$

Even for the largest possible separation, $A \sim 3000$, the spin period of the neutron star achieved in a CE event will be no larger than ~ 3 s. For the closest systems, $A \sim 10$ and $P_{rot} \sim 0.04$ s. These periods overlap with the range in periods displayed by binary radio pulsars with $P_{orb} = 0.1$–10 days, $e \le 0.05$, and $0.01\ \mathrm{s} \le P_{rot} \le 1$ s (e.g., 1744−24A, J2145−0750, and others in Fig. 9). These periods overlap also with the periods of single radio pulsars, and so spin-up in a common envelope followed by disruption of the binary when the secondary experiences a supernova explosion may account for some fraction of ordinary single radio pulsars.

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